Calculating and Predicting the Sound Frequency of a Tuba

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Purpose

The purpose of this project is to calculate the sound frequency of a tuba using its tube length and the open tube frequency formula. The frequency is compared with the music note frequency played by the tuba to find the possible sources of error. This project demonstrates that, to play very low notes (or low frequency), the tuba's tube length needs to be very long, which makes tuba a heavy instrument. This project also showed how the valves in tuba change the tube length, which alters the music note and sound frequency.

Hypothesis

If the tuba's tube length is known, then the sound frequency can be calculated because the tuba is an open air tube from the lip's vibration to the open end of the tube. The sound frequency formula for the open tube can be applied to calculate the frequencies from the measured tube length.   If the tube is longer, then the frequency of the sound generated by the tube is shorter because the wavelength of sound is longer, causing the number of vibrations per second to decrease. If the note's pitch gets higher when the tube length is shortened, then there is a relationship between the note, or frequency and tube length because the speed of sound is constant and is equal to the wavelength times the frequency.

Goal

The goal is to estimate and calculate the frequencies and wavelengths of several harmonics on a tuba tube length. The frequencies and wavelengths will be compared and the error will be discussed.

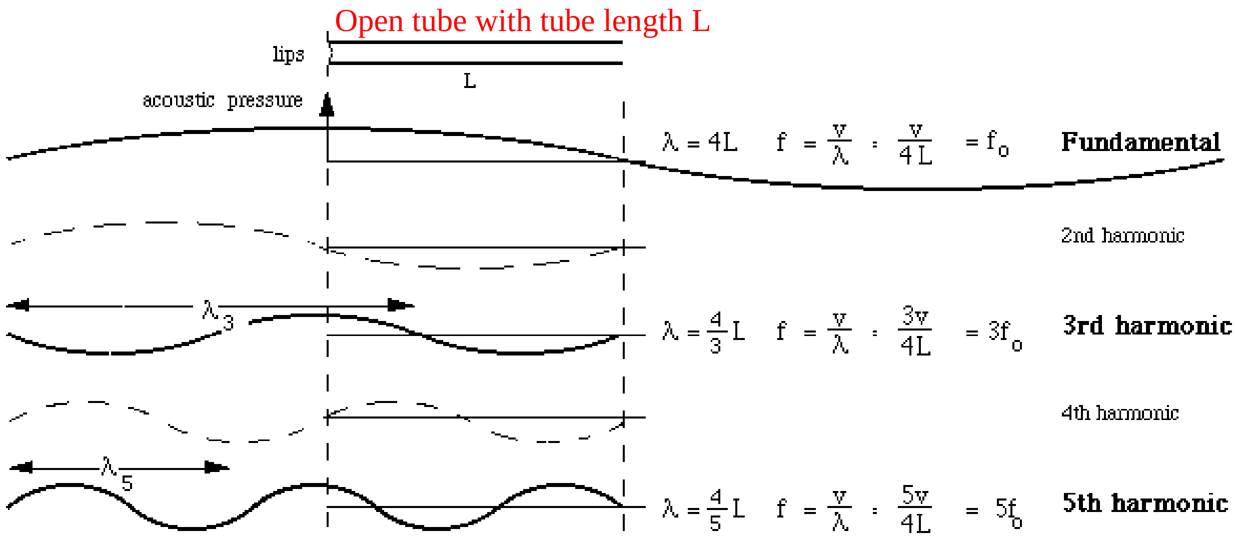
Background Information

In music, each note an instrument plays has a frequency. Frequency is the number of vibrations per second. If the note's pitch is higher, the frequency will be higher. If the note's pitch is lower, the frequency will be lower. There is a formula for calculating the frequency. The formula is

$$\text{Frequency} = \text{Speed of sound} / \text{Wavelength}$$

The speed of sound, v, is about 340 meters per second at sea level. The wavelength is the distance between two adjacent peaks in the sound wave. The frequency's unit is Hertz (Hz), which represents the number of vibration in a second.

For a given length of an open tube, L, as shown in the figure below, several frequencies, f, or harmonics can be generated with the high pressure in the lip side and low pressure in the other open end side of the tube. When tuba players play, the air going across the vibrating lips creates an area of high pressure.



