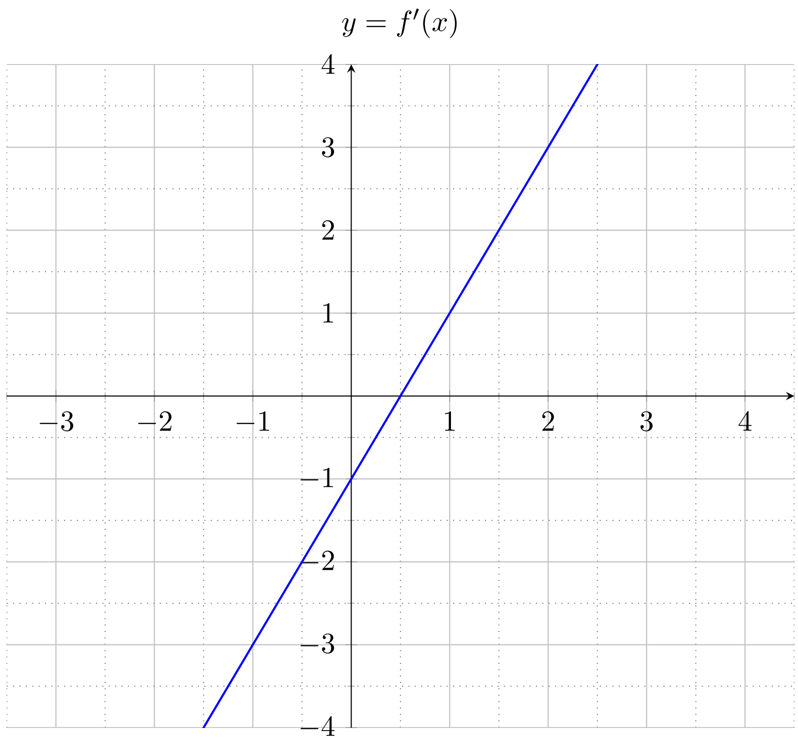
1. The purpose of this problem is to see how to construct a derivative function one point at a time by looking at a graph.
   Background review: estimating derivatives, one point at a time:
   - The derivative of a function at a point represents the slope (or rate of change) of a function at that point.
   - If you have a graph, you can estimate the derivative one point at a time by drawing the tangent line at that point, then calculating the slope of that tangent line (remember, slope is rise over run).

   (a) Go to the website http://www.shodor.org/interactivate/activities/Derivate/

   (b) Enter the function $y = x^2 - x - 2$. Use the tool to calculate the slope of the graph at each of the points x = -1, 0, 1, 2 and 2.5. Enter the values of the slope in the following table:

x	-1	0	1	2	2.5
slope of f at x	-3	-1	1	3	4

   (c) Now plot these points and connect them smoothly to see a graph of $f'(x)$



   (d) What do you think the formula for this graph is?
   Solution: It's a straight line with slope 2 and y-intercept $(0, -1)$, so $y = 2x - 1$

2. In this problem, you'll calculate the derivative of the same function as the previous problem, but this time you'll do it analytically (with formulas)

   (a) Calculate the derivative of $f(x) = x^2 - x - 2$.
   $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$
   Solution:

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{((x+h)^2 - (x+h) - 2) - (x^2 - x - 2)}{h} \\
&= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x - h - 2 - x^2 + x + 2}{h} \\
&= \lim_{h \to 0} \frac{2xh + h^2 - h}{h} \\
&= \lim_{h \to 0} \frac{h(2x + h - 1)}{h} \\
&= \lim_{h \to 0} (2x + h - 1) \\
&= 2x - 1.
\end{aligned}$$

   (b) Do your results from this problem match your results from the last problem?

   Solution: Yes

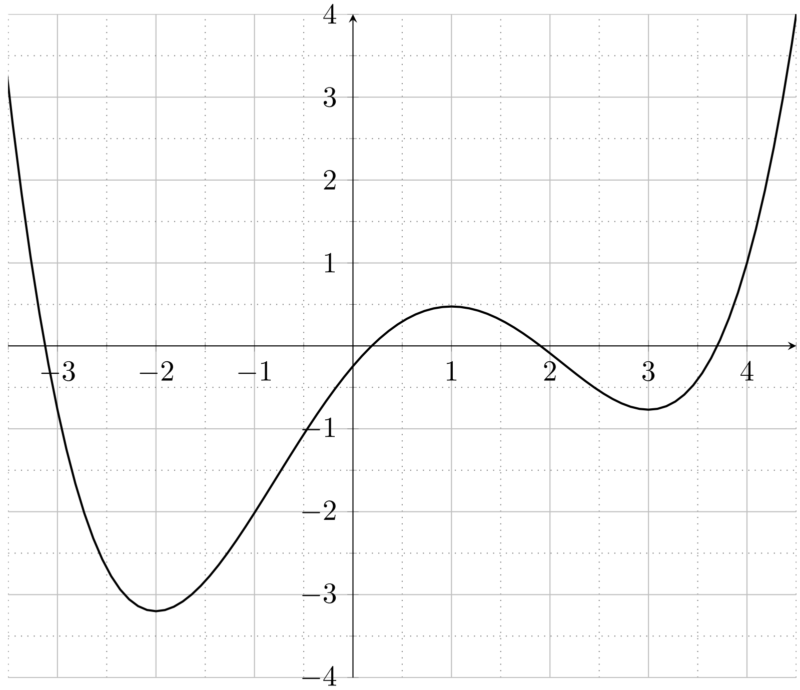
3. Khan academy has an exercise that gives a good visual and tactile experience of producing the derivative function point by point. Do at least one exercise on this website:

