Wind Power Fundamentals

Alexander Kalmikov, Ph. D.
Department of Earth, Atmospheric and Planetary Sciences
Massachusetts Institute of Technology,
77 Massachusetts Ave, Cambridge, MA 02139, USA.
Email: kalex@alum.mit.edu

Abstract:
Wind energy technology is based on the ability to capture the energy contained in air motion. Wind power quantifies the rate of this kinetic energy extraction. Wind power is also the rate of kinetic energy flow carried by the moving air. Because the motion is both the source of the energy and the means of its transport, the efficiency of wind power extraction is a balance of slowing down the wind while maintaining a sufficient flow. This chapter quantifies these fundamental concepts and discusses the nature of wind.

Key words: Wind energy flow rate, kinetic energy flux, wind power density, power coefficient, Betz Limit, capacity factor, source of wind energy.

1 Wind physics basics: what is wind and how wind is generated

Wind is atmospheric air in motion[1]. It is ubiquitous and one of the basic physical elements of our environment. Depending on the speed of the moving air, wind might feel light and etherial, being silent and invisible to the naked eye. Or, it can be a strong and destructive force, loud and visible as a result of the heavy debris it carries along. The velocity of the air motion defines the strength of wind and is directly related to the amount of energy in the wind, that is -- its **kinetic energy**. The source of this energy, however, is **solar radiation.** The electro-magnetic radiation from the sun unevenly heats the earth surface, stronger in the tropics and weaker in the high latitudes. Also, as a result of a differential absorption of sun light by soil, rock, water and vegetation, air in different regions warms up at different rate. This uneven heating is converted through convective processes to air motion, which is adjusted by the rotation of the earth. The convective processes are disturbances of the hydrostatic balance whereby otherwise stagnant air masses are displaced and move in reaction to forces induced by changes in air density and buoyancy due to temperature differences. Air is pushed from high to low pressure regions, balancing friction and inertial forces due to the rotation of the earth.

[1] We focus on air motions with net total displacement, excluding fast small amplitude oscillatory motions due to adiabatic pressure fluctuations associated with propagation of sound waves.

The patterns of differential earth surface heating as well as other thermal processes such as evaporation, precipitation, clouds, shade and variations of surface radiation absorption appear on different space and time scales. These are coupled with dynamical forces due to earth rotation and flow momentum redistribution to drive a variety of wind generation processes, leading to the existence of a large variety of wind phenomena. These winds can be categorized based on their spatial scale and physical generation mechanisms.

2 Wind types: brief overview of wind power meteorology

Wind systems span a wide range of spatial scales, from global circulation on the planetary scale, through synoptic scale weather systems, to mesoscale regional and microscale local winds. Table 2.1 lists the spatial scales of these broad wind type categories. Example of planetary circulations are sustained zonal flows such as the jet stream, trade winds and polar jets. Mesoscale winds include orographic and thermally induced circulations [1]. On the microscale wind systems include flow channeling by urban topography [2] as well as sub-mesoscale convective wind storm phenomena as an example.

Spatial scales	Wind types	Length scale
Planetary scale	global circulation	10000 km
Synoptic scale	weather systems	1000 km
Meso scale	regional orographic or thermally induced circulations	10 - 100 km
Micro scale	local flow modulation, boundary layer turbulent gusts	100 - 1000 m

Table 2.1 Spatial scales of wind systems and a sample of associated wind types.

A long list of various wind types can be assembled from scientific and colloquial names of different winds around the world. The associated physical phenomena enable a finer classification across the spatial scales. Generating physical mechanisms define geostrophic winds, thermal winds, gradient winds. Katabatic and anabatic winds are local topographic winds generated by cooling and heating of mountain slopes. Bora, Foehn and Chinook are locale specific names for strong downslope wind storms [3]. In Greenland -- Piteraq is a downslope storm as strong as a hurricane, with sustained wind speeds of 70 m s^{-1} (160 miles per hour). In coastal areas sea breeze and land breeze circulations are regular daily occurrences. Convective storms generate strong transient winds, with downdrafts which can be particularly dangerous (and not very useful for wind power harvesting). Disastrous hurricanes and typhoons, as well as smaller scale tornadoes are examples of very energetic and destructive wind systems. A micro scale version of these winds are gusts, dust devils and microbursts. Nocturnal jets appear in regular cycles in regions with specific vertical atmospheric structure. Atmospheric waves driven by gravity and modulated by topography are common in many places. Locale specific regional wind names include Santa Anas, nor'easters and ethesian winds, to mention just few.