On the diagram above, the odd harmonics are able to be played because of the high pressure at the end of the tube in which the player blows through. The even harmonics has no pressure at the beginning of the tube, making it impossible to play.

- *Fundamental (1st harmonic)*: This is the lowest note can be played by a specific tube length (L). The wavelength, λ, is equal to 4L. The frequency f equals v / 4L.
- *3rd harmonic*: This is the next high note from fundamental. The wavelength, λ , is equal to 4L / 3, which is the wavelength of the fundamental divided by 3. The frequency f = v / (4L / 3) = 3v / 4L.
- *5th harmonic*: This is the 3rd high note from fundamental. The wavelength λ = 4L / 5 and the frequency f = v / (4L / 5) = 5v / 4L.
- *7th harmonic and higher*: Tuba player can play about 8 notes with each tube length, which is 15 harmonics.

Frequency of Music Notes

The frequency of middle A is 440 Hz. The frequency of low A, which is an octave lower than middle A, is 220 or 440 divided by 2.

There are twelve half steps to in an octave and $1.0595^{12} = 2$. The frequency of 440 (middle A) is divided by 1.0595 sequentially to find the frequency of each note from middle A to low A, as listed on the table below.

Note	Middle A	Ab	G	Gb	F	E	Eb	D	Db	C	B	Bb	Low A
Freq.	440	415	392	370	349	330	311	294	277	262	247	233	220

=415/1.0595

=440/1.0595

Using this procedure, we can find the frequency of each music note, which is listed in the table of results.

Tuba Air Tube and Valves

The tuba has a long tube length to get low frequency sounds. The air is blown from the mouthpiece (see figure) and comes out from the open end of the tube.

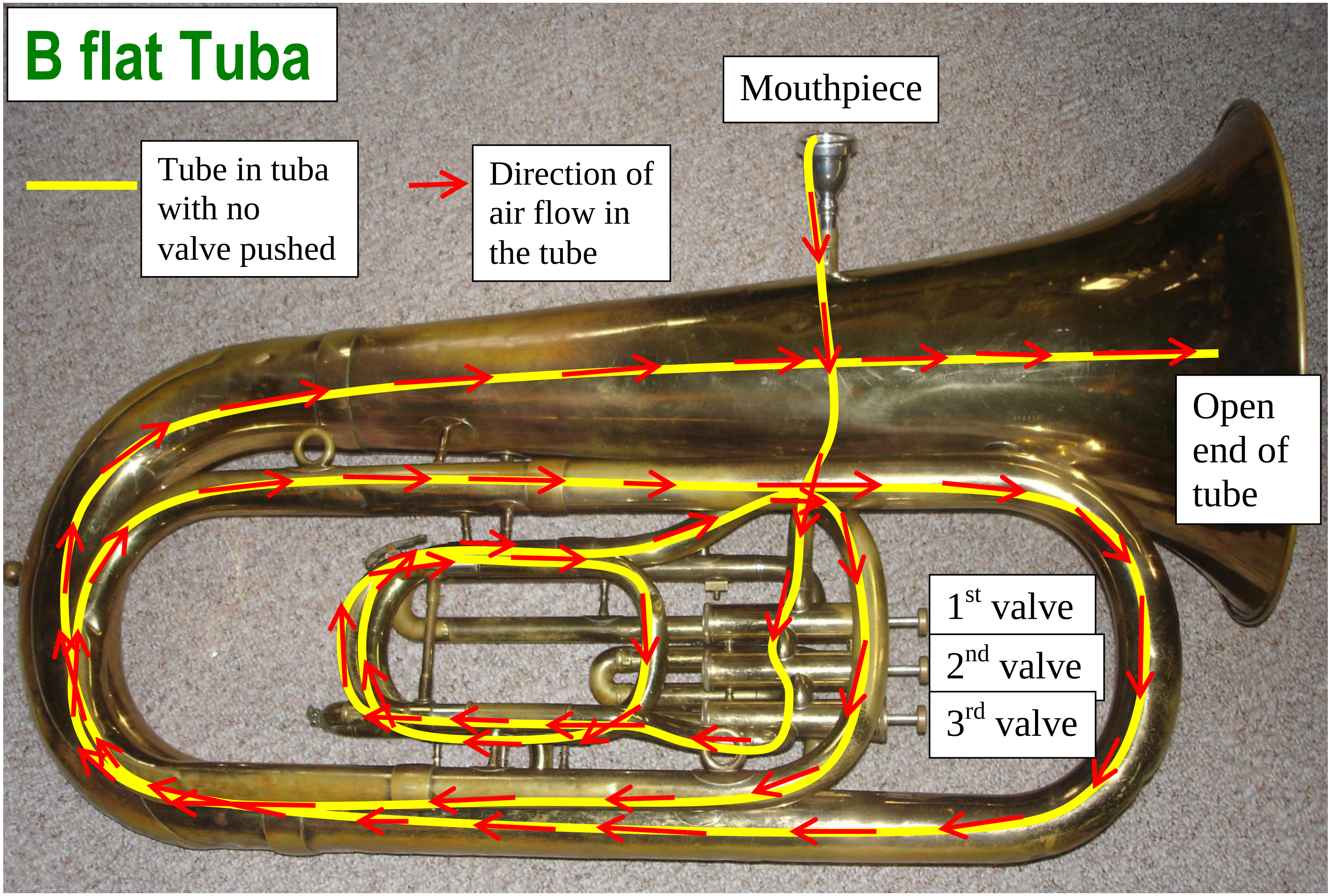
The tuba has three valves to alter the tube length. Without pushing any valves, the route of the tube is shown by the yellow color line, with the direction of the air flow indicated by red colored arrows. There are four loops of tubing without any valves pressed. Three valves, commonly called 1st, 2nd, and 3rd valve, are shown in the next figure. After pressing a valve, an extra loop of tube, marked by the green, blue, and red color lines in the figure, is added to make five loops of tube and increase the tube length.