   `http://www.tinyurl.com/math1300-week4`

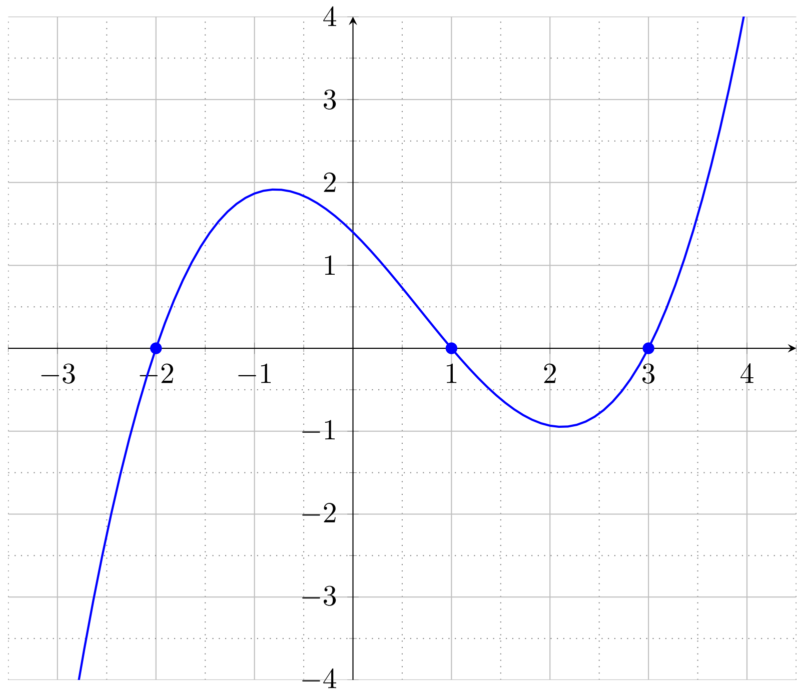
   In the space below, sketch the graph of $f(x)$ and $f'(x)$ from one of the exercises you did.

   Solution: Solutions vary.

4. Below is a graph of a function.



Without the aid of technology, use the graph of the function above to sketch a graph of its derivative function on the axes below.



5. Based on your experience above, what seems to be true about the relationship between $f(x)$ and $f'(x)$?

   (a) Where $f(x)$ is increasing, $f'(x)$ is ____positive____.

   (b) Where $f(x)$ is decreasing, $f'(x)$ is ____negative____.

6. Below are some more involved questions. We will be addressing these in the coming sections. Do you have any guesses for these?

   (a) Where $f(x)$ is concave up, $f'(x)$ is ____increasing____.

   (b) Where $f(x)$ is concave down, $f'(x)$ is ____decreasing____.

   (c) Where $f(x)$ is ____discontinuous____, ____has a cusp or corner____, or ____has a vertical tangent line____ $f'(x)$ is undefined.

   Solution: Note: discontinuities can include where $f(x)$ is undefined, has a vertical asymptote, has a jump discontinuity, or has a hole.

   (d) What will the derivative be when $f(x)$ has a relative high point (maximum) or relative low point (minimum)?

   Solution: At the relative extreme points of $f(x)$, the derivative will be zero or will be undefined.

   (e) If the derivative is 0 at a point, what are all the ways the original function could look?

   Solution: The top of a hill ($y = -x^2$ at $x = 0$), the bottom of a valley ($y = x^2$ at $x = 0$), or a horizontal tangent line that is neither a high point or a low point ($y = x^3$ at $x = 0$).

   (f) What about if the derivative has a relative maximum or minimum?

   Solution: Where the derivative has a relative maximum the original function has its steepest positive slope and where the derivative has a relative minimum the original function has its steepest negative slope. The function changes concavity there.