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8-1 Study Guide and Intervention

Multiplying Monomials

Multiply Monomials A **monomial** is a number, a variable, or a product of a number and one or more variables. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 **Simplify $(3x^6)(5x^2)$.**

$(3x^6)(5x^2) = (3)(5)(x^6 \cdot x^2)$ Associative Property
$= (3 \cdot 5)(x^{6+2})$ Product of Powers
$= 15x^8$ Simplify.

The product is $15x^8$.

Example 2 **Simplify $(-4a^3b)(3a^2b^5)$.**

$(-4a^3b)(3a^2b^5) = (-4)(3)(a^3 \cdot a^2)(b \cdot b^5)$
$= -12(a^{3+2})(b^{1+5})$
$= -12a^5b^6$

The product is $-12a^5b^6$.

Exercises

Simplify.

1. $y(y^5)$ y^6
2. $n^2 \cdot n^7$ n^9
3. $(-7x^2)(x^4)$ $-7x^6$
4. $x(x^2)(x^4)$ x^7
5. $m \cdot m^5$ m^6
6. $(-x^3)(-x^4)$ x^7
7. $(2a^2)(8a)$ $16a^3$
8. $(rs)(rs^3)(s^2)$ r^2s^6
9. $(x^2y)(4xy^3)$ $4x^3y^4$
10. $\frac{1}{3}(2a^3b)(6b^3)$ $4a^3b^4$
11. $(-4x^3)(-5x^7)$ $20x^{10}$
12. $(-3j^2k^4)(2jk^6)$ $-6j^3k^{10}$
13. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$ $a^3b^2c^7$
14. $(-5xy)(4x^2)(y^4)$ $-20x^3y^5$
15. $(10x^3yz^2)(-2xy^5z)$ $-20x^4y^6z^3$

Lesson 8-1

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8-1 Study Guide and Intervention *(continued)*

Multiplying Monomials

Powers of Monomials An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and all integers m and n, $(a^m)^n = a^{mn}$.
Power of a Product	For any number a and all integers m and n, $(ab)^m = a^mb^m$.

Example **Simplify $(-2ab^2)^3(a^2)^4$.**

$(-2ab^2)^3(a^2)^4 = (-2ab^2)^3(a^8)$ Power of a Power
$= (-2)^3(a^3)(b^2)^3(a^8)$ Power of a Product
$= (-2)^3(a^3)(a^8)(b^2)^3$ Commutative Property
$= (-2)^3(a^{11})(b^2)^3$ Product of Powers
$= -8a^{11}b^6$ Power of a Power

The product is $-8a^{11}b^6$.

Exercises

Simplify.

1. $(y^5)^2$ y^{10}
2. $(n^7)^4$ n^{28}
3. $(x^2)^5(x^3)$ x^{13}
4. $-3(ab^4)^3$ $-3a^3b^{12}$
5. $(-3ab^4)^3$ $-27a^3b^{12}$
6. $(4x^2b)^3$ $64x^6b^3$
7. $(4a^2)^2(b^3)$ $16a^4b^3$
8. $(4x)^2(b^3)$ $16x^2b^3$
9. $(x^2y^4)^5$ $x^{10}y^{20}$
10. $(2a^3b^2)(b^3)^2$ $2a^3b^8$
11. $(-4xy)^3(-2x^2)^3$ $512x^9y^3$
12. $(-3j^2k^3)^2(2j^2k)^3$ $72j^{10}k^9$
13. $(25a^2b)^3\left(\frac{1}{5}abc\right)^2$ $625a^8b^5c^2$
14. $(2xy)^2(-3x^2)(4y^4)$ $-48x^4y^6$
15. $(2x^3y^2z^2)^3(x^2z)^4$ $8x^{17}y^6z^{10}$
16. $(-2n^6y^5)(-6n^3y^2)(ny)^3$ $12n^{12}y^{10}$
17. $(-3a^3n^4)(-3a^3n)^4$ $-243a^{15}n^8$
18. $-3(2x)^4(4x^5y)^2$ $-768x^{14}y^2$

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8-1 Skills Practice

Multiplying Monomials

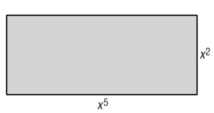
Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. 11 **Yes; 11 is a real number and an example of a constant.**
2. $a - b$ **No; This is the difference, not the product, of two variables.**
3. $\frac{p^2}{q^2}$ **No; This is the quotient, not the product, of two variables.**
4. y **Yes; Single variables are monomials.**
5. j^3k **Yes; This is the product of two variables.**
6. $2a + 3b$ **No; This is the sum of two monomials.**

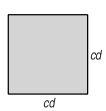
Simplify.

7. $a^2(a^3)(a^6)$ $\mathbf{a^{11}}$
8. $x(x^2)(x^7)$ $\mathbf{x^{10}}$
9. $(y^2z)(yz^2)$ $\mathbf{y^3z^3}$
10. $(\ell^2k^2)(\ell^3k)$ $\mathbf{\ell^5k^3}$
11. $(e^2f^4)(e^2f^2)$ $\mathbf{e^4f^6}$
12. $(cd^2)(c^3d^2)$ $\mathbf{c^4d^4}$
13. $(2x^2)(3x^5)$ $\mathbf{6x^7}$
14. $(5a^7)(4a^2)$ $\mathbf{20a^9}$
15. $(4xy^3)(3x^3y^5)$ $\mathbf{12x^4y^8}$
16. $(7a^5b^2)(a^2b^3)$ $\mathbf{7a^7b^5}$
17. $(-5m^3)(3m^8)$ $\mathbf{-15m^{11}}$
18. $(-2c^4d)(-4cd)$ $\mathbf{8c^5d^2}$
19. $(10^2)^3$ $\mathbf{10^6}$ **or 1,000,000**
20. $(p^3)^{12}$ $\mathbf{p^{36}}$
21. $(-6p)^2$ $\mathbf{36p^2}$
22. $(-3y)^3$ $\mathbf{-27y^3}$
23. $(3pq^2)^2$ $\mathbf{9p^2q^4}$
24. $(2b^3c^4)^2$ $\mathbf{4b^6c^8}$

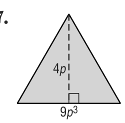
GEOMETRY **Express the area of each figure as a monomial.**

25.


$\mathbf{x^7}$

26.


$\mathbf{c^2d^2}$

27.


$\mathbf{18p^4}$

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8-1 Practice (Average)

Multiplying Monomials

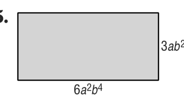
Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. $\frac{21a^2}{7b}$ **No; this involves the quotient, not the product, of variables.**
2. $\frac{b^3c^2}{2}$ **Yes; this is the product of a number, $\frac{1}{2}$, and two variables.**

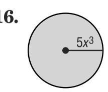
Simplify.

3. $(-5x^2y)(3x^4)$ $\mathbf{-15x^6y}$
4. $(2ab^2c^2)(4a^3b^2c^2)$ $\mathbf{8a^4b^4c^4}$
5. $(3cd^4)(-2c^2)$ $\mathbf{-6c^3d^4}$
6. $(4g^3h)(-2g^5)$ $\mathbf{-8g^8h}$
7. $(-15xy^4)\left(-\frac{1}{3}xy^3\right)$ $\mathbf{5x^2y^7}$
8. $(-xy)^3(xz)$ $\mathbf{-x^4y^3z}$
9. $(-18m^2n)^2\left(-\frac{1}{6}mn^2\right)$ $\mathbf{-54m^5n^4}$
10. $(0.2a^2b^3)^2$ $\mathbf{0.04a^4b^6}$
11. $\left(\frac{2}{3}p\right)^2$ $\mathbf{\frac{4}{9}p^2}$
12. $\left(\frac{1}{4}cd^3\right)^2$ $\mathbf{\frac{1}{16}c^2d^6}$
13. $(0.4k^3)^3$ $\mathbf{0.064k^9}$
14. $[(4^2)^2]^2$ $\mathbf{4^8}$ **or 65,536**

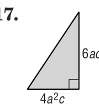
GEOMETRY **Express the area of each figure as a monomial.**

15.


$\mathbf{18a^3b^6}$

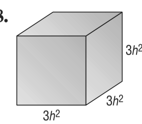
16.


$\mathbf{(25x^6)\pi}$

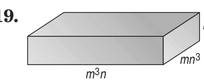
17.


$\mathbf{12a^3c^4}$

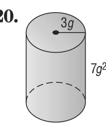
GEOMETRY **Express the volume of each solid as a monomial.**

18.


$\mathbf{27h^6}$

19.


$\mathbf{m^4n^5}$

20.


$\mathbf{(63g^4)\pi}$

21. **COUNTING** A panel of four light switches can be set in 2^4 ways. A panel of five light switches can set in twice this many ways. In how many ways can five light switches be set? $\mathbf{2^5}$ **or 32**

22. **HOBBIES** Tawa wants to increase her rock collection by a power of three this year and then increase it again by a power of two next year. If she has 2 rocks now, how many rocks will she have after the second year? $\mathbf{2^6}$ **or 64**

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8-1 Reading to Learn Mathematics

Multiplying Monomials

Pre-Activity **Why does doubling speed quadruple braking distance?**

Read the introduction to Lesson 8-1 at the top of page 410 in your textbook.

Find two examples in the table to verify the statement that when speed is doubled, the braking distance is quadrupled. Write your examples in the table.

Speed (miles per hour)	Braking Distance (feet)	Speed Doubled (miles per hour)	Braking Distance Quadrupled (feet)
20	**20**	**40**	**80**
30	**45**	**60**	**180**

Reading the Lesson

1. Describe the expression $3xy$ using the terms *monomial*, *constant*, *variable*, and *product*.

The monomial $3xy$ is the product of the constant 3 and the variables x and y.

2. Complete the chart by choosing the property that can be used to simplify each expression. Then simplify the expression.

Expression	Property	Expression Simplified
$3^5 \cdot 3^2$	<u>Product of Powers</u> Power of a Power Power of a Product	**3^7 or 2187**
$(a^3)^4$	Product of Powers <u>Power of a Power</u> Power of a Product	**a^{12}**
$(-4xy)^5$	Product of Powers Power of a Power <u>Power of a Product</u>	**$-1024x^5y^5$**

Helping You Remember

3. Write an example of each of the three properties of powers discussed in this lesson. Then, using the examples, explain how the property is used to simplify them.

Sample answer: For $z^2 \cdot z^5$, since the bases are the same, use the Product of Powers Property and add the exponents to get z^7. For $(a^4)^3$, use the Power of a Power Property. Multiply the exponents to get a^{12}. For $(3rs)^3$, use the Power of a Product Property. Raise the constant and each variable to the power to get $27r^3s^3$.

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8-1 Enrichment

An Wang

An Wang (1920–1990) was an Asian-American who became one of the pioneers of the computer industry in the United States. He grew up in Shanghai, China, but came to the United States to further his studies in science. In 1948, he invented a magnetic pulse controlling device that vastly increased the storage capacity of computers. He later founded his own company, Wang Laboratories, and became a leader in the development of desktop calculators and word processing systems. In 1988, Wang was elected to the National Inventors Hall of Fame.

Digital computers store information as numbers. Because the electronic circuits of a computer can exist in only one of two states, open or closed, the numbers that are stored can consist of only two digits, 0 or 1. Numbers written using only these two digits are called **binary numbers**. To find the decimal value of a binary number, you use the digits to write a *polynomial in 2*. For instance, this is how to find the decimal value of the number 1001101_2. (The subscript 2 indicates that this is a binary number.)

$$\begin{aligned}1001101_2 &= 1 \times \underline{2^6} + 0 \times \underline{2^5} + 0 \times \underline{2^4} + 1 \times \underline{2^3} + 1 \times \underline{2^2} + 0 \times \underline{2^1} + 1 \times \underline{2^0}\\ &= 1 \times \underline{64} + 0 \times \underline{32} + 0 \times \underline{16} + 1 \times \underline{8} + 1 \times \underline{4} + 0 \times \underline{2} + 1 \times \underline{1}\\ &= 64 + 0 + 0 + 8 + 4 + 0 + 1\\ &= 77\end{aligned}$$

Find the decimal value of each binary number.

1. 1111_2 **15**
2. 10000_2 **16**
3. 11000011_2 **195**
4. 10111001_2 **185**

Write each decimal number as a binary number.

5. 8 **1000**
6. 11 **1011**
7. 29 **11101**
8. 117 **1110101**

9. The chart at the right shows a set of decimal code numbers that is used widely in storing letters of the alphabet in a computer's memory. Find the code numbers for the letters of your name. Then write the code for your name using binary numbers. **Answers will vary.**

The American Standard Guide for Information Interchange (ASCII)

A	65	N	78	a	97	n	110
B	66	O	79	b	98	o	111
C	67	P	80	c	99	p	112
D	68	Q	81	d	100	q	113
E	69	R	82	e	101	r	114
F	70	S	83	f	102	s	115
G	71	T	84	g	103	t	116
H	72	U	85	h	104	u	117
I	73	V	86	i	105	v	118
J	74	W	87	j	106	w	119
K	75	X	88	k	107	x	120
L	76	Y	89	l	108	y	121
M	77	Z	90	m	109	z	122

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8-2 Study Guide and Intervention

Dividing Monomials

Quotients of Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 **Simplify $\frac{a^4b^7}{ab^2}$. Assume neither a nor b is equal to zero.**

$\frac{a^4b^7}{ab^2} = \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right)$ Group powers with the same base.

$= (a^{4-1})(b^{7-2})$ Quotient of Powers

$= a^3b^5$ Simplify.

The quotient is a^3b^5.