As shown on the Tube Routes for the Three Valve Tubes, 1st valve is equal to a whole step, which is equivalent to two half steps. 2nd valve is equal to a half step, and 3rd valve is equal to 1 ½ steps. Pressing combinations of the three valves and changing the harmonics, a tuba can cover at least 4 octaves of notes.

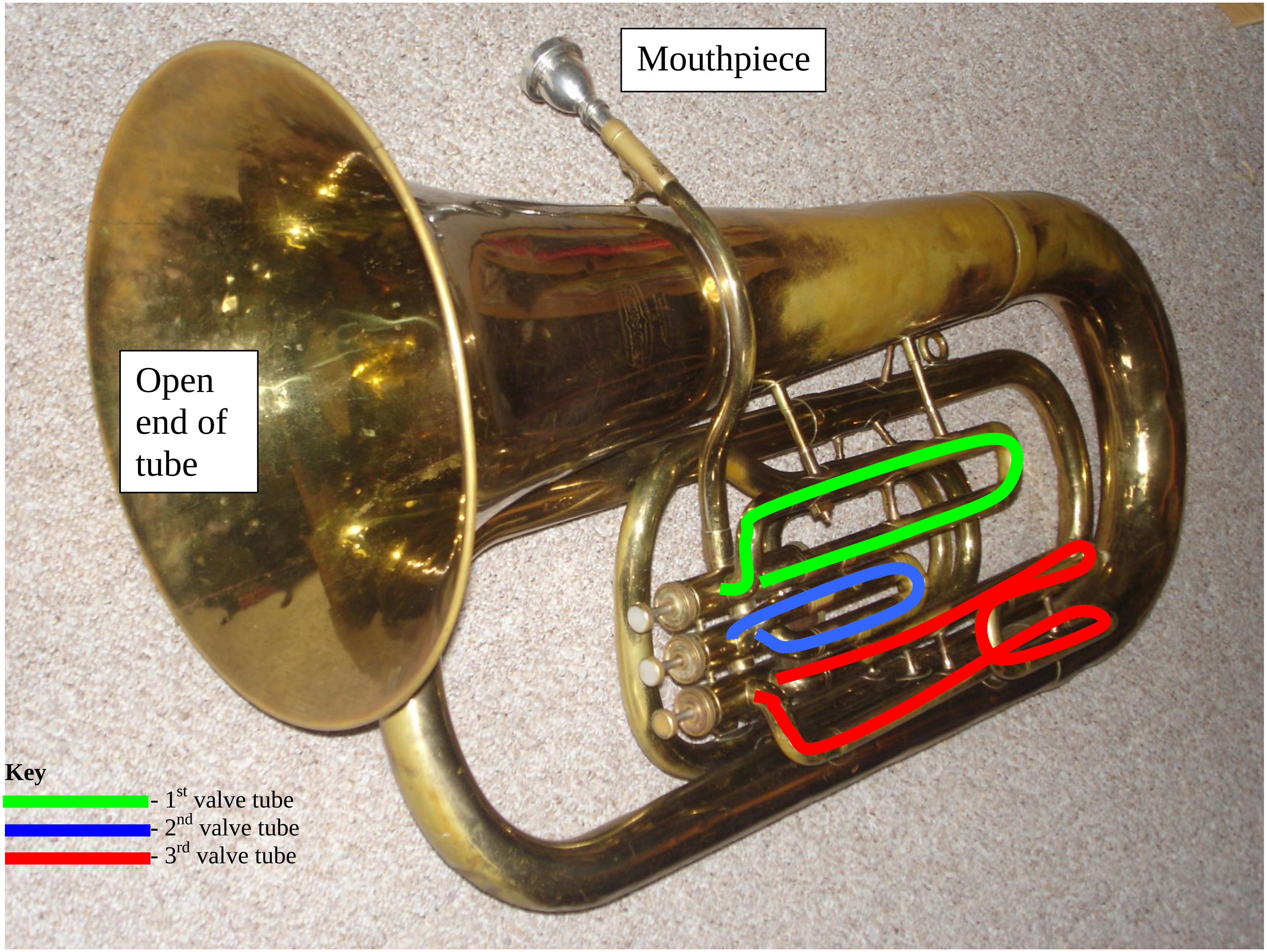
Using this background information, the hypothesis can be answered:

- If the length of the tube is known, then the sound frequency can be calculated because the tuba is an open tube from the mouthpiece to the bell.
- If the tube length is longer, then the wavelength will be longer because the frequency is decreased.
- If the music note's pitches gets higher when the tube length is shortened, then there is a relationship between the note and the tube length because the speed of sound is constant and is equal to the wavelength times the frequency.

B flat Tuba


▬ Tube in tuba with no valve pushed
→ Direction of air flow in the tube
Mouthpiece
Open end of tube
1st valve
2nd valve
3rd valve

Tube Routes for the Three Valve Tubes

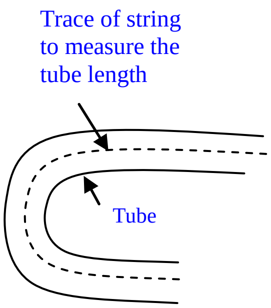


Materials

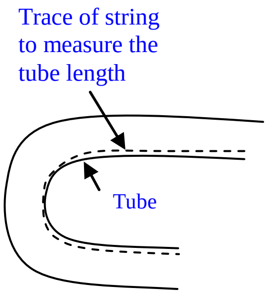
- B flat Tuba with the mouthpiece (see figures)
- String or yarn (to follow the tube around to measuring the tuba tube length)
- Measuring tape (to measure the length of the straightened string)
- Calculator (to calculate the wavelength and frequency)
- Pencil and paper (to record data)

Procedure

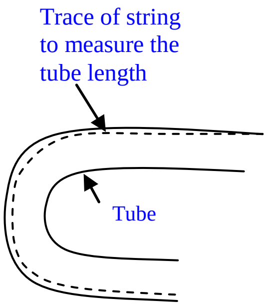
1. *Measure the average tube length*: Use the string and line in the center of the tube. The length of string represents the length of the tube. On the curves, place the string in the center of the tube (see figure on the right). Divide the tuba into sections and measure the length of each section separately. Add up the measurements of all the segments of tube in the tuba and record the total tube length onto the table.



2. *Measure the short tube length*: Line the string along inside the tube so it would measure the shortest path air could travel through the tube. It is marked as the dashed curve line (see figure on the right). Use the measuring tape to measure the length of the string. Add up all the length of all sections and record the distance of the short air length onto the table.



3. *Measure the long tube length*: Line the string along outside the tube so it would measure the shortest path air could travel through the tube. It is marked as the dashed curve line (see figure on the right). Use the measuring tape to measure the length of the string. Add up all the length of all sections and record the distance of the short air length onto the table.



