Assignment #7 – Solutions (Chapter 5)

7. Consider the data set shown in Table 5.1

Table 5.1.

Record	A	B	C	Class
1	0	0	0	+
2	0	0	1	−
3	0	1	1	−
4	0	1	1	−
5	0	0	1	+
6	1	0	1	+
7	1	0	1	−
8	1	0	1	−
9	1	1	1	+
10	1	0	1	+

(a) Estimate the conditional probabilities for $P(A|+)$, $P(B|+)$, $P(C|+)$, $P(A|-)$, $P(B|-)$, and $P(C|-)$.

Answer:

$P(A = 1|-) = 2/5 = 0.4$, $P(B = 1|-) = 2/5 = 0.4$, $P(C = 1|-) = 1$, $P(A = 0|-) = 3/5 = 0.6$,

$P(B = 0|-) = 3/5 = 0.6$, $P(C = 0|-) = 0$; $P(A = 1|+) = 3/5 = 0.6$, $P(B = 1|+) = 1/5 = 0.2$, $P(C = 1|+) = 4/5 = 0.8$,

$P(A = 0|+) = 2/5 = 0.4$, $P(B = 0|+) = 4/5 = 0.8$,

$P(C=0|+)=1/5=0.2$

(b) Use the estimate of conditional probabilities given in the previous question to predict the class label for a test sample ($A = 0, B = 1, C = 0$) using the naıve Bayes approach.

Answer:

Let $P(A = 0, B = 1, C = 0) = K$.

$$P(+|A = 0, B = 1, C = 0)$$
$$= \frac{P(A = 0, B = 1, C = 0|+) \times P(+)}{P(A = 0, B = 1, C = 0)}$$
$$= \frac{P(A = 0|+)P(B = 1|+)P(C = 0|+) \times P(+)}{K}$$
$$= .008/K.$$

$$P(-|A = 0, B = 1, C = 0)$$
$$= \frac{P(A = 0, B = 1, C = 0|-) \times P(-)}{P(A = 0, B = 1, C = 0)}$$
$$= \frac{P(A = 0|-) \times P(B = 1|-) \times P(C = 0|-) \times P(-)}{K}$$
$$= 0/K$$

The class label should be '+'.

(c) Estimate the conditional probabilities using the m-estimate approach, with $p = 1/2$ and $m = 4$.

Answer:

$P(A = 0|+) = (2 + 2)/(5 + 4) = 4/9$,

$P(A = 0|-) = (3 + 2)/(5 + 4) = 5/9$,

$P(B = 1|+) = (1 + 2)/(5 + 4) = 3/9$,

$P(B = 1|-) = (2 + 2)/(5 + 4) = 4/9$,

$P(C = 0|+) = (1 + 2)/(5 + 4) = 3/9$,

$P(C = 0|-) = (0 + 2)/(5 + 4) = 2/9$.

(d) Repeat part (b) using the conditional probabilities given in part (c).

Answer:

Let $P(A = 0, B = 1, C = 0) = K$

$P(+|A=0, B=1, C=0) = 0.0247/K$

$P(-|A = 0, B = 1, C = 0) = 0.0549/K$

The class label should be '-'.

(e) Compare the two methods for estimating probabilities. Which method is better and why?

Answer:

When one of the conditional probability is zero, the estimate for conditional probabilities using the m-estimate probability approach is better, since we don't want the entire expression to become zero.

8. Consider the data set shown in Table 5.11.

Instance	*A*	*B*	*C*	Class
1	0	0	1	−
2	1	0	1	+
3	0	1	0	−
4	1	0	0	−
5	1	0	1	+
6	0	0	1	+
7	1	1	0	−
8	0	0	0	−
9	0	1	0	+
10	1	1	1	+

(a) Estimate the conditional probabilities for $P(A = 1|+)$, $P(B = 1|+)$, $P(C = 1|+)$, $P(A = 1|-)$, $P(B = 1|-)$, and $P(C = 1|-)$ using the same approach as in the previous problem.

Answer:

$P(A = 1|+) = 0.6$, $P(B = 1|+) = 0.4$, $P(C = 1|+) = 0.8$, $P(A = 1|-) = 0.4$, $P(B = 1|-) = 0.4$, and $P(C = 1|-) = 0.2$

(b) Use the conditional probabilities in part (a) to predict the class label for a test sample $(A = 1, B = 1, C = 1)$ using the naıve Bayes approach.