Meteorology is the scientific field involved in the study and explanation of all these wind phenomena. It enables both a theoretical understanding and the practical forecasting capabilities of wind. Statistics of observed wind occurrences define wind climates in different regions. Mathematical and computer models are used for theoretical simulation, exploratory resource assessment and operational forecasting of winds. Meteorology literature, focusing on wind power is available, in the form of introductory texts and reviews [4-7].

3 Fundamental Equation of Wind Power: kinetic energy flux and wind power density

The fundamental equation of wind power answers the most basic quantitative question - ***how much energy is in the wind***. First we distinguish between concepts of *power* and *energy*. Power is the time-rate of energy. For example, we will need to know how much energy can be generated by a wind turbine per unit time. On a more homely front, the power of the wind is the rate of wind energy flow through an open window.

Wind energy depends on:

- amount of air (the volume of air in consideration)
- speed of air (the magnitude of its velocity)
- mass of air (related to its volume via density)

Wind power quantifies the amount of wind energy flowing through an area of interest per unit time. In other words, wind power is the flux of wind energy through an area of interest. *Flux* is a fundamental concept in fluid mechanics, measuring the rate of flow of any quantity carried with the moving fluid, by definition normalized per unit area. For example, *mass flux* is the rate of mass flow through an area of interest divided by this area. *Volume flux* is the volume flowing through area of interest per unit time and per unit area. Consider an area element A (Fig 2.1) and flow of magnitude U through this area[2]. The volume of air flowing through this area during unit time dt is given by the volume of the cylinder with cross section area A and length $U \cdot dt$, that is the volume $A \cdot U \cdot dt$. Therefore volume flow rate is $A \cdot U$, the volume flux is U. The mass flow rate is derived by multiplying the volume flow rate by the density of the flow ρ and is equal to the mass of that cylinder divided by unit time

$$\frac{dm}{dt} = \rho \cdot A \cdot U \qquad \textbf{(2.1)}$$

[2] Here we restrict the discussion to flow perpendicular to the area of interest. In general, flow is a vector quantify that can be oriented in any direction and only its component perpendicular to the area element is considered when quantifying the flux through that area.

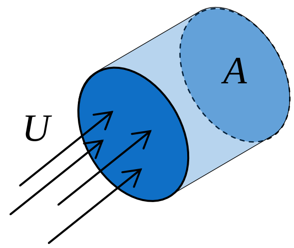

Figure 2.1 Schematics of air flow at velocity *U* through area *A*. The cylinder depicts the volume flowing in unit time *dt* through area *A*.

Wind energy by definition is the energy content of air flow due to its motion. This type of energy is called the **kinetic energy** and is a function of its mass and velocity, given by

$$KE = \tfrac{1}{2} \cdot m \cdot U^2 \qquad \textbf{(2.2)}$$

Wind power is the rate of kinetic energy flow. In derivation similar to the other flow rate quantities discussed above, the amount of kinetic energy flowing per unit time through a given area is equal the kinetic energy content of the cylinder in Fig (2.1).

$$P = \tfrac{1}{2} \cdot \frac{dm}{dt} \cdot U^2 \qquad \textbf{(2.3)}$$

Here mass flow rate (2.1) was substituted for air mass in (2.2). The resultant equation for wind power is

$$P = \tfrac{1}{2} \cdot \rho \cdot A \cdot U^3 \qquad \textbf{(2.4)}$$

This is a fundamental equation in wind power analysis. It exhibits a highly nonlinear cubic dependence on wind speed. E.g. doubling the wind speed leads to eight-fold increase in its available power. This explains why ambient wind speed is the major factor in considering wind energy. In Eq. (2.4), the power of the wind is a linear function of air density and as a result of the limited range of air density fluctuations, the density is of secondary importance. The power dependence on the area implies a nonlinear quadratic dependence on the radius of a wind turbine swept area, highlighting the advantages of longer wind turbine blades.