4. Measure the length of all the valve tubes. Record the lengths onto the table.
5. Play all **30** music notes possible from low A down in the tuba. Calculate the music note frequency and wavelength. Knowing that low A is 220 Hz and that dividing 220 by 1.0595 would give the frequency of the note a half step below low A. Record them in the table.

6. Each music note played has fingerings (use of valves in tuba). Identify the valve tube lengths and add to the tube length of the tuba with no valves pushed down to find the total tube length. Note the average and short tube lengths are used. Identify the harmonics of the note, by matching to the music note frequency, and use the tube length to calculate the frequency. Record the harmonics, fingering, wavelength, and frequency onto the table.
7. Draw charts to compare the wavelength and frequency calculated from tube lengths and music note.

Data Collection

Measured tube lengths:

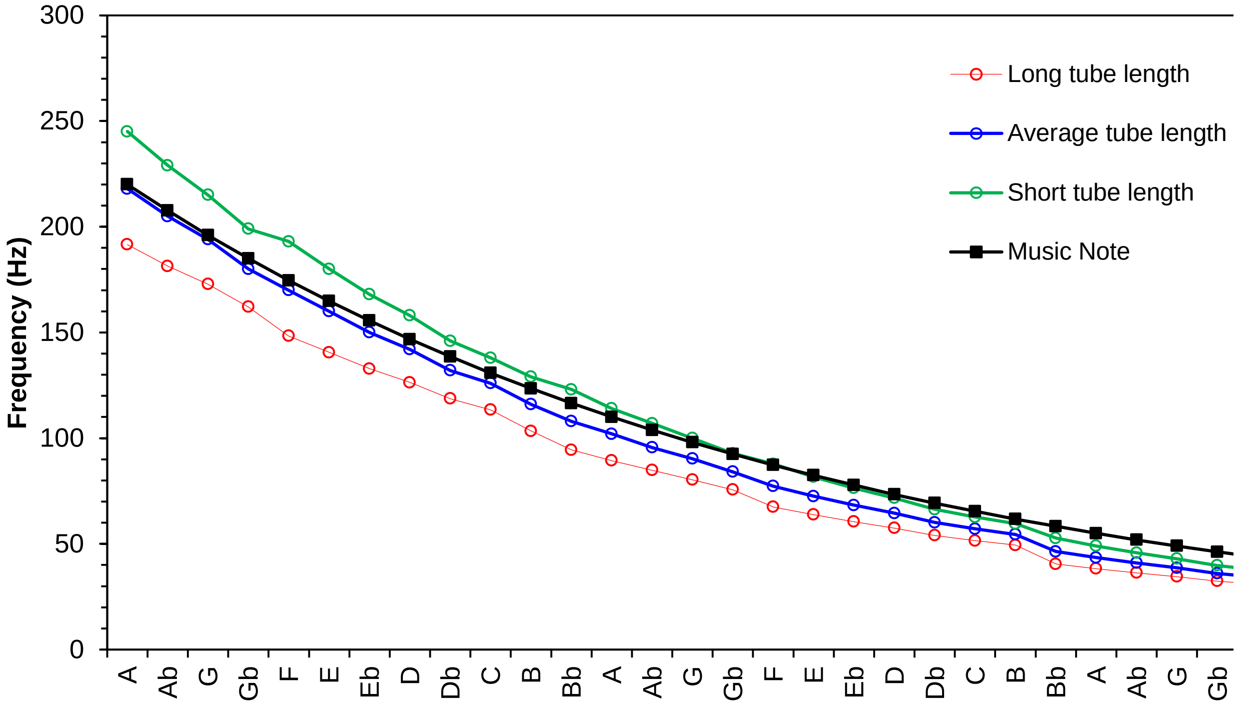
Longest length of tubing in the tuba without the valves. (cm)	666
Average length of tubing in the tuba without the valves. (cm)	550
Shortest length of tubing in the tuba without the valves. (cm)	484
Length of 1st valve tube (cm)	70
Length of 2nd valve tube (cm)	37.5
Length of 3rd valve tube (cm)	85

Calculated wavelengths and frequencies for tuba music notes:

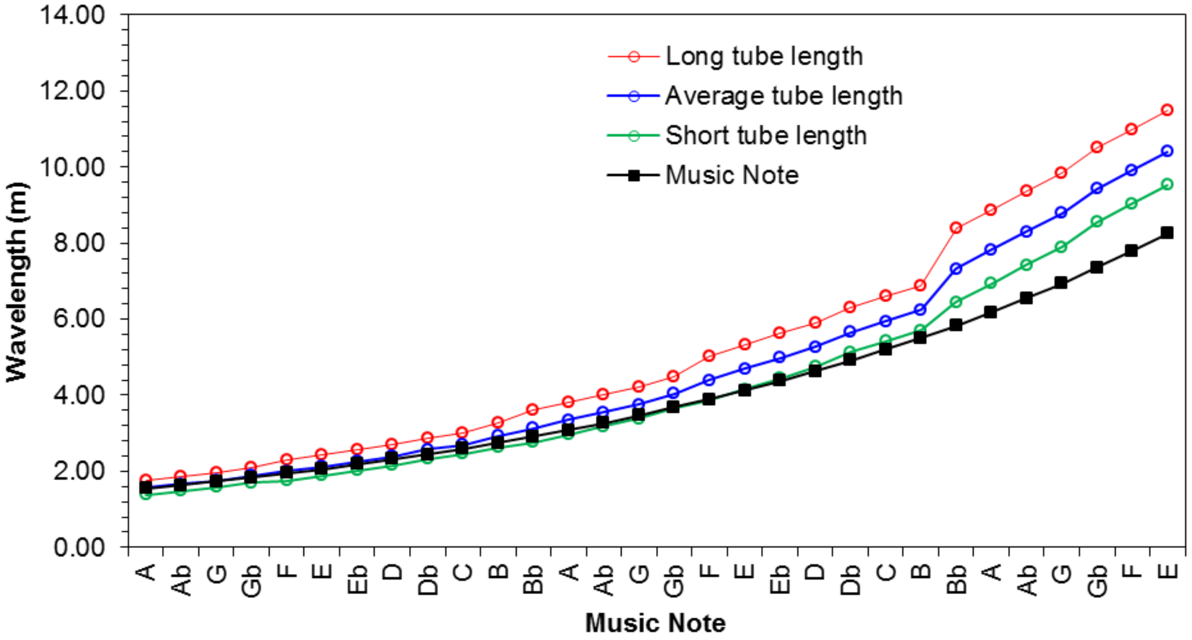
	Music note		**Tuba**										
	Calculation				**Average tube length (m)**			**Short tube length (m)**			**Long tube length (m)**		
Music note	Freq. (Hz)	Wavelength (m)	Harmonic	Tuba fingering	Tube length (m)	Freq. (Hz)	Wavelength (m)	Tube length (m)	Freq. (Hz)	Wavelength (m)	Tube length (m)	Freq. (Hz)	Wavelength (m)
A	220	1.55	15	2	5.86	218	1.56	5.2	245	1.39	6.66	192	1.78
Ab	207.7	1.64	15	1	6.22	205	1.66	5.56	229	1.48	7.02	181	1.87
G	196	1.73	15	1,2	6.59	194	1.76	5.93	215	1.58	7.39	173	1.97
Gb	185	1.84	15	2,3	7.07	180	1.89	6.41	199	1.71	7.87	162	2.10
F	174.6	1.95	11	0	5.5	170	2	4.84	193	1.76	6.3	148	2.29
E	164.8	2.06	11	2	5.86	160	2.13	5.2	180	1.89	6.66	141	2.42
Eb	155.6	2.19	11	1	6.22	150	2.26	5.56	168	2.02	7.02	133	2.55
D	146.8	2.32	11	1,2	6.59	142	2.39	5.93	158	2.15	7.39	126	2.69
Db	138.6	2.45	11	2,3	7.07	132	2.57	6.41	146	2.33	7.87	119	2.86
C	130.8	2.6	11	1,3	7.44	126	2.7	6.78	138	2.46	8.24	113	3.00
B	123.5	2.75	9	1,2	6.59	116	2.93	5.93	129	2.63	7.39	103	3.28
Bb	116.5	2.92	7	0	5.5	108	3.14	4.84	123	2.77	6.3	94.4	3.60
A	110	3.09	7	2	5.86	102	3.35	5.2	114	2.97	6.66	89.4	3.81
Ab	103.8	3.27	7	1	6.22	95.6	3.56	5.56	107	3.18	7.02	84.8	4.01
G	98	3.47	7	1,2	6.59	90.3	3.76	5.93	100	3.39	7.39	80.3	4.22