Example 2 **Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that b is not equal to zero.**

$\left(\frac{2a^3b^5}{3b^2}\right)^3 = \frac{(2a^3b^5)^3}{(3b^2)^3}$ Power of a Quotient

$= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3}$ Power of a Product

$= \frac{8a^9b^{15}}{27b^6}$ Power of a Power

$= \frac{8a^9b^9}{27}$ Quotient of Powers

The quotient is $\frac{8a^9b^9}{27}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{5^5}{5^2}$ **5^3 or 125**
2. $\frac{m^6}{m^4}$ **m^2**
3. $\frac{p^5n^4}{p^2n}$ **p^3n^3**
4. $\frac{a^2}{a}$ **a**
5. $\frac{x^5y^3}{x^5y^2}$ **y**
6. $\frac{-2y^7}{14y^5}$ **$-\frac{1}{7}y^2$**
7. $\frac{xy^6}{y^4x}$ **y^2**
8. $\left(\frac{2a^2b}{a}\right)^3$ **$8a^3b^3$**
9. $\left(\frac{4p^4q^4}{3p^2q^2}\right)^3$ **$\frac{64}{27}p^6q^6$**
10. $\left(\frac{2v^5w^3}{v^4w^3}\right)^4$ **$16v^4$**
11. $\left(\frac{3r^6s^3}{2r^5s}\right)^4$ **$\frac{81}{16}r^4s^8$**
12. $\frac{r^7s^7t^2}{s^3r^3t^2}$ **r^4s^4**

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8-2 Study Guide and Intervention *(continued)*

Dividing Monomials

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a, $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example **Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that the denominator is not equal to zero.**

$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$ Group powers with the same base.

$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$ Quotient of Powers and Negative Exponent Properties

$= \frac{1}{4}a^{-5}b^0c^5$ Simplify.

$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$ Negative Exponent and Zero Exponent Properties

$= \frac{c^5}{4a^5}$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{2^2}{2^{-3}}$ **2^5 or 32**
2. $\frac{m}{m^{-4}}$ **m^5**
3. $\frac{p^{-8}}{p^3}$ **$\frac{1}{p^{11}}$**
4. $\frac{b^{-4}}{b^{-5}}$ **b**
5. $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$ **$\frac{w}{4y^2}$**
6. $\frac{(a^2b^3)^2}{(ab)^{-2}}$ **a^6b^8**
7. $\frac{x^4y^0}{x^{-2}}$ **x^6**
8. $\frac{(6a^{-1}b)^2}{(b^2)^4}$ **$\frac{36}{a^2b^6}$**
9. $\frac{(3st)^2u^{-4}}{s^{-1}t^2u^7}$ **$\frac{9s^3}{u^{11}}$**
10. $\frac{s^{-3}t^{-5}}{(s^2t^3)^{-1}}$ **$\frac{1}{st^2}$**
11. $\left(\frac{4m^2n^2}{8m^{-1}\ell}\right)^0$ **1**
12. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$ **$-\frac{m^3}{32n^{10}}$**

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8-2 Skills Practice

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{6^5}{6^4}$ **6^1 or 6**
2. $\frac{9^{12}}{9^8}$ **9^4 or 6561**
3. $\frac{x^4}{x^2}$ **x^2**
4. $\frac{r^3s^2}{r^3s^4}$ **$\frac{1}{s^2}$**
5. $\frac{m}{m^3}$ **$\frac{1}{m^2}$**
6. $\frac{9d^7}{3d^6}$ **$3d$**
7. $\frac{12n^5}{36n}$ **$\frac{n^4}{3}$**
8. $\frac{w^4u^3}{w^4u}$ **u^2**
9. $\frac{a^3b^5}{ab^2}$ **a^2b^3**
10. $\frac{m^7n^2}{m^3n^2}$ **m^4**
11. $\frac{-21w^5u^2}{7w^4u^5}$ **$-\frac{3w}{u^3}$**
12. $\frac{32x^3y^2z^5}{-8xyz^2}$ **$-4x^2yz^3$**
13. $\left(\frac{4p^7}{7s^2}\right)^2$ **$\frac{16p^{14}}{49s^4}$**
14. 4^{-4} **$\frac{1}{256}$**
15. 8^{-2} **$\frac{1}{64}$**
16. $\left(\frac{5}{3}\right)^{-2}$ **$\frac{9}{25}$**
17. $\left(\frac{9}{11}\right)^{-1}$ **$\frac{11}{9}$**
18. $\frac{h^3}{h^{-6}}$ **h^9**
19. $k^0(k^4)(k^{-6})$ **$\frac{1}{k^2}$**
20. $k^{-1}(\ell^{-6})(m^3)$ **$\frac{m^3}{k\ell^6}$**
21. $\frac{f^{-7}}{f^4}$ **$\frac{1}{f^{11}}$**
22. $\left(\frac{16p^5q^2}{2p^3q^3}\right)^0$ **1**
23. $\frac{f^{-5}g^4}{h^{-2}}$ **$\frac{g^4h^2}{f^5}$**
24. $\frac{15x^6y^{-9}}{5xy^{-11}}$ **$3x^5y^2$**
25. $\frac{-15w^0u^{-1}}{5u^3}$ **$-\frac{3}{u^4}$**
26. $\frac{48x^6y^7z^5}{-6xy^5z^6}$ **$-\frac{8x^5y^2}{z}$**

Lesson 8-2

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8-2 Practice (Average)

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{8^8}{8^4}$ **8^4 or 4096**
2. $\frac{a^4b^6}{ab^3}$ **a^3b^3**
3. $\frac{xy^2}{xy}$ **y**
4. $\frac{m^5np}{m^4p}$ **mn**
5. $\frac{5c^2d^3}{-4c^2d}$ **$-\frac{5d^2}{4}$**
6. $\frac{8y^7z^6}{4y^6z^5}$ **$2yz$**
7. $\left(\frac{4f^3g}{3h^6}\right)^3$ **$\frac{64f^9g^3}{27h^{18}}$**
8. $\left(\frac{6w^5}{7p^6s^3}\right)^2$ **$\frac{36w^{10}}{49p^{12}s^6}$**
9. $\frac{-4c^2}{24c^5}$ **$-\frac{1}{6c^3}$**
10. $x^3(y^{-5})(x^{-8})$ **$\frac{1}{x^5y^5}$**
11. $p(q^{-2})(r^{-3})$ **$\frac{p}{q^2r^3}$**
12. 12^{-2} **$\frac{1}{144}$**
13. $\left(\frac{3}{7}\right)^{-2}$ **$\frac{49}{9}$**
14. $\left(\frac{4}{3}\right)^{-4}$ **$\frac{81}{256}$**
15. $\frac{22r^3s^2}{11r^2s^{-3}}$ **$2rs^5$**
16. $\frac{-15w^0u^{-1}}{5u^3}$ **$-\frac{3}{u^4}$**
17. $\frac{8c^3d^2f^4}{4c^{-1}d^2f^{-3}}$ **$2c^4f^7$**
18. $\left(\frac{x^{-3}y^5}{4^{-3}}\right)^0$ **1**
19. $\frac{6f^{-2}g^3h^5}{54f^{-2}g^{-5}h^3}$ **$\frac{g^8h^2}{9}$**
20. $\frac{-12t^{-1}u^5v^{-4}}{2t^{-3}uv^5}$ **$-\frac{6t^2u^4}{v^9}$**
21. $\frac{r^4}{(3r)^3}$ **$\frac{r}{27}$**
22. $\frac{m^{-2}n^{-5}}{(m^4n^3)^{-1}}$ **$\frac{m^2}{n^2}$**
23. $\frac{(j^{-1}k^3)^{-4}}{j^3k^3}$ **$\frac{j}{k^{15}}$**
24. $\frac{(2a^{-2}b)^{-3}}{5a^2b^4}$ **$\frac{a^4}{40b^7}$**
25. $\left(\frac{q^{-1}r^3}{qr^{-2}}\right)^{-5}$ **$\frac{q^{10}}{r^{25}}$**
26. $\left(\frac{7c^{-3}d^3}{c^5de^{-4}}\right)^{-1}$ **$\frac{c^8}{7d^2e^4}$**
27. $\left(\frac{2x^3y^2z}{3x^4yz^{-2}}\right)^{-2}$ **$\frac{9x^2}{4y^2z^6}$**

28. **BIOLOGY** A lab technician draws a sample of blood. A cubic millimeter of the blood contains 22^3 white blood cells and 22^5 red blood cells. What is the ratio of white blood cells to red blood cells? **$\frac{1}{484}$**

29. **COUNTING** The number of three-letter "words" that can be formed with the English alphabet is 26^3. The number of five-letter "words" that can be formed is 26^5. How many times more five-letter "words" can be formed than three-letter "words"? **676**

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8-2 Reading to Learn Mathematics

Dividing Monomials

Pre-Activity **How can you compare pH levels?**

Read the introduction to Lesson 8-2 at the top of page 417 in your textbook.

- In the formula $c = \left(\frac{1}{10}\right)^{pH}$, identify the base and the exponent.

  base = $\frac{1}{10}$, exponent = pH

- How do you think c will change as the exponent increases?

  ***c* will decrease.**

Reading the Lesson

1. Explain what the statement $\frac{a^m}{a^n} = a^{m-n}$ means.

To divide two powers that have the same base, subtract the exponents.

2. To find c in the formula $c = \left(\frac{1}{10}\right)^{pH}$, you can find the power of the numerator, the power of the denominator, and divide. This is an example of what property?
Power of a Quotient Property

3. Use the Quotient of Powers Property to explain why $3^0 = 1$. **Sample answer: $\frac{3^4}{3^4} = 1$. The Quotient of Powers Property says that when you divide two powers that have the same base, you subtract the exponents. So $\frac{3^4}{3^4} = 3^0$.**

4. Consider the expression 4^{-3}.

a. Explain why the expression 4^{-3} is not simplified. **An expression involving exponents is not considered simplified if the expression contains negative exponents.**

b. Define the term reciprocal. **The reciprocal of a number is 1 divided by the number.**

c. 4^{-3} is the reciprocal of what power of 4? **4^3**

d. What is the simplified form of 4^{-3}? **$\frac{1}{4^3}$ or $\frac{1}{64}$**

Helping You Remember

5. Describe how you would help a friend who needs to simplify the expression $\frac{4x^2}{2x^5}$.

Divide the constants and group powers with the same base to get $\left(\frac{4}{2}\right)\left(\frac{x^2}{x^5}\right)$. Use the Quotient of Powers Property to get $(2)(x^{2-5})$ or $(2)(x^{-3})$. To simplify $(2)(x^{-3})$, use the Negative Exponent Property to get $(2)\left(\frac{1}{x^3}\right)$, or $\frac{2}{x^3}$.

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8-2 Enrichment

Patterns with Powers

Use your calculator, if necessary, to complete each pattern.

a.		b.		c.	
2^{10} =	**1024**	5^{10} =	**9,765,625**	4^{10} =	**1,048,576**
2^9 =	**512**	5^9 =	**1,953,125**	4^9 =	**262,144**
2^8 =	**256**	5^8 =	**390,625**	4^8 =	**65,536**
2^7 =	**128**	5^7 =	**78,125**	4^7 =	**16,384**
2^6 =	**64**	5^6 =	**15,625**	4^6 =	**4096**
2^5 =	**32**	5^5 =	**3125**	4^5 =	**1024**
2^4 =	**16**	5^4 =	**625**	4^4 =	**256**
2^3 =	**8**	5^3 =	**125**	4^3 =	**64**
2^2 =	**4**	5^2 =	**25**	4^2 =	**16**
2^1 =	**2**	5^1 =	**5**	4^1 =	**4**

Study the patterns for a, b, and c above. Then answer the questions.

1. Describe the pattern of the exponents from the top of each column to the bottom.
The exponents decrease by one from each row to the one below.

2. Describe the pattern of the powers from the top of the column to the bottom. **To get each power, divide the power on the row above by the base (2, 5, or 4).**

3. What would you expect the following powers to be?
2^0 **1** 5^0 **1** 4^0 **1**

4. Refer to Exercise 3. Write a rule. Test it on patterns that you obtain using 22, 25, and 24 as bases. **Any nonzero number to the zero power equals one.**

Study the pattern below. Then answer the questions.

$0^3 = 0$ $0^2 = 0$ $0^1 = 0$ $0^0 =$ __?__ 0^{-1} does not exist. 0^{-2} does not exist. 0^{-3} does not exist.

5. Why do 0^{-1}, 0^{-2}, and 0^{-3} not exist?
Negative exponents are not defined unless the base is nonzero.

6. Based upon the pattern, can you determine whether 0^0 exists?
No, since the pattern $0^n = 0$ breaks down for $n < 1$.

7. The symbol 0^0 is called an **indeterminate**, which means that it has no unique value. Thus it does not exist as a unique real number. Why do you think that 0^0 cannot equal 1?
Answers will vary. One answer is that if $0^0 = 1$, then $1 = \frac{1}{1} = \frac{1^0}{0^0} = \left(\frac{1}{0}\right)^0$, which is a false result, since division by zero is not allowed. Thus, 0^0 cannot equal 1.

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8-3 Study Guide and Intervention

Scientific Notation

Scientific Notation Keeping track of place value in very large or very small numbers written in standard form may be difficult. It is more efficient to write such numbers in scientific notation. A number is expressed in scientific notation when it is written as a product of two factors, one factor that is greater than or equal to 1 and less than 10 and one factor that is a power of ten.

Scientific Notation	A number is in scientific notation when it is in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer.

Example 1 **Express 3.52×10^4 in standard notation.**

$3.52 \times 10^4 = 3.52 \times 10{,}000$
$= 35{,}200$

The decimal point moved 4 places to the right.

Example 2 **Express 6.21×10^{-5} in standard notation.**

$6.21 \times 10^{-5} = 6.21 \times \frac{1}{10^5}$
$= 6.21 \times 0.00001$
$= 0.0000621$

The decimal point moved 5 places to the left.

Example 3 **Express 37,600,000 in scientific notation.**

$37{,}600{,}000 = 3.76 \times 10^7$

The decimal point moved 7 places so that it is between the 3 and the 7. Since $37{,}600{,}000 > 1$, the exponent is positive.

Example 4 **Express 0.0000549 in scientific notation.**

$0.0000549 = 5.49 \times 10^{-5}$

The decimal point moved 5 places so that it is between the 5 and the 4. Since $0.0000549 < 1$, the exponent is negative.

Exercises

Express each number in standard notation.