It is customary to normalize ambient wind power dividing by the area of interest; that is in terms of specific power flow. This leads to the definition of kinetic wind energy flux, known as the **wind power density (*WDP*)**. Similarly to the definitions of flux and flow rate definitions above, wind energy flux is wind energy flow rate per unit area is given by:

$$WPD \equiv \frac{P}{A} = \tfrac{1}{2} \cdot \rho \cdot U^3 \qquad \textbf{(2.5)}$$

Wind power density is used to compare wind resources independent of wind turbine size and is the quantitative basis for the standard classification [8] of wind resource at the

National Renewable Energy Laboratory (NREL) of the USA. Mean wind power density has advantages over mean wind speed for comparing sites with different probability distribution skewness, because of the cubic nonlinear dependence of wind power on wind speed (see Fig. 11 in reference [9] and discussion therein). Further technical details of this classification system were originally introduced in reference[10]. Typical values of wind power classes with the corresponding power densities and mean wind speeds are presented in Table 2.2.

Wind Power Classification			
Wind Power Class	Resource Potential	Wind Power Density / W m^{-2}	Wind Speed / m s^{-1}
1	Poor	0 - 200	0.0 - 5.9
2	Marginal	200 - 300	5.9 - 6.7
3	Fair	300 - 400	6.7 - 7.4
4	Good	500 - 600	7.4 - 7.9
5	Excellent	500 - 600	7.9 - 8.4
6	Outstanding	600 - 800	8.4 - 9.3
7	Superb	> 800	> 9.3

Table 2.2 Wind power classes measured at 50 m above ground according to NREL wind power density based classification. Wind speed corresponding to each class is the mean wind speed based on Rayleigh probability distribution of equivalent mean wind power density at 1500 m elevation above sea level. Data adopted from [11].

4 Wind power capture: efficiency in extracting wind power

In the previous section we considered the total wind power content of ambient air flow. Fundamentally, not all this power is available for utilization. The efficiency in wind power extraction is quantified by the **Power Coefficient** (C_p) which is the ratio of power extracted by the turbine to the total power of the wind resource $C_p = P_T / P_{wind}$.
Turbine power capture therefore is given by

$$P_T = \tfrac{1}{2} \cdot \rho \cdot A \cdot U^3 \cdot C_p \qquad \textbf{(2.6)}$$

which is always smaller than P_{wind}. In fact, there exists a theoretical upper limit on the maximum extractable power fraction - known as the **Betz Limit**. According to Betz theory [12] the maximum possible power coefficient C_p = 16/27, that is, 59% efficiency is the best a conventional wind turbine can do in extracting power from the wind. The reason why higher, e.g. 100%, efficiency is not possible is due to the fluid mechanical nature of wind power, dependent on the continuous flow of air in motion. If, hypothetically speaking, 100% of kinetic energy was extracted, then the flow of air would be reduced to a complete stop and no velocity would remain available to sustain the flow through the energy extraction device, irrespective of the specific wind turbine technology used. The maximum extraction efficiency is achieved at the optimum balance of the largest wind slowdown that still maintains sufficiently fast flow past the turbine. (See references [13,14] for further technical details and an historic account of Betz limit derivations by contemporary researchers).

Another key metric of wind power efficiency is the **Capacity Factor** (*CF)* quantifying the fraction of the installed generating capacity that actually generates power.

$$CF = \frac{E_{actual}}{E_{ideal}} = \frac{time \cdot \overline{P}}{time \cdot P_N} = \frac{\overline{P}}{P_N} \quad \textbf{(2.7)}$$

The *CF* is the ratio of the actual generated energy to the energy which could potentially be generated by the system in consideration under ideal environmental conditions. Considering that energy is the product of its time-rate, that is, the power with the elapsed time, this energy ratio is equal the ratio of average power $\overline{P}$ to the nominal power of the system P_N. For a single wind turbine this nominal power is equal to its nameplate capacity, typically the maximum power it can generate under favorable wind conditions. Considering a typical power curve for a turbine (Fig. 2.2) this is the flat region for strong wind just below the cut-out wind speed.

Equivalently, *CF* can be regarded as the fraction of the year the turbine generator is operating at rated power (nominal capacity), that is, the fraction of the effective time relative to the total time

$$CF = \frac{E_{actual}}{E_{ideal}} = \frac{E_{actual}}{time \cdot P_N} = \frac{E_{actual} / P_N}{time} = \frac{time_{effective}}{time} \quad \textbf{(2.8)}$$

Therefore, total annual energy generation can be calculated by multiplying turbine (or wind plant) rated power P_N by time length of one year and by *CF*.

$$E_{actual} = P_N \cdot time_{effective} = P_N \cdot time \cdot CF \quad \textbf{(2.9)}$$

A typical value of *CF* for an economically viable project is 30%, reaching about 50% in regions with a very good wind resource. The *CF* is based on both the characteristics of the turbine and the site – integrating the power curve with the wind resource variability (Fig 2.2) produces the actual generation or the average power. This highlights the dependence on power production of wind variability and the importance of wind meteorology and climatology for wind power forecasting and resource assessment.