Answer:

Let $R : (A = 1, B = 1, C = 1)$ be the test record. To determine its class, we need to compute $P(+|R)$ and $P(-|R)$. Using Bayes Theorem

$P(+|R) = P(R|+)P(+)/P(R)$ and $P(-|R) = P(R|-)P(-)/P(R)$. Since $P(+) = P(-) = 0.5$ and $P(R)$ is constant, R can be classified by comparing $P(+|R)$ and $P(-|R)$.

For this question,

$$P(R|+) = P(A = 1|+) \times P(B = 1|+) \times P(C = 1|+) = 0.192$$

$$P(R|-) = P(A = 1|-) \times P(B = 1|-) \times P(C = 1|-) = 0.032$$

Since $P(R|+)$ is larger, the record is assigned to (+) class.

(c) Compare $P(A = 1)$, $P(B = 1)$, and $P(A = 1, B = 1)$. State the relationships between A and B.

Answer:

$P(A = 1) = 0.5$, $P(B = 1) = 0.4$ and $P(A = 1, B = 1) = P(A) \times P(B) = 0.2$. Therefore, A and B are independent.

(d) Repeat the analysis in part (c) using $P(A = 1)$, $P(B = 0)$, and $P(A = 1, B = 0)$.

Answer:

$P(A = 1) = 0.5$, $P(B = 0) = 0.6$, and $P(A = 1, B = 0) = P(A = 1) \times P(B = 0) = 0.3$. A and B are still independent.

(e) Compare $P(A = 1, B = 1|Class = +)$ against $P(A = 1|Class = +)$ and $P(B = 1|Class = +)$. Are the variables conditionally independent given the class?

Answer:

Compare $P(A = 1, B = 1|+) = 0.2$ against $P(A = 1|+) = 0.6$ and $P(B = 1|Class = +) = 0.4$. Since the product of $P(A = 1|+)$ and $P(A = 1|-)$ is not the same as $P(A = 1, B = 1|+)$, A and B are not conditionally independent given the class.

10. Repeat the analysis shown in Example 5.3 for finding the location of a decision boundary using the following information:

a. The prior probabilities are P (Crocodile) = 2 × P (Alligator). **Answer:**
We need to find $\tilde{x}$ that satisfies P[X=$\tilde{x}$|Crocodile] x P[Crocodile] = P[X=$\tilde{x}$|Alligator] x P[Alligator]. Using the Gaussian density function for the first term in each expression and solving the equation, we obtain $\tilde{x}$=12.5758.

b. The prior probabilities are P (Alligator) = 2 × P (Crocodile). **Answer:**
Using the formula as before, we obtain $\hat{x}$ = 14.3754.
.

c. The prior probabilities are the same, but their standard deviations are different; i.e., σ(Crocodile) = 4 and σ(Alligator) = 2.
Answer: Using the formula as before, we obtain a quadratic equation with two solutions: $\hat{x}$ = 7.624, and 14.375. In this case, the decision is crocodile when X is less than or equal to 7.624, alligator if X is between 7.624 and 14.375; otherwise it is a crocodile. This can be easily seen if you draw the two Gaussian curves and by inspecting their intersection points.

12. Given the Bayesian network shown in Figure 5.4, compute the following probabilities:

(a) P (B = good, F = empty, G = empty, S =yes)

Answer:

$$
\begin{aligned}
& P\,(B = good, F = empty, G = empty, S = yes) \\
= \; & P\,(B = good) \times P\,(F = empty) \times P\,(G = empty | B = good, F = empty) \\
& \times P\,(S = yes | B = good, F = empty) \\
= \; & 0.9 \times 0.2 \times 0.8 \times 0.2 = 0.0288.
\end{aligned}
$$

(b) P (B = bad, F = empty, G = not empty, S = no).

Answer:

$$
\begin{aligned}
& P\,(B = bad, F = empty, G = not\ \ empty, S = no) \\
= \; & P\,(B = bad) \times P\,(F = empty) \times P\,(G = not\ \ empty | B = bad, F = empty) \\
& \times P\,(S = no | B = bad, F = empty) \\
& 0.1 \times 0.2 \times 0.1 \times 1.0 = 0.002.
\end{aligned}
$$

(c) Given that the battery is bad, compute the probability that the car will start.

Answer:

$$P\,(S = yes | B = bad)$$

=P[S=yes, B=bad]/P[B=bad]=0.1x0.1x0.8/0.1=0.08