1. 3.65×10^5
365,000

2. 7.02×10^{-4}
0.000702

3. 8.003×10^8
800,300,000

4. 7.451×10^6
7,451,000

5. 5.91×10^0
5.91

6. 7.99×10^{-1}
0.799

7. 8.9354×10^{10}
89,354,000,000

8. 8.1×10^{-9}
0.0000000081

9. 4×10^{15}
4,000,000,000,000,000

Express each number in scientific notation.

10. 0.0000456
4.56×10^{-5}

11. 0.00001
1×10^{-5}

12. 590,000,000
5.9×10^8

13. 0.00000000012
1.2×10^{-10}

14. 0.000080436
8.0436×10^{-5}

15. 0.03621
3.621×10^{-2}

16. 433×10^4
4.33×10^6

17. 0.0042×10^{-3}
4.2×10^{-6}

18. 50,000,000,000
5×10^{10}

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8-3 Study Guide and Intervention *(continued)*

Scientific Notation

Products and Quotients with Scientific Notation You can use properties of powers to compute with numbers written in scientific notation.

Example 1 **Evaluate $(6.7 \times 10^3)(2 \times 10^{-5})$. Express the result in scientific and standard notation.**

$(6.7 \times 10^3)(2 \times 10^{-5}) = (6.7 \times 2)(10^3 \times 10^{-5})$	Associative Property
$= 13.4 \times 10^{-2}$	Product of Powers
$= (1.34 \times 10^1) \times 10^{-2}$	$13.4 = 1.34 \times 10^1$
$= 1.34 \times (10^1 \times 10^{-2})$	Associative Property
$= 1.34 \times 10^{-1}$ or 0.134	Product of Powers

The solution is 1.34×10^{-1} or 0.134.

Example 2 **Evaluate $\frac{1.5088 \times 10^8}{4.1 \times 10^5}$. Express the result in scientific and standard notation.**

$\frac{1.5088 \times 10^8}{4.1 \times 10^5} = \left(\frac{1.5088}{4.1}\right)\left(\frac{10^8}{10^5}\right)$	Associative Property
$= 0.368 \times 10^3$	Quotient of Powers
$= (3.68 \times 10^{-1}) \times 10^3$	$0.368 = 3.68 \times 10^{-1}$
$= 3.68 \times (10^{-1} \times 10^3)$	Associative Property
$= 3.68 \times 10^2$ or 368	Product of Powers

The solution is 3.68×10^2 or 368.

Exercises

Evaluate. Express each result in scientific and standard notation.

1. $\frac{1.4 \times 10^4}{2 \times 10^2}$
7×10; 70

2. $\frac{3 \times 10^{-12}}{2 \times 10^{-15}}$
1.5×10^3; 1500

3. $(3.2 \times 10^{-2})(2.0 \times 10^2)$
6.4×10^0; 6.4

4. $\frac{1.2672 \times 10^{-8}}{2.4 \times 10^{-12}}$
5.28×10^3; 5280

5. $(7.7 \times 10^5)(2.1 \times 10^2)$
1.617×10^8; 161,700,000

6. $\frac{9.72 \times 10^8}{7.2 \times 10^{10}}$
1.35×10^{-2}; 0.0135

7. $(3.3 \times 10^5)(1.5 \times 10^{-4})$
4.95×10^1; 49.5

8. $\frac{3.3 \times 10^{-12}}{1.1 \times 10^{-14}}$
3×10^2; 300

9. $\frac{4 \times 10^{-4}}{2.5 \times 10^2}$
1.6×10^{-6}; 0.0000016

10. FUEL CONSUMPTION North America burned 4.5×10^{16} BTU of petroleum in 1998. At this rate, how many BTU's will be burned in 9 years? **Source:** *The New York Times 2001 Almanac*
4.05×10^{17}

11. OIL PRODUCTION If the United States produced 6.25×10^9 barrels of crude oil in 1998, and Canada produced 1.98×10^9 barrels, what is the quotient of their production rates? Write a statement using this quotient. **Source:** *The New York Times 2001 Almanac*
About 3.16; Sample answer: The United States produces more than 3 times the crude oil of Canada.

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8-3 Skills Practice

Scientific Notation

Express each number in standard notation.

1. 4×10^3 **4000**

2. 2×10^8 **200,000,000**

3. 3.2×10^5 **320,000**

4. 3×10^{-6} **0.000003**

5. 9×10^{-2} **0.09**

6. 4.7×10^{-7} **0.00000047**

ASTRONOMY Express the number in each statement in standard notation.

7. The diameter of Jupiter is 1.42984×10^5 kilometers. **142,984**

8. The surface density of the main ring around Jupiter is 5×10^{-6} grams per centimeter squared. **0.000005**

9. The minimum distance from Mars to Earth is 5.45×10^7 kilometers. **54,500,000**

Express each number in scientific notation.

10. 41,000,000 **4.1×10^7**

11. 65,100 **6.51×10^4**

12. 283,000,000 **2.83×10^8**

13. 264,701 **2.64701×10^5**

14. 0.019 **1.9×10^{-2}**

15. 0.000007 **7×10^{-6}**

16. 0.000010035 **1.0035×10^{-5}**

17. 264.9 **2.649×10^2**

18. 150×10^2 **1.5×10^4**

Evaluate. Express each result in scientific and standard notation.

19. $(3.1 \times 10^7)(2 \times 10^{-5})$ **6.2×10^2; 620**

20. $(5 \times 10^{-2})(1.4 \times 10^{-4})$ **7.0×10^{-6}; 0.000007**

21. $(3 \times 10^3)(4.2 \times 10^{-1})$ **1.26×10^3; 1260**

22. $(3 \times 10^{-2})(5.2 \times 10^9)$ **1.56×10^8; 156,000,000**

23. $(2.4 \times 10^2)(4 \times 10^{-10})$ **9.6×10^{-8}; 0.000000096**

24. $(1.5 \times 10^{-4})(7 \times 10^{-5})$ **1.05×10^{-8}; 0.0000000105**

25. $\frac{5.1 \times 10^6}{1.5 \times 10^2}$ **3.4×10^4; 34,000**

26. $\frac{7.2 \times 10^{-5}}{4 \times 10^{-3}}$ **1.8×10^{-2}; 0.018**

Lesson 8-3

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8-3 Practice (Average)

Scientific Notation

Express each number in standard notation.

1. 7.3×10^7 **73,000,000**

2. 2.9×10^3 **2900**

3. 9.821×10^{12} **9,821,000,000,000**

4. 3.54×10^{-1} **0.354**

5. 7.3642×10^4 **73,642**

6. 4.268×10^{-6} **0.000004268**

PHYSICS Express the number in each statement in standard notation.

7. An electron has a negative charge of 1.6×10^{-19} Coulomb. **0.00000000000000000016**

8. In the middle layer of the sun's atmosphere, called the chromosphere, the temperature averages 2.78×10^4 degrees Celsius. **27,800**

Express each number in scientific notation.

9. 915,600,000,000 **9.156×10^{11}**

10. 6387 **6.387×10^3**

11. 845,320 **8.4532×10^5**

12. 0.00000000814 **8.14×10^{-9}**

13. 0.00009621 **9.621×10^{-5}**

14. 0.003157 **3.157×10^{-3}**

15. 30,620 **3.062×10^4**

16. 0.0000000000112 **1.12×10^{-11}**

17. 56×10^7 **5.6×10^8**

18. 4740×10^5 **4.74×10^8**

19. 0.076×10^{-3} **7.6×10^{-5}**

20. 0.0057×10^3 **5.7 or 5.7×10^0**

Evaluate. Express each result in scientific and standard notation.

21. $(5 \times 10^{-2})(2.3 \times 10^{12})$ **1.15×10^{11}; 115,000,000,000**

22. $(2.5 \times 10^{-3})(6 \times 10^{15})$ **1.5×10^{13}; 15,000,000,000,000**

23. $(3.9 \times 10^3)(4.2 \times 10^{-11})$ **1.638×10^{-7}; 0.0000001638**

24. $(4.6 \times 10^{-4})(3.1 \times 10^{-1})$ **1.426×10^{-4}; 0.0001426**

25. $\frac{3.12 \times 10^3}{1.56 \times 10^{-3}}$ **2.0×10^6; 2,000,000**

26. $\frac{6.72 \times 10^3}{4.2 \times 10^8}$ **1.6×10^{-5}; 0.000016**

27. $\frac{1.17 \times 10^2}{5 \times 10^{-1}}$ **2.34×10^2; 234**

28. $\frac{1.82 \times 10^5}{9.1 \times 10^7}$ **2.0×10^{-3}; 0.002**

29. $\frac{1.68 \times 10^4}{8.4 \times 10^{-4}}$ **2.0×10^7; 20,000,000**

30. $\frac{2.015 \times 10^{-3}}{3.1 \times 10^2}$ **6.5×10^{-6}; 0.0000065**

31. BIOLOGY A cubic millimeter of human blood contains about 5×10^6 red blood cells. An adult human body may contain about 5×10^6 cubic millimeters of blood. About how many red blood cells does such a human body contain? **about 2.5×10^{13} or 25 trillion**

32. POPULATION The population of Arizona is about 4.778×10^6 people. The land area is about 1.14×10^5 square miles. What is the population density per square mile? **about 42 people per square mile**

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8-3 Reading to Learn Mathematics

Scientific Notation

Pre-Activity **Why is scientific notation important in astronomy?**

Read the introduction to Lesson 8-3 at the top of page 425 in your textbook.

In the table, each mass is written as the product of a number and a power of 10. Look at the first factor in each product. How are these factors alike?
They are all greater than 1 and less than 10.

Reading the Lesson

1. Is the number 0.0543×10^4 in scientific notation? Explain.
No; the first factor is less than 1.

2. Complete each sentence to change from scientific notation to standard notation.
   a. To express 3.64×10^6 in standard notation, move the decimal point **6** places to the **right**.
   b. To express 7.825×10^{-3} in standard notation, move the decimal point **3** places to the **left**.

3. Complete each sentence to change from standard notation to scientific notation.
   a. To express 0.0007865 in scientific notation, move the decimal point **4** places to the right and write **7.865×10^{-4}**.
   b. To express 54,000,000,000 in scientific notation, move the decimal point **10** places to the left and write **5.4×10^{10}**.

4. Write *positive* or *negative* to complete each sentence.
   a. **Positive** powers of 10 are used to express very large numbers in scientific notation.
   b. **Negative** powers of 10 are used to express very small numbers in scientific notation.

Helping You Remember

5. Describe the method you would use to estimate how many times greater the mass of Saturn is than the mass of Pluto.
Divide 5.69×10^{26} by 1.27×10^{22}. Since $5.69 \div 1.27 \approx 4.48$ and $10^{26} \div 10^{22}$ is 10^4, the mass of Mars is about 4.48×10^4 times the mass of Pluto.

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8-3 Enrichment

Converting Metric Units

Scientific notation is convenient to use for unit conversions in the metric system.

Example 1 **How many kilometers are there in 4,300,000 meters?**

Divide the measure by the number of meters (1000) in one kilometer. Express both numbers in scientific notation.

$$\frac{4.3 \times 10^6}{1 \times 10^3} = 4.3 \times 10^3$$

The answer is 4.3 3 103 km.

Example 2 **Convert 3700 grams into milligrams.**

Multiply by the number of milligrams (1000) in 1 gram.

$(3.7 \times 10^3)(1 \times 10^3) = 3.7 \times 10^6$

There are 3.7×10^6 mg in 3700 g.

Complete the following. Express each answer in scientific notation.

1. 250,000 m = **2.5×10^2** km
2. 375 km = **3.75×10^5** m
3. 247 m = **2.47×10^4** cm
4. 5000 m = **5×10^6** mm
5. 0.0004 km = **4×10^{-1}** m
6. 0.01 mm = **1×10^{-5}** m
7. 6000 m = **6×10^6** mm
8. 340 cm = **3.4×10^{-3}** km
9. 52,000 mg = **5.2×10^1** g
10. 420 kL = **4.2×10^5** L

Solve.

11. The planet Mars has a diameter of 6.76×10^3 km. What is the diameter of Mars in meters? Express the answer in both scientific and decimal notation. **6,760,000 m; 6.76×10^6 m**

12. The distance from earth to the sun is 149,590,000 km. Light travels 3.0×10^8 meters per second. How long does it take light from the sun to reach the earth in minutes? Round to the nearest hundredth. **8.31 min**

13. A light-year is the distance that light travels in one year. (See Exercise 12.) How far is a light year in kilometers? Express your answer in scientific notation. Round to the nearest hundredth. **9.46×10^{12} km**

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8-4 Study Guide and Intervention

Polynomials

Degree of a Polynomial A **polynomial** is a monomial or a sum of monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. Polynomials with more than three terms have no special name. The **degree** of a monomial is the sum of the exponents of all its variables. The **degree of the polynomial** is the same as the degree of the monomial term with the highest degree.

Example **State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*. Then give the degree of the polynomial.**

Expression	Polynomial?	Monomial, Binomial, or Trinomial?	Degree of the Polynomial
$3x - 7xyz$	Yes. $3x - 7xyz = 3x + (-7xyz)$, which is the sum of two monomials	binomial	3
-25	Yes. -25 is a real number.	monomial	0
$7n^3 + 3n^{-4}$	No. $3n^{-4} = \frac{3}{n^4}$, which is not a monomial	none of these	—
$9x^3 + 4x + x + 4 + 2x$	Yes. The expression simplifies to $9x^3 + 7x + 4$, which is the sum of three monomials	trinomial	3

Exercises

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, *binomial*, or *trinomial*.

1. 36 yes; monomial
2. $\frac{3}{q^2} + 5$ no
3. $7x - x + 5$ yes; binomial
4. $8g^2h - 7gh + 2$ yes; trinomial
5. $\frac{1}{4y^2} + 5y - 8$ no
6. $6x + x^2$ yes; binomial

Find the degree of each polynomial.

