An Investigation of Using Keywords to Solve Word Problems

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Abstract

In mathematics, the keyword strategy involves identifying a keyword (e.g., *altogether*) and using that keyword to determine the operation needed to find a word problem's solution. We analyzed 747 high-stakes released items across Grades 3, 4, 5, 6, 7, and 8 from the Partnership for Assessment for Readiness for College and Careers and Smarter Balanced assessments. Of these, 57 items did not involve written text. Of the 690 text-based items, we classified 69% as directive word problems and 31% as routine word problems. For all 690 items, we identified any keywords (e.g., *total*, *each*, *share*) appearing in the text of the word problem. We categorized the 214 routine word problems by schema: total, difference, change, equal groups, comparison, ratios or proportions, or product of measures. We identified keywords within these problems and determined whether a keyword and its implied operation matched the correct problem solution. For single-step routine word problems, we determined keywords featured within the problem led to a correct problem solution with less than a 50% match rate. For multi-step routine word problems, the match rate was less than 10%. These low match rates indicate that keywords are an ineffective word-problem strategy that educators should avoid.

Keywords: keywords; instruction; mathematics; word problems

An Investigation of Using Keywords to Solve Word Problems

Competence with word-problem solving in mathematics serves as an important predictor of adult employment and wages (Every Child a Chance Trust, 2009; Hein et al., 2013). The correlation between word-problem solving and success in adulthood offers an important reason why current mathematics curricula and college- and career-ready mathematics standards emphasize word-problem solving as a necessary skill (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Solving word problems, however, presents a major challenge for many elementary and middle school students (Fuchs et al., 2014; Sharpe et al., 2014). Word-problem solving proves difficult because of the numerous steps involved in solving a problem from start to finish (Powell, 2011).

In this manuscript, we examine the strategy of using keywords to solve word problems (Karp et al., 2019; Verschaffel et al., 2000). The keyword strategy involves students identifying a specific word from a mathematics word problem and tying the word to an operation (Mevarech, 1999). For example, students may see the word *altogether* and assume they should add or associate the word *more than* with the operation of subtraction. Although many researchers have opposed the keyword strategy (Briars & Larkin, 1984; Clement & Bernhard, 2005; Karp et al., 2019; Powell & Fuchs, 2018; Sowder, 1989; Stigler et al., 1990), empirical evidence on the effectiveness of using the keyword strategy remains lacking. Therefore, the primary goal of this study was to analyze the released word problems from high-stakes mathematics assessments to understand the word problems students are expected to solve, quantify the prevalence of keywords, and examine the match rate of the keyword strategy. We define "match rate" as the likelihood of providing an equation representing the structure of the word problem that could lead to solving a word problem correctly. We understand that, even with a correct equation, a student could make a computation mistake or involve irrelevant information when solving the equation. Our use of "match rate" should be interpreted as the likelihood of solving a word problem correctly when linking a keyword to an operation without confounds such as computational error.

Word-Problem Solving

Mathematics word problems refer to the text-based mathematical problems that require students to read and understand the problem, interpret the relations among the problem's parts, determine which operations to use, and carry out the planned computations (Boonen et al. 2016; Krawec et al., 2012; Powell, 2011). Students in elementary and middle school solve word problems that classify as directive, routine, or non-routine (Powell & Fuchs, 2018).

Directive word problems are text-based word problems that direct a student to do something. Examples of directive word problems include: "Enter the length, in millimeters, of the pencil," "Click all the shapes that are quadrilaterals," or "Mia placed point P on the number line. Give the value of P as a fraction."

In a routine word problem, students read a text-based scenario, sometimes with accompanying pictures, tables, or graphs, and use information from the scenario to answer a word-problem question. Students manipulate numbers from the text to derive a problem solution. Routine word problems can involve a single-step (e.g., "A baker has 159 cups of brown sugar and 264 cups of white sugar. How many total cups of sugar does the baker have?") or multiple steps (e.g., "Hayley has 272 beads. She buys 38 more beads. She will use 89 beads to make bracelets and the rest to make necklaces. She will use 9 beads for each necklace. What is the greatest number of necklaces Hayley can make?").

Non-routine word problems cannot be solved by straightforward strategies and may not have a single correct answer. For example, students may see a menu of pizza toppings with the question, “How many different pizza choices do customers have if they select from four toppings?” (Beghetto, 2017, p. 990). Because non-routine word problems are very uncommon on high-stakes assessments, we do not provide a further discussion about non-routine word problems.