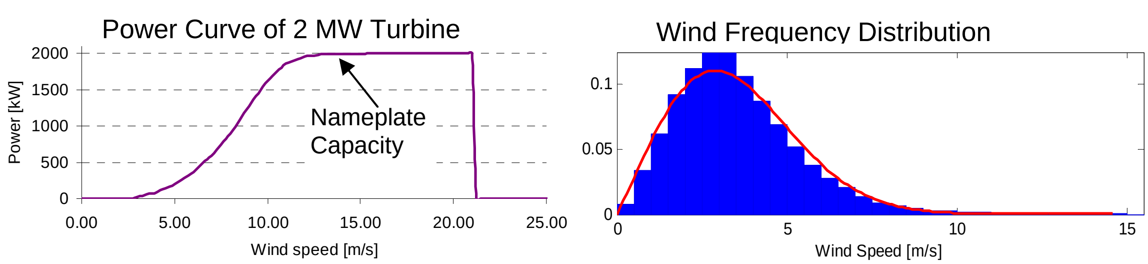

Figure 2.2 Typical wind turbine power curve (left panel) and the statistics of wind variability (right panel) given by a histogram and Weibull probability density fit.

5 Conclusion

Wind power is concerned with the utilization of kinetic wind energy. This is the energy contained in air motion itself. Since this is a form of mechanical energy of a moving fluid, its quantification requires elements of fluid mechanics. We reviewed the concepts of kinetic energy flux and derived the fundamental equation of wind power - quantifying the rate of wind energy flow. Standard metrics of wind power resource and utilization efficiency were introduced. The nature of wind was discussed with a brief overview of wind power meteorology.

References

[1] Pielke Sr, Roger A. *Mesoscale meteorological modeling*. Vol. 98. Academic press, 2013.

[2] Kalmikov, A., Uncovering MIT wind myths through micro-climatological CFD analysis, arXiv:1310.3538, 2013.

[3] Durran, Dale R. "Downslope winds." *Encyclopedia of Atmospheric Sciences*, 644-650, 2003.

[4] Emeis, S., *Wind Energy Meteorology*. Springer Berlin Heidelberg, Berlin, Heidelberg, 2013.

[5] Petersen, E. L., N. G. Mortensen, L. Landberg, J. Højstrup, and H. P. Frank, 1998: Wind power meteorology. Part I: climate and turbulence. *Wind Energy*, **1**, 25–45.

[6] Petersen, E. L., N. G. Mortensen, L. Landberg, J. Højstrup, and H. P. Frank, 1998: Wind power meteorology. Part II: siting and models. *Wind Energy*, **1**, 55–72.

[7] Banta, R. M., Y. L. Pichugina, N. D. Kelley, R. M. Hardesty, and W. A. Brewer, 2013: Wind energy meteorology: Insight into wind properties in the turbine-rotor layer of the atmosphere from high-resolution doppler lidar. *Bull. Am. Meteorol. Soc.*, **94**, 883–902.

[8] <http://rredc.nrel.gov/wind/pubs/atlas/appendix_A.html>

[9] Elliott, D. L., et al. *Wind Energy Resource Assessment of The Caribbean and Central America*. No. PNL-6234. Pacific Northwest National Laboratory (PNNL), Richland, WA (United States), 1987. doi:10.2172/971424.

[10] Elliott, D.L., and W.R. Barchet. *Wind Energy Resources Atlas. Volume 1. Northwest Region*. United States: N. p., 1980. doi:10.2172/5319588.

[11] <https://www.nrel.gov/gis/data-wind.html>

[12] Betz, A., The Maximum of the Theoretically Possible Exploitation of Wind by Means of a Wind Motor, Wind Engineering, 37, No. 4, 441-446, 2013.

Translation of: Das Maximum der theoretisch möglichen Ausnützung des Windes durch Windmotoren, Zeitschrift für das gesamte Turbinenwesen, Heft 26, 1920.

[13] van Kuik G.A.M., The Betz-Joukovsky limit: On the contributions to rotor aerodynamics by the British, German and Russian scientific schools, Wind Energy, 15, 335–344, 2012.

[14] van Kuik G.A.M., The Lanchester-Betz-Joukovsky Limit, Wind Energy, 10, 289–291, 2007.