7. $4x^2y^3z$ 6
8. $-2abc$ 3
9. $15m$ 1
10. $s + 5t$ 1
11. 22 0
12. $18x^2 + 4yz - 10y$ 2
13. $x^4 - 6x^2 - 2x^3 - 10$ 4
14. $2x^3y^2 - 4xy^3$ 5
15. $-2r^8s^4 + 7r^2s - 4r^7s^6$ 13
16. $9x^2 + yz^8$ 9
17. $8b + bc^5$ 6
18. $4x^4y - 8zx^2 + 2x^5$ 5
19. $4x^2 - 1$ 2
20. $9abc + bc - d^5$ 5
21. $h^3m + 6h^4m^2 - 7$ 6

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8-4 Study Guide and Intervention *(continued)*

Polynomials

Write Polynomials in Order The terms of a polynomial are usually arranged so that the powers of one variable are in **ascending** (increasing) order or **descending** (decreasing) order.

Example 1 **Arrange the terms of each polynomial so that the powers of x are in ascending order.**

a. $x^4 - x^2 + 5x^3$
$-x^2 + 5x^3 + x^4$

b. $8x^3y - y^2 + 6x^2y + xy^2$
$-y^2 + xy^2 + 6x^2y + 8x^3y$

Example 2 **Arrange the terms of each polynomial so that the powers of x are in descending order.**

a. $x^4 + 4x^5 - x^2$
$4x^5 + x^4 - x^2$

b. $-6xy + y^3 - x^2y^2 + x^4y^2$
$x^4y^2 - x^2y^2 - 6xy + y^3$

Exercises

Arrange the terms of each polynomial so that the powers of x are in ascending order.

1. $5x + x^2 + 6$
$6 + 5x + x^2$
2. $6x + 9 - 4x^2$
$9 + 6x - 4x^2$
3. $4xy + 2y + 6x^2$
$2y + 4xy + 6x^2$
4. $6y^2x - 6x^2y + 2$
$2 + 6y^2x - 6x^2y$
5. $x^4 + x^3 + x^2$
$x^2 + x^3 + x^4$
6. $2x^3 - x + 3x^7$
$-x + 2x^3 + 3x^7$
7. $-5cx + 10c^2x^3 + 15cx^2$
$-5cx + 15cx^2 + 10c^2x^3$
8. $-4nx - 5n^3x^3 + 5$
$5 - 4nx - 5n^3x^3$
9. $4xy + 2y + 5x^2$
$2y + 4xy + 5x^2$

Arrange the terms of each polynomial so that the powers of x are in descending order.

10. $2x + x^2 - 5$
$x^2 + 2x - 5$
11. $20x - 10x^2 + 5x^3$
$5x^3 - 10x^2 + 20x$
12. $x^2 + 4yx - 10x^5$
$-10x^5 + x^2 + 4yx$
13. $9bx + 3bx^2 - 6x^3$
$-6x^3 + 3bx^2 + 9bx$
14. $x^3 + x^5 - x^2$
$x^5 + x^3 - x^2$
15. $ax^2 + 8a^2x^5 - 4$
$8a^2x^5 + ax^2 - 4$
16. $3x^3y - 4xy^2 - x^4y^2 + y^5$
$-x^4y^2 + 3x^3y - 4xy^2 + y^5$
17. $x^4 + 4x^3 - 7x^5 + 1$
$-7x^5 + x^4 + 4x^3 + 1$
18. $-3x^6 - x^5 + 2x^8$
$2x^8 - 3x^6 - x^5$
19. $-15cx^2 + 8c^2x^5 + cx$
$8c^2x^5 - 15cx^2 + cx$
20. $24x^2y - 12x^3y^2 + 6x^4$
$6x^4 - 12x^3y^2 + 24x^2y$
21. $-15x^3 + 10x^4y^2 + 7xy^2$
$10x^4y^2 - 15x^3 + 7xy^2$

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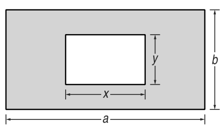
8-4 Skills Practice

Polynomials

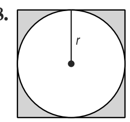
State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

1. $5mn + n^2$ yes; binomial
2. $4by + 2b - by$ yes; binomial
3. -32 yes; monomial
4. $\frac{3x}{7}$ yes; monomial
5. $5x^2 - 3x^{-4}$ no
6. $2c^2 + 8c + 9 - 3$ yes; trinomial

GEOMETRY Write a polynomial to represent the area of each shaded region.

7. 

$ab - xy$

8. 

$4r^2 - \pi r^2$

Find the degree of each polynomial.

9. 12 0
10. $3r^4$ 4
11. $b + 6$ 1
12. $4a^3 - 2a$ 3
13. $5abc - 2b^2 + 1$ 3
14. $8x^5y^4 - 2x^8$ 9

Arrange the terms of each polynomial so that the powers of *x* are in ascending order.

15. $3x + 1 + 2x^2$ $1 + 3x + 2x^2$
16. $5x - 6 + 3x^2$ $-6 + 5x + 3x^2$
17. $9x^2 + 2 + x^3 + x$ $2 + x + 9x^2 + x^3$
18. $-3 + 3x^3 - x^2 + 4x$ $-3 + 4x - x^2 + 3x^3$
19. $7r^5x + 21r^4 - r^2x^2 - 15x^3$ $21r^4 + 7r^5x - r^2x^2 - 15x^3$
20. $3a^2x^4 + 14a^2 - 10x^3 + ax^2$ $14a^2 + ax^2 - 10x^3 + 3a^2x^4$

Arrange the terms of each polynomial so that the powers of *x* are in descending order.

21. $x^2 + 3x^3 + 27 - x$ $3x^3 + x^2 - x + 27$
22. $25 - x^3 + x$ $-x^3 + x + 25$
23. $x - 3x^2 + 4 + 5x^3$ $5x^3 - 3x^2 + x + 4$
24. $x^2 + 64 - x + 7x^3$ $7x^3 + x^2 - x + 64$
25. $2cx + 32 - c^3x^2 + 6x^3$ $6x^3 - c^3x^2 + 2cx + 32$
26. $13 - x^3y^3 + x^2y^2 + x$ $-x^3y^3 + x^2y^2 + x + 13$

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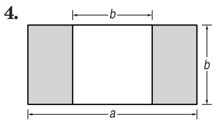
8-4 Practice (Average)

Polynomials

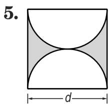
State whether each expression is a polynomial. If the expression is a polynomial, identify it as a *monomial*, a *binomial*, or a *trinomial*.

1. $7a^2b + 3b^2 - a^2b$ yes; binomial
2. $\frac{1}{5}y^3 + y^2 - 9$ yes; trinomial
3. $6g^2h^3k$ yes; monomial

GEOMETRY Write a polynomial to represent the area of each shaded region.

4. 

$ab - b^2$

5. 

$d^2 - \frac{1}{4}\pi d^2$

Find the degree of each polynomial.

6. $x + 3x^4 - 21x^2 + x^3$ 4
7. $3g^2h^3 + g^3h$ 5
8. $-2x^2y + 3xy^3 + x^2$ 4
9. $5n^3m - 2m^3 + n^2m^4 + n^2$ 6
10. $a^3b^2c + 2a^5c + b^3c^2$ 6
11. $10s^2t^2 + 4st^2 - 5s^3t^2$ 5

Arrange the terms of each polynomial so that the powers of *x* are in ascending order.

12. $8x^2 - 15 + 5x^5$ $-15 + 8x^2 + 5x^5$
13. $10bx - 7b^2 + x^4 + 4b^2x^3$ $-7b^2 + 10bx + 4b^2x^3 + x^4$
14. $-3x^3y + 8y^2 + xy^4$ $8y^2 + xy^4 - 3x^3y$
15. $7ax - 12 + 3ax^3 + a^2x^2$ $-12 + 7ax + a^2x^2 + 3ax^3$

Arrange the terms of each polynomial so that the powers of *x* are in descending order.

16. $13x^2 - 5 + 6x^3 - x$ $6x^3 + 13x^2 - x - 5$
17. $4x + 2x^5 - 6x^3 + 2$ $2x^5 - 6x^3 + 4x + 2$
18. $g^2x - 3gx^3 + 7g^3 + 4x^2$ $-3gx^3 + 4x^2 + g^2x + 7g^3$
19. $-11x^2y^3 + 6y - 2xy + 2x^4$ $2x^4 - 11x^2y^3 - 2xy + 6y$
20. $7a^2x^2 + 17 - a^3x^3 + 2ax$ $-a^3x^3 + 7a^2x^2 + 2ax + 17$
21. $12rx^3 + 9r^6 + r^2x + 8x^6$ $8x^6 + 12rx^3 + r^2x + 9r^6$

22. **MONEY** Write a polynomial to represent the value of t ten-dollar bills, f fifty-dollar bills, and h one-hundred-dollar bills. $10t + 50f + 100h$

23. **GRAVITY** The height above the ground of a ball thrown up with a velocity of 96 feet per second from a height of 6 feet is $6 + 96t - 16t^2$ feet, where t is the time in seconds. According to this model, how high is the ball after 7 seconds? Explain.
−106 ft; The height is negative because the model does not account for the ball hitting the ground when the height is 0 feet.

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8-4 Reading to Learn Mathematics

Polynomials

Pre-Activity **How are polynomials useful in modeling data?**

Read the introduction to Lesson 8-4 at the top of page 432 in your textbook.

- How many terms does $t^4 - 9t^3 + 26t - 18t + 76$ have?
  five
- What could you call a polynomial with just one term?
  a monomial

Reading the Lesson

1. What is the meaning of the prefixes *mono-*, *bi-*, and *tri-*?
***Mono-* means one, *bi-* means two, and *tri-* means three.**

2. Write examples of words that begin with the prefixes *mono-*, *bi-*, and *tri-*.
Sample answer: monocycle (one wheel), bicycle (two wheels), tricycle (three wheels)

3. Complete the table.

	monomial	binomial	trinomial	polynomial with more than three terms
Example	$3r^2t$	$2x^2 + 3x$	$5x^2 + 3x + 2$	$7s^2 + s^4 + 2s^3 - s + 5$
Number of Terms	**1**	**2**	**3**	**5**

4. What is the degree of the monomial $3xy^2z$? **4**

5. What is the degree of the polynomial $4x^4 + 2x^3y^3 + y^2 + 14$? Explain how you found your answer.
6; Since $0 + 4 = 4$, $4x^4$ has degree 4; since $0 + 3 + 3 = 6$, $2x^3y^3$ has degree 6; y^2 has degree 2; and 14 has degree 0. The highest degree of these terms is 6.

Helping You Remember

6. Use a dictionary to find the meaning of the terms *ascending* and *descending*. Write their meanings and then describe a situation in your everyday life that relates to them.
ascending: going, growing, or moving upward; descending: moving from a higher to a lower place; Sample answer: climbing stairs, hiking

Lesson 8-4

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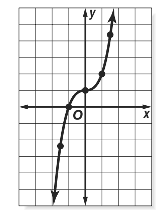
8-4 Enrichment

Polynomial Functions

Suppose a linear equation such as $23x + y = 4$ is solved for y. Then an equivalent equation, $y = 3x + 4$, is found. Expressed in this way, y is a function of x, or $f(x) = 3x + 4$. Notice that the right side of the equation is a binomial of degree 1.

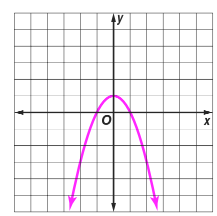
Higher-degree polynomials in x may also form functions. An example is $f(x) = x^3 + 1$, which is a polynomial function of degree 3. You can graph this function using a table of ordered pairs, as shown at the right.

x	y
$-1\frac{1}{2}$	$-2\frac{3}{8}$
-1	0
0	1
1	2
$1\frac{1}{2}$	$4\frac{3}{8}$

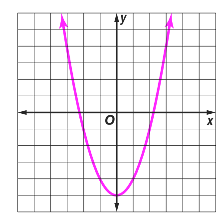


For each of the following polynomial functions, make a table of values for x and $y = f(x)$. Then draw the graph on the grid.

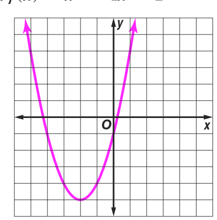
1. $f(x) = 1 - x^2$



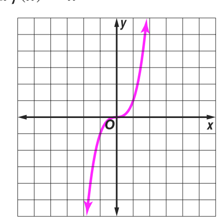
2. $f(x) = x^2 - 5$



3. $f(x) = x^2 + 4x - 1$



4. $f(x) = x^3$



Answers

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8-5 Study Guide and Intervention

Adding and Subtracting Polynomials

Add Polynomials To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms vertically. **Like terms** are monomial terms that are either identical or differ only in their coefficients, such as $3p$ and $-5p$ or $2x^2y$ and $8x^2y$.

Example 1 **Find $(2x^2 + x - 8) + (3x - 4x^2 + 2)$.**

Horizontal Method

Group like terms.

$(2x^2 + x - 8) + (3x - 4x^2 + 2)$
$= [(2x^2 + (-4x^2)] + (x + 3x) + [(-8) + 2)]$
$= -2x^2 + 4x - 6.$

The sum is $-2x^2 + 4x - 6$.

Example 2 **Find $(3x^2 + 5xy) + (xy + 2x^2)$.**

Vertical Method

Align like terms in columns and add.

$$\begin{array}{rl} & 3x^2 + 5xy \\ (+) & 2x^2 + xy \\ \hline & 5x^2 + 6xy \end{array}$$

Put the terms in descending order.

The sum is $5x^2 + 6xy$.

Exercises

Find each sum.