Gb	92.5	3.68	7	2,3	7.07	84.1	4.04	6.41	92.8	3.67	7.87	75.6	4.50
F	87.3	3.89	5	0	5.5	77.3	4.4	4.84	87.8	3.87	6.3	67.5	5.04
E	82.4	4.13	5	2	5.86	72.5	4.69	5.2	81.7	4.16	6.66	63.8	5.33
Eb	77.8	4.37	5	1	6.22	68.3	4.98	5.56	76.4	4.45	7.02	60.6	5.62
D	73.4	4.63	5	1,2	6.59	64.5	5.27	5.93	71.7	4.74	7.39	57.5	5.91
Db	69.3	4.91	5	2,3	7.07	60.1	5.66	6.41	66.3	5.13	7.87	54.0	6.30
C	65.4	5.2	5	1,3	7.44	57.1	5.95	6.78	62.7	5.42	8.24	51.5	6.59
B	61.7	5.51	5	1,2,3	7.8	54.5	6.24	7.14	59.5	5.71	8.6	49.4	6.88
low Bb	58.3	5.83	3	0	5.5	46.4	7.33	4.84	52.7	6.45	6.3	40.5	8.40
A	55	6.18	3	2	5.86	43.5	7.82	5.2	49	6.94	6.66	38.3	8.88
Ab	51.9	6.55	3	1	6.22	41	8.3	5.56	45.8	7.42	7.02	36.3	9.36
G	49	6.94	3	1,2	6.59	38.7	8.78	5.93	43	7.9	7.39	34.5	9.85
Gb	46.2	7.35	3	2,3	7.07	36	9.43	6.41	39.8	8.55	7.87	32.4	10.50
F	43.7	7.79	3	1,3	7.44	34.3	9.92	6.78	37.6	9.04	8.24	31.0	11.00
E	41.2	8.25	3	1,2,3	7.8	32.7	10.4	7.14	35.7	9.52	8.6	29.6	11.50

Frequency of Music Notes on the Tuba



Wavelength of Music Notes on the Tuba



Sources of Error

There are some possible sources of error. The tuba is a very low note music instrument and has LONG, 5 to 8 m, tube length. The pitches will vary a lot due to the long tube length. The **temperature** of the surrounding air will also affect the pitch. If the air is cold, the pitch will go flat, or lower because the air molecules are closer to each other, increasing the amount of air in the tuba. If the air is hotter, the air molecules will spread apart more, thereby decreasing the amount of air in the tuba, making it go sharp, or higher in pitch. An adjustment on the tuning slide is used to slightly change the tube length and the pitch.

Error is larger at the lower notes (low frequency, high wavelength). Since most tuba parts are between 55 to 160 Hz, the tuning slide would need to be pushed in to decrease the tube length a little bit. This makes sure the correct music note is between the note measured with the short and long tube length. This will make the tuba more in tune between 55 to 160 Hz, but higher pitches will be sharp and out of tune. Good tuba players make sure the high notes are in tune by adjusting the tuning slides or lips while playing.

There is also a very big difference between B and Bb notes, caused by the difference between having no valves pressed down and all the valves pressed down and different harmonics. Since tuba players usually tune the Bb, tuba players need to listen and make sure the B, a half step above, is in tune when it is played.

Conclusion

The first hypothesis was supported by the data because as the notes decreases, the length of the wavelength increases as seen on the graph -- Wavelength of Music Notes on the Tuba. The increasing wavelength will cause the frequency to decrease, and result in fewer vibrations per second, as seen on the graph -- Frequency of the Music Notes on the Tuba. Instruments like the tuba has very long tube length unlike other brass instruments, for example, the trumpet or the French horn. This is why Tubas has extremely low pitches. The second hypothesis was also supported because notes on the tuba were able to be predicted using only the tube length and harmonic. The third hypothesis was supported. According to the two graphs, as the notes decrease, the frequency decrease, but the wavelength increases.

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