1. $(4a - 5) + (3a + 6)$
$7a + 1$

2. $(6x + 9) + (4x^2 - 7)$
$4x^2 + 6x + 2$

3. $(6xy + 2y + 6x) + (4xy - x)$
$10xy + 5x + 2y$

4. $(x^2 + y^2) + (-x^2 + y^2)$
$2y^2$

5. $(3p^2 - 2p + 3) + (p^2 - 7p + 7)$
$4p^2 - 9p + 10$

6. $(2x^2 + 5xy + 4y^2) + (-xy - 6x^2 + 2y^2)$
$-4x^2 + 4xy + 6y^2$

7. $(5p + 2q) + (2p^2 - 8q + 1)$
$2p^2 + 5p - 6q + 1$

8. $(4x^2 - x + 4) + (5x + 2x^2 + 2)$
$6x^2 + 4x + 6$

9. $(6x^2 + 3x) + (x^2 - 4x - 3)$
$7x^2 - x - 3$

10. $(x^2 + 2xy + y^2) + (x^2 - xy - 2y^2)$
$2x^2 + xy - y^2$

11. $(2a - 4b - c) + (-2a - b - 4c)$
$-5b - 5c$

12. $(6xy^2 + 4xy) + (2xy - 10xy^2 + y^2)$
$-4xy^2 + 6xy + y^2$

13. $(2p - 5q) + (3p + 6q) + (p - q)$
$6p$

14. $(2x^2 - 6) + (5x^2 + 2) + (-x^2 - 7)$
$6x^2 - 11$

15. $(3z^2 + 5z) + (z^2 + 2z) + (z - 4)$
$4z^2 + 8z - 4$

16. $(8x^2 + 4x + 3y^2 + y) + (6x^2 - x + 4y)$
$14x^2 + 3x + 3y^2 + 5y$

Lesson 8-5

NAME ______________________ DATE __________ PERIOD _____

8-5 Study Guide and Intervention *(continued)*

Adding and Subtracting Polynomials

Subtract Polynomials You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

Example **Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.**

Horizontal Method

Use additive inverses to rewrite as addition. Then group like terms.

$(3x^2 + 2x - 6) - (2x + x^2 + 3)$
$= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]$
$= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]$
$= 2x^2 + (-9)$
$= 2x^2 - 9$

The difference is $2x^2 - 9$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rl} & 3x^2 + 2x - 6 \\ (-) & x^2 + 2x + 3 \\ \hline \end{array}$$

$$\begin{array}{rl} & 3x^2 + 2x - 6 \\ (+) & -x^2 - 2x - 3 \\ \hline & 2x^2 - 9 \end{array}$$

The difference is $2x^2 - 9$.

Exercises

Find each difference.

1. $(3a - 5) - (5a + 1)$
$-2a - 6$

2. $(9x + 2) - (-3x^2 - 5)$
$3x^2 + 9x + 7$

3. $(9xy + y - 2x) - (6xy - 2x)$
$3xy + y$

4. $(x^2 + y^2) - (-x^2 + y^2)$
$2x^2$

5. $(6p^2 + 4p + 5) - (2p^2 - 5p + 1)$
$4p^2 + 9p + 4$

6. $(6x^2 + 5xy - 2y^2) - (-xy - 2x^2 - 4y^2)$
$8x^2 + 6xy + 2y^2$

7. $(8p - 5q) - (-6p^2 + 6q - 3)$
$6p^2 + 8p - 11q + 3$

8. $(8x^2 - 4x - 3) - (-2x - x^2 + 5)$
$9x^2 - 2x - 8$

9. $(3x^2 - 2x) - (3x^2 + 5x - 1)$
$-7x + 1$

10. $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$
$5x^2 + 4xy + 7y^2$

11. $(2h - 6j - 2k) - (-7h - 5j - 4k)$
$9h - j + 2k$

12. $(9xy^2 + 5xy) - (-2xy - 8xy^2)$
$17xy^2 + 7xy$

13. $(2a - 8b) - (-3a + 5b)$
$5a - 13b$

14. $(2x^2 - 8) - (-2x^2 - 6)$
$4x^2 - 2$

15. $(6z^2 + 4z + 2) - (4z^2 + z)$
$2z^2 + 3z + 2$

16. $(6x^2 - 5x + 1) - (-7x^2 - 2x + 4)$
$13x^2 - 3x - 3$

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8-5 Skills Practice

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(2x + 3y) + (4x + 9y)$ **$6x + 12y$**

2. $(6s + 5t) + (4t + 8s)$ **$14s + 9t$**

3. $(5a + 9b) - (2a + 4b)$ **$3a + 5b$**

4. $(11m - 7n) - (2m + 6n)$ **$9m - 13n$**

5. $(m^2 - m) + (2m + m^2)$ **$2m^2 + m$**

6. $(x^2 - 3x) - (2x^2 + 5x)$ **$-x^2 - 8x$**

7. $(d^2 - d + 5) - (2d + 5)$ **$d^2 - 3d$**

8. $(2e^2 - 5e) + (7e - 3e^2)$ **$-e^2 + 2e$**

9. $(5f + g - 2) + (-2f + 3)$ **$3f + g + 1$**

10. $(6k^2 + 2k + 9) + (4k^2 - 5k)$ **$10k^2 - 3k + 9$**

11. $(x^3 - x + 1) - (3x - 1)$ **$x^3 - 4x + 2$**

12. $(b^2 + ab - 2) - (2b^2 + 2ab)$ **$-b^2 - ab - 2$**

13. $(7z^2 + 4 - z) - (-5 + 3z^2)$ **$4z^2 - z + 9$**

14. $(5 + 4n + 2m) + (-6m - 8)$ **$-3 + 4n - 4m$**

15. $(4t^2 + 2) + (-4 + 2t)$ **$4t^2 + 2t - 2$**

16. $(3g^3 + 7g) - (4g + 8g^3)$ **$-5g^3 + 3g$**

17. $(2a^2 + 8a + 4) - (a^2 - 3)$ **$a^2 + 8a + 7$**

18. $(3x^2 - 7x + 5) - (-x^2 + 4x)$ **$4x^2 - 11x + 5$**

19. $(7z^2 + z + 1) - (-4z + 3z^2 - 3)$ **$4z^2 + 5z + 4$**

20. $(2c^2 + 7c + 4) + (c^2 + 1 - 9c)$ **$3c^2 - 2c + 5$**

21. $(n^2 + 3n + 2) - (2n^2 - 6n - 2)$ **$-n^2 + 9n + 4$**

22. $(a^2 + ab - 3b^2) + (b^2 + 4a^2 - ab)$ **$5a^2 - 2b^2$**

23. $(\ell^2 - 5\ell - 6) + (2\ell^2 + 5 + \ell)$ **$3\ell^2 - 4\ell - 1$**

24. $(2m^2 + 5m + 1) - (4m^2 - 3m - 3)$ **$-2m^2 + 8m + 4$**

25. $(x^2 - 6x + 2) - (-5x^2 + 7x - 4)$ **$6x^2 - 13x + 6$**

26. $(5b^2 - 9b - 5) + (b^2 - 6 + 2b)$ **$6b^2 - 7b - 11$**

27. $(2x^2 - 6x - 2) + (x^2 + 4x) + (3x^2 + x + 5)$ **$6x^2 - x + 3$**

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8-5 Practice (Average)

Adding and Subtracting Polynomials

Find each sum or difference.

1. $(4y + 5) + (-7y - 1)$ **$-3y + 4$**

2. $(-x^2 + 3x) - (5x + 2x^2)$ **$-3x^2 - 2x$**

3. $(4k^2 + 8k + 2) - (2k + 3)$ **$4k^2 + 6k - 1$**

4. $(2m^2 + 6m) + (m^2 - 5m + 7)$ **$3m^2 + m + 7$**

5. $(2w^2 - 3w + 1) + (4w - 7)$ **$2w^2 + w - 6$**

6. $(g^3 + 2g^2) - (6g - 4g^2 + 2g^3)$ **$-g^3 + 6g^2 - 6g$**

7. $(5a^2 + 6a + 2) - (7a^2 - 7a + 5)$ **$-2a^2 + 13a - 3$**

8. $(-4p^2 - p + 9) + (p^2 + 3p - 1)$ **$-3p^2 + 2p + 8$**

9. $(x^3 - 3x + 1) - (x^3 + 7 - 12x)$ **$9x - 6$**

10. $(6c^2 - c + 1) - (-4 + 2c^2 + 8c)$ **$4c^2 - 9c + 5$**

11. $(-b^3 + 8bc^2 + 5) - (7bc^2 - 2 + b^3)$ **$-2b^3 + bc^2 + 7$**

12. $(5n^2 - 3n + 2) + (-n + 2n^2 - 4)$ **$7n^2 - 4n - 2$**

13. $(4y^2 + 2y - 8) - (7y^2 + 4 - y)$ **$-3y^2 + 3y - 12$**

14. $(w^2 - 4w - 1) + (-5 + 5w^2 - 3w)$ **$6w^2 - 7w - 6$**

15. $(4u^2 - 2u - 3) + (3u^2 - u + 4)$ **$7u^2 - 3u + 1$**

16. $(5b^2 - 8 + 2b) - (b + 9b^2 + 5)$ **$-4b^2 + b - 13$**

17. $(4d^2 + 2d + 2) + (5d^2 - 2 - d)$ **$9d^2 + d$**

18. $(8x^2 + x - 6) - (-x^2 + 2x - 3)$ **$9x^2 - x - 3$**

19. $(3h^2 + 7h - 1) - (4h + 8h^2 + 1)$ **$-5h^2 + 3h - 2$**

20. $(4m^2 - 3m + 10) + (m^2 + m - 2)$ **$5m^2 - 2m + 8$**

21. $(x^2 + y^2 - 6) - (5x^2 - y^2 - 5)$ **$-4x^2 + 2y^2 - 1$**

22. $(7t^2 + 2 - t) + (t^2 - 7 - 2t)$ **$8t^2 - 3t - 5$**

23. $(k^3 - 2k^2 + 4k + 6) - (-4k + k^2 - 3)$ **$k^3 - 3k^2 + 8k + 9$**

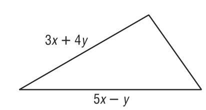
24. $(9j^2 + j + jk) + (-3j^2 - jk - 4j)$ **$6j^2 - 3j$**

25. $(2x + 6y - 3z) + (4x + 6z - 8y) + (x - 3y + z)$ **$7x - 5y + 4z$**

26. $(6f^2 - 7f - 3) - (5f^2 - 1 + 2f) - (2f^2 - 3 + f)$ **$-f^2 - 10f + 1$**

27. BUSINESS The polynomial $s^3 - 70s^2 + 1500s - 10{,}800$ models the profit a company makes on selling an item at a price s. A second item sold at the same price brings in a profit of $s^3 - 30s^2 + 450s - 5000$. Write a polynomial that expresses the total profit from the sale of both items. **$2s^3 - 100s^2 + 1950s - 15{,}800$**

28. GEOMETRY The measures of two sides of a triangle are given. If P is the perimeter, and $P = 10x + 5y$, find the measure of the third side. **$2x + 2y$**



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8-5 Reading to Learn Mathematics

Adding and Subtracting Polynomials

Pre-Activity **How can adding polynomials help you model sales?**

Read the introduction to Lesson 8-5 at the top of page 439 in your textbook.

What operation would you use to find how much more the traditional toy sales R were than the video games sales V?

subtraction

Reading the Lesson

1. Use the example $(-3x^3 + 4x^2 + 5x + 1) + (-5x^3 - 2x^2 + 2x - 7)$.

a. Show what is meant by grouping like terms horizontally.

$[-3x^3 + (-5x^3)] + [(4x^2 + (-2x^2)] + (5x + 2x) + [1 + (-7)]$

b. Show what is meant by aligning like terms vertically.

$$\begin{array}{r} -3x^3 + 4x^2 + 5x + 1 \\ \underline{(+)\ -5x^3 - 2x^2 + 2x - 7} \end{array}$$

c. Choose one method, then add the polynomials.

$-8x^3 + 2x^2 + 7x - 6$

2. How is subtracting a polynomial like subtracting a rational number?

You subtract by adding the additive inverse.

3. An algebra student got the following exercise wrong on his homework. What was his error?

$(3x^5 - 3x^4 + 2x^3 - 4x^2 + 5) - (2x^5 - x^3 + 2x^2 - 4)$

$= [3x^5 + (-2x^5)] + (-3x^4) + [2x^3 + (-x^3)] + [-4x^2 + (-2x^2)] + (5 + 4)$

$= x^5 - 3x^4 + x^3 - 6x^2 + 9$

He did not add the additive inverse of $-x^3$.

Helping You Remember

4. How is adding and subtracting polynomials vertically like adding and subtracting decimals vertically?

Aligning like terms when adding or subtracting polynomials is like using place value to align digits when you add or subtract decimals.

Lesson 8-5

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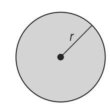
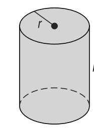
8-5 Enrichment

Circular Areas and Volumes

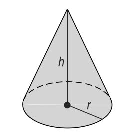
Area of Circle $A = \pi r^2$



Volume of Cylinder $V = \pi r^2 h$

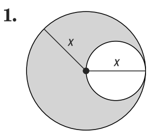


Volume of Cone $V = \frac{1}{3}\pi r^2 h$



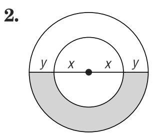
Write an algebraic expression for each shaded area. (Recall that the diameter of a circle is twice its radius.)

1.



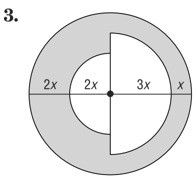
$\pi x^2 - \pi\left(\frac{x}{2}\right)^2 = \pi x^2$

2.



$\frac{\pi}{2}(y^2 + 2xy)$

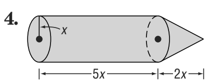
3.



$\frac{19}{2}\pi x^2$

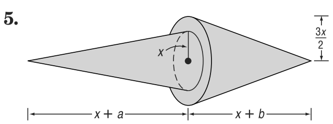
Write an algebraic expression for the total volume of each figure.

4.



$5\frac{2}{3}\pi x^3$

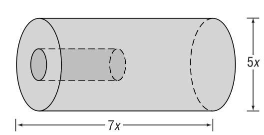
5.



$\frac{\pi}{12}[13x^3 + (4a + 9b)x^2]$

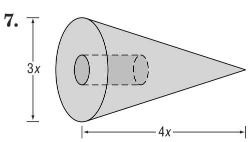
Each figure has a cylindrical hole with a radius of 2 inches and a height of 5 inches. Find each volume.

6.



$\frac{175\pi}{4}x^3 - 20\pi \text{ in}^3$

7.



$3\pi x^3 - 20\pi \text{ in}^3$

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8-6 Study Guide and Intervention

Multiplying a Polynomial by a Monomial

Product of Monomial and Polynomial The Distributive Property can be used to multiply a polynomial by a monomial. You can multiply horizontally or vertically. Sometimes multiplying results in like terms. The products can be simplified by combining like terms.

Example 1 **Find $-3x^2(4x^2 + 6x - 8)$.**

Horizontal Method

$-3x^2(4x^2 + 6x - 8)$
$= -3x^2(4x^2) + (-3x^2)(6x) - (-3x^2)(8)$
$= -12x^4 + (-18x^3) - (-24x^2)$
$= -12x^4 - 18x^3 + 24x^2$

Vertical Method

$$\begin{array}{r} 4x^2 + 6x - 8 \\ (\times) \qquad -3x^2 \\ \hline -12x^4 - 18x^3 + 24x^2 \end{array}$$

The product is $-12x^4 - 18x^3 + 24x^2$.

Example 2 **Simplify $-2(4x^2 + 5x) - x(x^2 + 6x)$.**

$-2(4x^2 + 5x) - x(x^2 + 6x)$
$= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x)$
$= -8x^2 + (-10x) + (-x^3) + (-6x^2)$
$= (-x^3) + [-8x^2 + (-6x^2)] + (-10x)$
$= -x^3 - 14x^2 - 10x$

Exercises

Find each product.

1. $x(5x + x^2)$ $5x^2 + x^3$

2. $x(4x^2 + 3x + 2)$ $4x^3 + 3x^2 + 2x$

3. $-2xy(2y + 4x^2)$ $-4xy^2 - 8x^3y$

4. $-2g(g^2 - 2g + 2)$ $-2g^3 + 4g^2 - 4g$

5. $3x(x^4 + x^3 + x^2)$ $3x^5 + 3x^4 + 3x^3$

6. $-4x(2x^3 - 2x + 3)$ $-8x^4 + 8x^2 - 12x$

7. $-4cx(10 + 3x)$ $-40cx - 12cx^2$

8. $3y(-4x - 6x^3 - 2y)$ $-12xy - 18x^3y - 6y^2$

9. $2x^2y^2(3xy + 2y + 5x)$ $6x^3y^3 + 4x^2y^3 + 10x^3y^2$

Simplify.

10. $x(3x - 4) - 5x$ $3x^2 - 9x$

11. $-x(2x^2 - 4x) - 6x^2$ $-2x^3 - 2x^2$

12. $6a(2a - b) + 2a(-4a + 5b)$ $4a^2 + 4ab$

13. $4r(2r^2 - 3r + 5) + 6r(4r^2 + 2r + 8)$ $32r^3 + 68r$

14. $4n(3n^2 + n - 4) - n(3 - n)$ $12n^3 + 5n^2 - 19n$

15. $2b(b^2 + 4b + 8) - 3b(3b^2 + 9b - 18)$ $-7b^3 - 19b^2 + 70b$

16. $-2z(4z^2 - 3z + 1) - z(3z^2 + 2z - 1)$ $-11z^3 + 4z^2 - z$

17. $2(4x^2 - 2x) - 3(-6x^2 + 4) + 2x(x - 1)$ $28x^2 - 6x - 12$

Lesson 8-6

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8-6 Study Guide and Intervention (continued)

Multiplying a Polynomial by a Monomial

Solve Equations with Polynomial Expressions Many equations contain polynomials that must be added, subtracted, or multiplied before the equation can be solved.

Example **Solve $4(n - 2) + 5n = 6(3 - n) + 19$.**

$4(n - 2) + 5n = 6(3 - n) + 19$	Original equation
$4n - 8 + 5n = 18 - 6n + 19$	Distributive Property
$9n - 8 = 37 - 6n$	Combine like terms.
$15n - 8 = 37$	Add 6*n* to both sides.
$15n = 45$	Add 8 to both sides.
$n = 3$	Divide each side by 15.

The solution is 3.

Exercises

Solve each equation.

1. $2(a - 3) = 3(-2a + 6)$ 3

2. $3(x + 5) - 6 = 18$ 3

3. $3x(x - 5) - 3x^2 = -30$ 2

4. $6(x^2 + 2x) = 2(3x^2 + 12)$ 2

5. $4(2p + 1) - 12p = 2(8p + 12)$ -1

6. $2(6x + 4) + 2 = 4(x - 4)$ $-3\frac{1}{4}$

7. $-2(4y - 3) - 8y + 6 = 4(y - 2)$ 1

8. $c(c + 2) - c(c - 6) = 10c - 12$ 6

9. $3(x^2 - 2x) = 3x^2 + 5x - 11$ 1

10. $2(4x + 3) + 2 = -4(x + 1)$ -1

11. $3(2h - 6) - (2h + 1) = 9$ 7

12. $3(y + 5) - (4y - 8) = -2y + 10$ -13

13. $3(2a - 6) - (-3a - 1) = 4a - 2$ $2\frac{3}{5}$

14. $5(2x^2 - 1) - (10x^2 - 6) = -(x + 2)$ -3

15. $3(x + 2) + 2(x + 1) = -5(x - 3)$ $\frac{7}{10}$

16. $4(3p^2 + 2p) - 12p^2 = 2(8p + 6)$ $-\frac{3}{2}$

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8-6 Skills Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. $a(4a + 3)$ $4a^2 + 3a$

2. $-c(11c + 4)$ $-11c^2 - 4c$

3. $x(2x - 5)$ $2x^2 - 5x$

4. $2y(y - 4)$ $2y^2 - 8y$

5. $-3n(n^2 + 2n)$ $-3n^3 - 6n^2$

6. $4h(3h - 5)$ $12h^2 - 20h$

7. $3x(5x^2 - x + 4)$ $15x^3 - 3x^2 + 12x$

8. $7c(5 - 2c^2 + c^3)$ $35c - 14c^3 + 7c^4$

9. $-4b(1 - 9b - 2b^2)$ $-4b + 36b^2 + 8b^3$

10. $6y(-5 - y + 4y^2)$ $-30y - 6y^2 + 24y^3$

11. $2m^2(2m^2 + 3m - 5)$ $4m^4 + 6m^3 - 10m^2$

12. $-3n^2(-2n^2 + 3n + 4)$ $6n^4 - 9n^3 - 12n^2$

Simplify.

13. $w(3w + 2) + 5w$ $3w^2 + 7w$

14. $f(5f - 3) - 2f$ $5f^2 - 5f$

15. $-p(2p - 8) - 5p$ $-2p^2 + 3p$

16. $y^2(-4y + 5) - 6y^2$ $-4y^3 - y^2$

17. $2x(3x^2 + 4) - 3x^3$ $3x^3 + 8x$

18. $4a(5a^2 - 4) + 9a$ $20a^3 - 7a$

19. $4b(-5b - 3) - 2(b^2 - 7b - 4)$ $-22b^2 + 2b + 8$

20. $3m(3m + 6) - 3(m^2 + 4m + 1)$ $6m^2 + 6m - 3$

Solve each equation.

21. $3(a + 2) + 5 = 2a + 4$ -7

22. $2(4x + 2) - 8 = 4(x + 3)$ 4

23. $5(y + 1) + 2 = 4(y + 2) - 6$ -5

24. $4(b + 6) = 2(b + 5) + 2$ -6

25. $6(m - 2) + 14 = 3(m + 2) - 10$ -2

26. $3(c + 5) - 2 = 2(c + 6) + 2$ 1

Lesson 8-6

NAME ______________________ DATE __________ PERIOD _____

8-6 Practice (Average)

Multiplying a Polynomial by a Monomial

Find each product.

1. $2h(-7h^2 - 4h)$ $-14h^3 - 8h^2$

2. $6pq(3p^2 + 4q)$ $18p^3q + 24pq^2$

3. $-2u^2n(4u - 2n)$ $-8u^3n + 4u^2n^2$

4. $5jk(3jk + 2k)$ $15j^2k^2 + 10jk^2$

5. $-3rs(-2s^2 + 3r)$ $6rs^3 - 9r^2s$

6. $4mg^2(2mg + 4g)$ $8m^2g^3 + 16mg^3$

7. $-\frac{1}{4}m(8m^2 + m - 7)$ $-2m^3 - \frac{1}{4}m^2 + \frac{7}{4}m$

8. $-\frac{2}{3}n^2(-9n^2 + 3n + 6)$ $6n^4 - 2n^3 - 4n^2$

Simplify.

9. $-2\ell(3\ell - 4) + 7\ell$ $-6\ell^2 + 15\ell$

10. $5w(-7w + 3) + 2w(-2w^2 + 19w + 2)$ $-4w^3 + 3w^2 + 19w$

11. $6t(2t - 3) - 5(2t^2 + 9t - 3)$ $2t^2 - 63t + 15$

12. $-2(3m^3 + 5m + 6) + 3m(2m^2 + 3m + 1)$ $9m^2 - 7m - 12$

13. $-3g(7g - 2) + 3(g^2 + 2g + 1) - 3g(-5g + 3)$ $-3g^2 + 3g + 3$

14. $4z^2(z - 7) - 5z(z^2 - 2z - 2) + 3z(4z - 2)$ $-z^3 - 6z^2 + 4z$

Solve each equation.

15. $5(2s - 1) + 3 = 3(3s + 2)$ 8

16. $3(3u + 2) + 5 = 2(2u - 2)$ -3

17. $4(8n + 3) - 5 = 2(6n + 8) + 1$ $\frac{1}{2}$

18. $8(3b + 1) = 4(b + 3) - 9$ $-\frac{1}{4}$

19. $h(h - 3) - 2h = h(h - 2) - 12$ 4

20. $w(w + 6) + 4w = -7 + w(w + 9)$ -7

21. $t(t + 4) - 1 = t(t + 2) + 2$ $\frac{3}{2}$

22. $u(u - 5) + 8u = u(u + 2) - 4$ -4

23. NUMBER THEORY Let x be an integer. What is the product of twice the integer added to three times the next consecutive integer? $5x + 3$

INVESTMENTS For Exercises 24–26, use the following information.

Kent invested \$5,000 in a retirement plan. He allocated x dollars of the money to a bond account that earns 4% interest per year and the rest to a traditional account that earns 5% interest per year.

24. Write an expression that represents the amount of money invested in the traditional account. $5,000 - x$

25. Write a polynomial model in simplest form for the total amount of money T Kent has invested after one year. (*Hint:* Each account has $A + IA$ dollars, where A is the original amount in the account and I is its interest rate.) $T = 5,250 - 0.01x$

26. If Kent put \$500 in the bond account, how much money does he have in his retirement plan after one year? \$5,245

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8-6 Reading to Learn Mathematics

Multiplying a Polynomial by a Monomial

Pre-Activity **How is finding the product of a monomial and a polynomial related to finding the area of a rectangle?**

Read the introduction to Lesson 8-6 at the top of page 444 in your textbook.

You may recall that the formula for the area of a rectangle is $A = \ell w$. In this rectangle, ℓ = **$x + 3$** and w = **$2x$**. How would you substitute these values in the area formula?

$A = (x + 3)(2x)$

Reading the Lesson

1. Refer to Lesson 8-6.

a. How is the Distributive Property used to multiply a polynomial by a monomial?

The monomial is multiplied by each term in the polynomial.

b. Use the Distributive Property to complete the following.

$2y^2(3y^2 + 2y - 7) = 2y^2(\underline{3y^2}) + 2y^2(\underline{2y}) - 2y^2(\underline{7})$

$= \underline{6y^4} + \underline{4y^3} - \underline{14y^2}$

$-3x^3(x^3 - 2x^2 + 3) = \underline{-3x^3(x^3)} - \underline{(-3x^3)(2x^2)} + \underline{(-3x^3)(3)}$

$= \underline{-3x^6} + \underline{6x^5} - \underline{9x^3}$

2. What is the difference between simplifying an expression and solving an equation?

Simplifying an expression is combining like terms. Solving an equation is finding the value of the variable that makes the equation true.

Helping You Remember

3. Use the equation $2x(x - 5) + 3x(x + 3) = 5x(x + 7) - 9$ to show how you would explain the process of solving equations with polynomial expressions to another algebra student.

Use the Distributive Property.	$2x^2 - 10x + 3x^2 + 9x = 5x^2 + 35x - 9$
Combine like terms.	$5x^2 - x = 5x^2 + 35x - 9$
Subtract $5x^2$ from both sides.	$-x = 35x - 9$
Subtract $35x$ from both sides.	$-36x = -9$
Divide each side by −36.	$x = 0.25$

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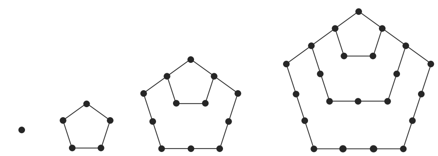
8-6 Enrichment

Figurate Numbers

The numbers below are called **pentagonal numbers**. They are the numbers of dots or disks that can be arranged as pentagons.



1. Find the product $\frac{1}{2}n(3n - 1)$. **$\frac{3n^2}{2} - \frac{n}{2}$**

2. Evaluate the product in Exercise 1 for values of n from 1 through 4. **1, 5, 12, 22**

3. What do you notice? **They are the first four pentagonal numbers.**

4. Find the next six pentagonal numbers. **35, 51, 70, 92, 117, 145**

5. Find the product $\frac{1}{2}n(n + 1)$. **$\frac{n^2}{2} + \frac{n}{2}$**

6. Evaluate the product in Exercise 5 for values of n from 1 through 5. On another sheet of paper, make drawings to show why these numbers are called the triangular numbers. **1, 3, 6, 10, 15**

7. Find the product $n(2n - 1)$. **$2n^2 - n$**

8. Evaluate the product in Exercise 7 for values of n from 1 through 5. Draw these hexagonal numbers. **1, 6, 15, 28, 45**

9. Find the first 5 square numbers. Also, write the general expression for any square number. **1, 4, 9, 16, 25; n^2**

The numbers you have explored above are called the plane figurate numbers because they can be arranged to make geometric figures. You can also create solid figurate numbers.

10. If you pile 10 oranges into a pyramid with a triangle as a base, you get one of the tetrahedral numbers. How many layers are there in the pyramid? How many oranges are there in the bottom layers? **3 layers; 6**

11. Evaluate the expression $\frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$ for values of n from 1 through 5 to find the first five tetrahedral numbers. **1, 4, 10, 20, 35**

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8-7 Study Guide and Intervention

Multiplying Polynomials

Multiply Binomials To multiply two binomials, you can apply the Distributive Property twice. A useful way to keep track of terms in the product is to use the FOIL method as illustrated in Example 2.

Example 1 **Find $(x + 3)(x - 4)$.**

Horizontal Method

$(x + 3)(x - 4)$
$= x(x - 4) + 3(x - 4)$
$= (x)(x) + x(-4) + 3(x) + 3(-4)$
$= x^2 - 4x + 3x - 12$
$= x^2 - x - 12$

Vertical Method

$$\begin{array}{r} x + 3 \\ (\times) \quad x - 4 \\ \hline -4x - 12 \\ x^2 + 3x \quad\quad \\ \hline x^2 - x - 12 \end{array}$$

The product is $x^2 - x - 12$.

Example 2 **Find $(x - 2)(x + 5)$ using the FOIL method.**

$(x - 2)(x + 5)$
First, Outer, Inner, Last
$= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)$
$= x^2 + 5x + (-2x) - 10$
$= x^2 + 3x - 10$

The product is $x^2 + 3x - 10$.

Exercises

Find each product.

1. $(x + 2)(x + 3)$ $x^2 + 5x + 6$

2. $(x - 4)(x + 1)$ $x^2 - 3x - 4$

3. $(x - 6)(x - 2)$ $x^2 - 8x + 12$

4. $(p - 4)(p + 2)$ $p^2 - 2p - 8$

5. $(y + 5)(y + 2)$ $y^2 + 7y + 10$

6. $(2x - 1)(x + 5)$ $2x^2 + 9x - 5$

7. $(3n - 4)(3n - 4)$ $9n^2 - 24n + 16$

8. $(8m - 2)(8m + 2)$ $64m^2 - 4$

9. $(k + 4)(5k - 1)$ $5k^2 + 19k - 4$

10. $(3x + 1)(4x + 3)$ $12x^2 + 13x + 3$

11. $(x - 8)(-3x + 1)$ $-3x^2 + 25x - 8$

12. $(5t + 4)(2t - 6)$ $10t^2 - 22t - 24$

13. $(5m - 3n)(4m - 2n)$ $20m^2 - 22mn + 6n^2$

14. $(a - 3b)(2a - 5b)$ $2a^2 - 11ab + 15b^2$

15. $(8x - 5)(8x + 5)$ $64x^2 - 25$

16. $(2n - 4)(2n + 5)$ $4n^2 + 2n - 20$

17. $(4m - 3)(5m - 5)$ $20m^2 - 35m + 15$

18. $(7g - 4)(7g + 4)$ $49g^2 - 16$

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8-7 Study Guide and Intervention *(continued)*

Multiplying Polynomials

Multiply Polynomials The Distributive Property can be used to multiply any two polynomials.

Example **Find $(3x + 2)(2x^2 - 4x + 5)$.**

$(3x + 2)(2x^2 - 4x + 5)$
$= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5)$ Distributive Property
$= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10$ Distributive Property
$= 6x^3 - 8x^2 + 7x + 10$ Combine like terms.

The product is $6x^3 - 8x^2 + 7x + 10$.

Exercises

Find each product.

1. $(x + 2)(x^2 - 2x + 1)$ $x^3 - 3x + 2$

2. $(x + 3)(2x^2 + x - 3)$ $2x^3 + 7x^2 - 9$

3. $(2x - 1)(x^2 - x + 2)$ $2x^3 - 3x^2 + 5x - 2$

4. $(p - 3)(p^2 - 4p + 2)$ $p^3 - 7p^2 + 14p - 6$

5. $(3k + 2)(k^2 + k - 4)$ $3k^3 + 5k^2 - 10k - 8$

6. $(2t + 1)(10t^2 - 2t - 4)$ $20t^3 + 6t^2 - 10t - 4$

7. $(3n - 4)(n^2 + 5n - 4)$ $3n^3 + 11n^2 - 32n + 16$

8. $(8x - 2)(3x^2 + 2x - 1)$ $24x^3 + 10x^2 - 12x + 2$

9. $(2a + 4)(2a^2 - 8a + 3)$ $4a^3 - 8a^2 - 26a + 12$

10. $(3x - 4)(2x^2 + 3x + 3)$ $6x^3 + x^2 - 3x - 12$

11. $(n^2 + 2n - 1)(n^2 + n + 2)$ $n^4 + 3n^3 + 3n^2 + 3n - 2$

12. $(t^2 + 4t - 1)(2t^2 - t - 3)$ $2t^4 + 7t^3 - 9t^2 - 11t + 3$

13. $(y^2 - 5y + 3)(2y^2 + 7y - 4)$ $2y^4 - 3y^3 - 33y^2 + 41y - 12$

14. $(3b^2 - 2b + 1)(2b^2 - 3b - 4)$ $6b^4 - 13b^3 - 4b^2 + 5b - 4$

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8-7 Skills Practice

Multiplying Polynomials

Find each product.

1. $(m + 4)(m + 1)$ $m^2 + 5m + 4$

2. $(x + 2)(x + 2)$ $x^2 + 4x + 4$

3. $(b + 3)(b + 4)$ $b^2 + 7b + 12$

4. $(t + 4)(t - 3)$ $t^2 + t - 12$

5. $(r + 1)(r - 2)$ $r^2 - r - 2$

6. $(z - 5)(z + 1)$ $z^2 - 4z - 5$

7. $(3c + 1)(c - 2)$ $3c^2 - 5c - 2$

8. $(2x - 6)(x + 3)$ $2x^2 - 18$

9. $(d - 1)(5d - 4)$ $5d^2 - 9d + 4$

10. $(2\ell + 5)(\ell - 4)$ $2\ell^2 - 3\ell - 20$

11. $(3n - 7)(n + 3)$ $3n^2 + 2n - 21$

12. $(q + 5)(5q - 1)$ $5q^2 + 24q - 5$

13. $(3b + 3)(3b - 2)$ $9b^2 + 3b - 6$

14. $(2m + 2)(3m - 3)$ $6m^2 - 6$

15. $(4c + 1)(2c + 1)$ $8c^2 + 6c + 1$

16. $(5a - 2)(2a - 3)$ $10a^2 - 19a + 6$

17. $(4h - 2)(4h - 1)$ $16h^2 - 12h + 2$

18. $(x - y)(2x - y)$ $2x^2 - 3xy + y^2$

19. $(e + 4)(e^2 + 3e - 6)$ $e^3 + 7e^2 + 6e - 24$

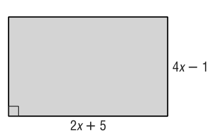
20. $(t + 1)(t^2 + 2t + 4)$ $t^3 + 3t^2 + 6t + 4$

21. $(k + 4)(k^2 + 3k - 6)$ $k^3 + 7k^2 + 6k - 24$

22. $(m + 3)(m^2 + 3m + 5)$ $m^3 + 6m^2 + 14m + 15$

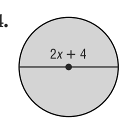
GEOMETRY Write an expression to represent the area of each figure.

23.



$8x^2 + 18x - 5$ units^2

24.



$(x^2 + 4x + 4)\pi$ units^2

Lesson 8-7

NAME ______________________ DATE ________ PERIOD _____

8-7 Practice (Average)

Multiplying Polynomials

Find each product.

1. $(q + 6)(q + 5)$ $q^2 + 11q + 30$

2. $(x + 7)(x + 4)$ $x^2 + 11x + 28$

3. $(s + 5)(s - 6)$ $s^2 - s - 30$

4. $(n - 4)(n - 6)$ $n^2 - 10n + 24$

5. $(a - 5)(a - 8)$ $a^2 - 13a + 40$

6. $(w - 6)(w - 9)$ $w^2 - 15w + 54$

7. $(4c + 6)(c - 4)$ $4c^2 - 10c - 24$

8. $(2x - 9)(2x + 4)$ $4x^2 - 10x - 36$

9. $(4d - 5)(2d - 3)$ $8d^2 - 22d + 15$

10. $(4b + 3)(3b - 4)$ $12b^2 - 7b - 12$

11. $(4m + 2)(4m - 3)$ $16m^2 - 4m - 6$

12. $(5c - 5)(7c + 9)$ $35c^2 + 10c - 45$

13. $(6a - 3)(7a - 4)$ $42a^2 - 45a + 12$

14. $(6h - 3)(4h - 2)$ $24h^2 - 24h + 6$

15. $(2x - 2)(5x - 4)$ $10x^2 - 18x + 8$

16. $(3a - b)(2a - b)$ $6a^2 - 5ab + b^2$

17. $(4g + 3h)(2g + 3h)$ $8g^2 + 18gh + 9h^2$

18. $(4x + y)(4x + y)$ $16x^2 + 8xy + y^2$

19. $(m + 5)(m^2 + 4m - 8)$ $m^3 + 9m^2 + 12m - 40$

20. $(t + 3)(t^2 + 4t + 7)$ $t^3 + 7t^2 + 19t + 21$

21. $(2h + 3)(2h^2 + 3h + 4)$ $4h^3 + 12h^2 + 17h + 12$

22. $(3d + 3)(2d^2 + 5d - 2)$ $6d^3 + 21d^2 + 9d - 6$

23. $(3q + 2)(9q^2 - 12q + 4)$ $27q^3 - 18q^2 - 12q + 8$

24. $(3r + 2)(9r^2 + 6r + 4)$ $27r^3 + 36r^2 + 24r + 8$

25. $(3c^2 + 2c - 1)(2c^2 + c + 9)$ $6c^4 + 7c^3 + 27c^2 + 17c - 9$

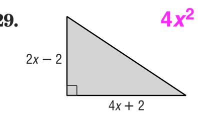
26. $(2\ell^2 + \ell + 3)(4\ell^2 + 2\ell - 2)$ $8\ell^4 + 8\ell^3 + 10\ell^2 + 4\ell - 6$

27. $(2x^2 - 2x - 3)(2x^2 - 4x + 3)$ $4x^4 - 12x^3 + 8x^2 + 6x - 9$

28. $(3y^2 + 2y + 2)(3y^2 - 4y - 5)$ $9y^4 - 6y^3 - 17y^2 - 18y - 10$

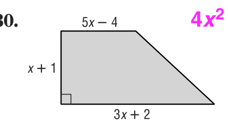
GEOMETRY Write an expression to represent the area of each figure.

29.



$4x^2 - 2x - 2$ units^2

30.



$4x^2 + 3x - 1$ units^2

31. NUMBER THEORY Let x be an even integer. What is the product of the next two consecutive even integers? $x^2 + 6x + 8$

32. GEOMETRY The volume of a rectangular pyramid is one third the product of the area of its base and its height. Find an expression for the volume of a rectangular pyramid whose base has an area of $3x^2 + 12x + 9$ square feet and whose height is $x + 3$ feet. $x^3 + 7x^2 + 15x + 9$ feet^3

Answers

NAME ______________________ DATE ________ PERIOD _____

8-7 Reading to Learn Mathematics

Multiplying Polynomials

Pre-Activity **How is multiplying binomials similar to multiplying two-digit numbers?**

Read the introduction to Lesson 8-7 at the top of page 452 in your textbook.

In your own words, explain how the distributive property is used twice to multiply two-digit numbers.

The ones of the first factor are multiplied by the tens and the ones of the other factor. Then the tens of the first factor are multiplied by the tens and ones of the other factor.

Reading the Lesson

1. How is multiplying binomials similar to multiplying two-digit numbers?

Binomials have two terms and each term of one binomial is multiplied by each term of the other binomial.

2. Complete the table using the FOIL method.

		Product of First Terms	+	Product of Outer Terms	+	Product of Inner Terms	+	Product of Last Terms
$(x + 5)(x - 3)$	=	$(x)(x)$	+	$(x)(-3)$	+	$(5)(x)$	+	$(5)(-3)$
	=	x^2	+	$-3x$	+	$5x$	+	-15
	=	x^2	+	$2x$	−	15		
$(3y + 6)(y - 2)$	=	$(3y)(y)$	+	$(3y)(-2)$	+	$(6)(y)$	+	$(6)(-2)$
	=	$3y^2$	+	$-6y$	+	$6y$	+	-12
	=	$3y^2$	−	12				

Helping You Remember

3. Think of a method for remembering all the product combinations used in the FOIL method for multiplying two binomials. Describe your method using words or a diagram.

Sample answer: Imagine that the two binomials are written on the floor. For FOIL, think of all the possible ways you could have your left foot on a term of the first binomial and your right foot on a term of the second binomial. Your feet could be on the first terms, the outer terms, the inner terms, or the last terms.

Lesson 8-7

NAME ______________________ DATE ________ PERIOD _____

8-7 Enrichment

Pascal's Triangle

This arrangement of numbers is called Pascal's Triangle. It was first published in 1665, but was known hundreds of years earlier.

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

1. Each number in the triangle is found by adding two numbers. What two numbers were added to get the 6 in the 5th row?

3 and 3

2. Describe how to create the 6th row of Pascal's Triangle.

The first and last numbers are 1. Evaluate 1 + 4, 4 + 6, 6 + 4, and 4 + 1 to find the other numbers.

3. Write the numbers for rows 6 through 10 of the triangle.

Row 6: **1 5 10 10 5 1**

Row 7: **1 6 15 20 15 6 1**

Row 8: **1 7 21 35 35 21 7 1**

Row 9: **1 8 28 56 70 56 28 8 1**

Row 10: **1 9 36 84 126 126 84 36 9 1**

Multiply to find the expanded form of each product.

4. $(a + b)^2$ **$a^2 + 2ab + b^2$**

5. $(a + b)^3$ **$a^3 + 3a^2b + 3ab^2 + b^3$**

6. $(a + b)^4$ **$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$**

Now compare the coefficients of the three products in Exercises 4–6 with Pascal's Triangle.

7. Describe the relationship between the expanded form of $(a + b)^n$ and Pascal's Triangle.

The coefficients of the expanded form are found in row $n + 1$ of Pascal's Triangle.

8. Use Pascal's Triangle to write the expanded form of $(a + b)^6$.

$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

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8-8 Study Guide and Intervention

Special Products

Squares of Sums and Differences Some pairs of binomials have products that follow specific patterns. One such pattern is called the *square of a sum*. Another is called the *square of a difference*.

Square of a sum	$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
Square of a difference	$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Example 1 **Find $(3a + 4)(3a + 4)$.**

Use the square of a sum pattern, with $a = 3a$ and $b = 4$.

$$(3a + 4)(3a + 4) = (3a)^2 + 2(3a)(4) + (4)^2 = 9a^2 + 24a + 16$$

The product is $9a^2 + 24a + 16$.

Example 2 **Find $(2z - 9)(2z - 9)$.**

Use the square of a difference pattern with $a = 2z$ and $b = 9$.

$$(2z - 9)(2z - 9) = (2z)^2 - 2(2z)(9) + (9)(9) = 4z^2 - 36z + 81$$

The product is $4z^2 - 36z + 81$.

Exercises

Find each product.

1. $(x - 6)^2$ — $x^2 - 12x + 36$
2. $(3p + 4)^2$ — $9p^2 + 24p + 16$
3. $(4x - 5)^2$ — $16x^2 - 40x + 25$
4. $(2x - 1)^2$ — $4x^2 - 4x + 1$
5. $(2h + 3)^2$ — $4h^2 + 12h + 9$
6. $(m + 5)^2$ — $m^2 + 10m + 25$
7. $(c + 3)^2$ — $c^2 + 6c + 9$
8. $(3 - p)^2$ — $9 - 6p + p^2$
9. $(x - 5y)^2$ — $x^2 - 10xy + 25y^2$
10. $(8y + 4)^2$ — $64y^2 + 64y + 16$
11. $(8 + x)^2$ — $64 + 16x + x^2$
12. $(3a - 2b)^2$ — $9a^2 - 12ab + 4b^2$
13. $(2x - 8)^2$ — $4x^2 - 32x + 64$
14. $(x^2 + 1)^2$ — $x^4 + 2x^2 + 1$
15. $(m^2 - 2)^2$ — $m^4 - 4m^2 + 4$
16. $(x^3 - 1)^2$ — $x^6 - 2x^3 + 1$
17. $(2h^2 - k^2)^2$ — $4h^4 - 4h^2k^2 + k^4$
18. $\left(\frac{1}{4}x + 3\right)^2$ — $\frac{1}{16}x^2 + \frac{3}{2}x + 9$
19. $(x - 4y^2)^2$ — $x^2 - 8xy^2 + 16y^4$
20. $(2p + 4q)^2$ — $4p^2 + 16pq + 16q^2$
21. $\left(\frac{2}{3}x - 2\right)^2$ — $\frac{4}{9}x^2 - \frac{8}{3}x + 4$

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Lesson 8-8

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8-8 Study Guide and Intervention *(continued)*

Special Products

Product of a Sum and a Difference There is also a pattern for the product of a sum and a difference of the same two terms, $(a + b)(a - b)$. The product is called the **difference of squares**.

Product of a Sum and a Difference	$(a + b)(a - b) = a^2 - b^2$

Example **Find $(5x + 3y)(5x - 3y)$.**

$(a + b)(a - b) = a^2 - b^2$	Product of a Sum and a Difference
$(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2$	$a = 5x$ and $b = 3y$
$= 25x^2 - 9y^2$	Simplify.

The product is $25x^2 - 9y^2$.

Exercises

Find each product.

1. $(x - 4)(x + 4)$ — $x^2 - 16$
2. $(p + 2)(p - 2)$ — $p^2 - 4$
3. $(4x - 5)(4x + 5)$ — $16x^2 - 25$
4. $(2x - 1)(2x + 1)$ — $4x^2 - 1$
5. $(h + 7)(h - 7)$ — $h^2 - 49$
6. $(m - 5)(m + 5)$ — $m^2 - 25$
7. $(2c - 3)(2c + 3)$ — $4c^2 - 9$
8. $(3 - 5q)(3 + 5q)$ — $9 - 25q^2$
9. $(x - y)(x + y)$ — $x^2 - y^2$
10. $(y - 4x)(y + 4x)$ — $y^2 - 16x^2$
11. $(8 + 4x)(8 - 4x)$ — $64 - 16x^2$
12. $(3a - 2b)(3a + 2b)$ — $9a^2 - 4b^2$
13. $(3y - 8)(3y + 8)$ — $9y^2 - 64$
14. $(x^2 - 1)(x^2 + 1)$ — $x^4 - 1$
15. $(m^2 - 5)(m^2 + 5)$ — $m^4 - 25$
16. $(x^3 - 2)(x^3 + 2)$ — $x^6 - 4$
17. $(h^2 - k^2)(h^2 + k^2)$ — $h^4 - k^4$
18. $\left(\frac{1}{4}x + 2\right)\left(\frac{1}{4}x - 2\right)$ — $\frac{1}{16}x^2 - 4$
19. $(3x - 2y^2)(3x + 2y^2)$ — $9x^2 - 4y^4$
20. $(2p - 5s)(2p + 5s)$ — $4p^2 - 25s^2$
21. $\left(\frac{4}{3}x - 2y\right)\left(\frac{4}{3}x + 2y\right)$ — $\frac{16}{9}x^2 - 4y^2$

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Answers

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8-8 Skills Practice

Special Products

Find each product.

1. $(n + 3)^2$ $n^2 + 6n + 9$
2. $(x + 4)(x + 4)$ $x^2 + 8x + 16$
3. $(y - 7)^2$ $y^2 - 14y + 49$
4. $(t - 3)(t - 3)$ $t^2 - 6t + 9$
5. $(b + 1)(b - 1)$ $b^2 - 1$
6. $(a - 5)(a + 5)$ $a^2 - 25$
7. $(p - 4)^2$ $p^2 - 8p + 16$
8. $(z + 3)(z - 3)$ $z^2 - 9$
9. $(\ell + 2)(\ell + 2)$ $\ell^2 + 4\ell + 4$
10. $(r - 1)(r - 1)$ $r^2 - 2r + 1$
11. $(3g + 2)(3g - 2)$ $9g^2 - 4$
12. $(2m - 3)(2m + 3)$ $4m^2 - 9$
13. $(6 + u)^2$ $36 + 12u + u^2$
14. $(r + s)^2$ $r^2 + 2rs + s^2$
15. $(3q + 1)(3q - 1)$ $9q^2 - 1$
16. $(c - e)^2$ $c^2 - 2ce + e^2$
17. $(2k - 2)^2$ $4k^2 - 8k + 4$
18. $(w + 3h)^2$ $w^2 + 6wh + 9h^2$
19. $(3p - 4)(3p + 4)$ $9p^2 - 16$
20. $(t + 2u)^2$ $t^2 + 4tu + 4u^2$
21. $(x - 4y)^2$ $x^2 - 8xy + 16y^2$
22. $(3b + 7)(3b - 7)$ $9b^2 - 49$
23. $(3y - 3g)(3y + 3g)$ $9y^2 - 9g^2$
24. $(s^2 + r^2)^2$ $s^4 + 2s^2r^2 + r^4$
25. $(2k + m^2)^2$ $4k^2 + 4km^2 + m^4$
26. $(3u^2 - n)^2$ $9u^4 - 6u^2n + n^2$

27. **GEOMETRY** The length of a rectangle is the sum of two whole numbers. The width of the rectangle is the difference of the same two whole numbers. Using these facts, write a verbal expression for the area of the rectangle. **The area is the square of the larger number minus the square of the smaller number.**

Lesson 8-8

NAME ______________________ DATE __________ PERIOD _____

8-8 Practice (Average)

Special Products

Find each product.

1. $(n + 9)^2$ $n^2 + 18n + 81$
2. $(q + 8)^2$ $q^2 + 16q + 64$
3. $(\ell - 10)^2$ $\ell^2 - 20\ell + 100$
4. $(r - 11)^2$ $r^2 - 22r + 121$
5. $(p + 7)^2$ $p^2 + 14p + 49$
6. $(b + 6)(b - 6)$ $b^2 - 36$
7. $(z + 13)(z - 13)$ $z^2 - 169$
8. $(4e + 2)^2$ $16e^2 + 16e + 4$
9. $(5w - 4)^2$ $25w^2 - 40w + 16$
10. $(6h - 1)^2$ $36h^2 - 12h + 1$
11. $(3s + 4)^2$ $9s^2 + 24s + 16$
12. $(7v - 2)^2$ $49v^2 - 28v + 4$
13. $(7k + 3)(7k - 3)$ $49k^2 - 9$
14. $(4d - 7)(4d + 7)$ $16d^2 - 49$
15. $(3g + 9h)(3g - 9h)$ $9g^2 - 81h^2$
16. $(4q + 5t)(4q - 5t)$ $16q^2 - 25t^2$
17. $(a + 6u)^2$ $a^2 + 12au + 36u^2$
18. $(5r + s)^2$ $25r^2 + 10rs + s^2$
19. $(6c - m)^2$ $36c^2 - 12cm + m^2$
20. $(k - 6y)^2$ $k^2 - 12ky + 36y^2$
21. $(u - 7p)^2$ $u^2 - 14up + 49p^2$
22. $(4b - 7v)^2$ $16b^2 - 56bv + 49v^2$
23. $(6n + 4p)^2$ $36n^2 + 48np + 16p^2$
24. $(5q + 6s)^2$ $25q^2 + 60qs + 36s^2$
25. $(6a - 7b)(6a + 7b)$ $36a^2 - 49b^2$
26. $(8h + 3d)(8h - 3d)$ $64h^2 - 9d^2$
27. $(9x + 2y^2)^2$ $81x^2 + 36xy^2 + 4y^4$
28. $(3p^3 + 2m)^2$ $9p^6 + 12p^3m + 4m^2$
29. $(5a^2 - 2b)^2$ $25a^4 - 20a^2b + 4b^2$
30. $(4m^3 - 2t)^2$ $16m^6 - 16m^3t + 4t^2$
31. $(6e^3 - c)^2$ $36e^6 - 12e^3c + c^2$
32. $(2b^2 - g)(2b^2 + g)$ $4b^4 - g^2$
33. $(2v^2 + 3e^2)(2v^2 + 3e^2)$ $4v^4 + 12v^2e^2 + 9e^4$

34. **GEOMETRY** Janelle wants to enlarge a square graph that she has made so that a side of the new graph will be 1 inch more than twice the original side s. What trinomial represents the area of the enlarged graph? $4s^2 + 4s + 1$

GENETICS For Exercises 35 and 36, use the following information.

In a guinea pig, pure black hair coloring B is dominant over pure white coloring b. Suppose two hybrid Bb guinea pigs, with black hair coloring, are bred.

35. Find an expression for the genetic make-up of the guinea pig offspring. $0.25BB + 0.50Bb + 0.25bb$

36. What is the probability that two hybrid guinea pigs with black hair coloring will produce a guinea pig with white hair coloring? **25%**

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8-8 Reading to Learn Mathematics

Special Products

Pre-Activity **When is the product of two binomials also a binomial?**

Read the introduction to Lesson 8-8 at the top of page 458 in your textook.

What is a meant by the term *trinomial product*?

a three-term polynomial answer when multiplying polynomials

Reading the Lesson

1. Refer to the Key Concepts boxes on pages 458, 459, and 460.

a. When multiplying two binomials, there are three special products. What are the three special products that may result when multiplying two binomials?

square of a sum, square of a difference, product of a sum and a difference

b. Explain what is meant by the name of each special product.

square of a sum: squaring the sum of two monomials; square of a difference: squaring the difference of two monomials; product of a sum and a difference: the product of the sum and the difference of the same two terms

c. Use the examples in the Key Concepts boxes to complete the table.

	Symbols	Product	Example	Product
Square of a Sum	$(a + b)^2$	$a^2 + 2ab + b^2$	$(x + 7)^2$	$x^2 + 14x + 49$
Square of a Difference	$(a - b)^2$	$a^2 - 2ab + b^2$	$(x - 4)^2$	$x^2 - 8x + 16$
Product of a Sum and a Difference	$(a + b)(a - b)$	$a^2 - b^2$	$(x + 9)(x - 9)$	$x^2 - 81$

2. What is another phrase that describes the product of the sum and difference of two terms? **difference of squares**

Helping You Remember

3. Explain how FOIL can help you remember how many terms are in the special products studied in this lesson. **For the square of a sum and the square of a difference, the inner and outer products are equal, so there are are only three terms. For the product of the sum and difference of two terms, two of the products for FOIL are opposites. That means that the final product has only two terms.**

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8-8 Enrichment

Sums and Differences of Cubes

Recall the formulas for finding some special products:

Perfect-square trinomials: $(a + b)^2 = a^2 + 2ab + b^2$ or
$(a - b)^2 = a^2 - 2ab + b^2$

Difference of two squares: $(a + b)(a - b) = a^2 - b^2$

A pattern also exists for finding the cube of a sum $(a + b)^3$.

1. Find the product of $(a + b)(a + b)(a + b)$.
$a^3 + 3a^2b + 3ab^2 + b^3$

2. Use the pattern from Exercise 1 to evaluate $(x + 2)^3$.
$x^3 + 6x^2 + 12x + 8$

3. Based on your answer to Exercise 1, predict the pattern for the cube of a difference $(a - b)^3$.
$a^3 - 3a^2b + 3ab^2 - b^3$

4. Find the product of $(a - b)(a - b)(a - b)$ and compare it to your answer for Exercise 3.
$a^3 - 3a^2b + 3ab^2 - b^3$

5. Use the pattern from Exercise 4 to evaluate $(x + 4)^3$.
$x^3 + 12x^2 + 48x + 64$

Find each product.

6. $(x + 6)^3$
$x^3 + 18x^2 + 108x + 216$

7. $(x - 10)^3$
$x^3 - 30x^2 + 300x - 1000$

8. $(3x - y)^3$
$27x^3 - 27x^2y + 9xy^2 - y^3$

9. $(2x - y)^3$
$8x^3 - 12x^2y + 6xy^2 - y^3$

10. $(4x + 3y)^3$
$64x^3 + 144x^2y + 108xy^2 + 27y^3$

11. $(5x + 2)^3$
$125x^3 + 150x^2 + 60x + 8$