Difficulty with Word-Problem Solving

Word-problem solving differs from other types of mathematics problems, such as computation, in which students carry out a mathematical operation of a problem presented with mathematical notation (Boonen et al., 2016). Word-problem solving is a complex process composed of multiple phases. To solve a word problem, students need to interpret a text-based scenario by reading the scenario; identifying important information within the text or accompanying visual (e.g., graph or chart); determining how to approach solving the problem; organizing information into a graphic organizer, equation, or picture; constructing an equation(s); performing the calculation(s); and checking whether the response is reasonable (Freeman-Green et al., 2015; Kajamies et al., 2010; Wang et al., 2016). Difficulty with any part of the word-problem solving process can lead to incorrect responses and low scores on word-problem measures (Kingsdorf & Krawec, 2014; Kintsch & Greeno, 1985; Mayer et al., 1984).

Furthermore, even when many students can solve simple word problems that require calculation of a sum or difference, students often struggle with more complex problems such as those that involve solving for an unknown addend, subtrahend, or difference (García et al., 2006; Van Dooren et al., 2010) or those that feature multiplication, division, fractions, or decimals (Fuchs et al., 2016). Word-problem difficulty is exacerbated by problems requiring more than one step to develop a solution, problems with difficult readability, and problems containing extraneous or insufficient information (Jarosz & Jaeger, 2019; Jitendra et al., 2013; Walkington et al., 2018; Wang et al., 2016).

Strategies for Solving Word Problems

To help students with word-problem solving, researchers have deemed several strategies as effective. For example, using a cognitive strategy to set up and solve word problems proves valuable (Krawec et al., 2012; Montague et al., 2011) by targeting cognitive and meta-cognitive processes that help students successfully work through a word problem from start to finish. For example, Montague et al. (2011) used the cognitive strategy of Read, Paraphrase, Visualize, Hypothesize, Estimate, Compute, and Check with middle-school students, and they determined that students, including students with disabilities, benefitted from use of the cognitive strategy. As another example, Freeman-Green et al. (2015) used the SOLVE strategy: Study the problem, Organize the facts, Line up a plan, Verify your plan with action, and Evaluate your answer. Fuchs et al. (2014) used RUN: Read the problem, Underline the labels, and Name the problem type.

Another evidence-based word-problem strategy involves using graphic organizers or drawings to set up and solve word problems. van Garderen (2007) helped students improve word-problem performance with line diagrams and part/whole diagrams. Swanson et al. (2014) encouraged students to use diagrams to understand how parts comprise a whole and how to compare quantities within different types of word problems. Similarly, Sharp and Dennis (2017) helped students improve their word-problem solving by drawing models to represent the problem. Jitendra et al. (2013) used different graphic organizers to help students organize information from ratio, percent, and proportion word problems. Flores et al. (2016) used the concrete-pictorial-abstract approach (using hands-on tools and pictures to understand the numbers and symbols of mathematics) to teach students about word-problem solving.

Schema Strategy

Another widely-researched strategy for improving word-problem performance is schema instruction, which involves students categorizing and solving word problems based on problem types (e.g., Fuchs et al., 2010; Griffin et al., 2018; Jitendra et al., 2013; Peltier et al., 2020; Xin & Zhang, 2009). Only a limited number of schemas appear regularly in mathematics textbooks or on high-stakes tests and more broadly in real-life mathematics (Marshall, 1995). By explicitly teaching the schema (i.e., underlying structure) of a word problem, teachers can help students to connect unfamiliar word problems to previously-learned materials (Marshall, 1995). Using the schema strategy to solve problems has led to improved word-problem performance for students (Fuchs et al., 2010; Jitendra et al., 2013; Powell et al., 2021). Researchers have described word-problem schemas for decades as a way for students to understand common patterns in the operations (Carpenter et al., 1981; Cooper & Sweller, 1987: Kintsch & Greeno, 1987), and authors of mathematics teaching textbooks feature the schemas as essential for development sense about the operations (Van de Walle et al., 2019). Schema instruction has been rigorously evaluated and has been recommended as an evidence-based practice for improving word-problem solving (Cook et al., 2019; Gersten et al., 2009; Jitendra et al., 2015; Peltier & Vannest, 2017). For this reason, we used the schema framework to classify the released word problems in our study.

Schemas fall into additive or multiplicative categories (Carpenter, 1981; Kintsch & Greeno, 1985; Van de Walle et al., 2019; Willis & Fuson, 1988; see Table 1 for an overview of the schemas with accompanying examples). Additive schemas involve addition or subtraction and include: (a) total (i.e., part-part-whole or combine), in which two or more separate parts are put together to make a total (e.g., "Maria has 7 blue crayons and 9 red crayons. How many crayons does Maria have in all?"); (b) difference (i.e., compare), in which two sets are compared for a difference (e.g., "Justin has 12 marbles. Jorge has 19 marbles. How many more marbles does Jorge have than Justin?"); and (c) change (i.e., join or separate), in which a starting amount increases or decreases over time because something happens to change the starting amount (e.g., "Lia had some money. Then, she spent $14 at the movies and has $21 left. How much money did Lia have?").

Multiplicative schemas involve multiplication or division and include: (a) equal groups (i.e., array), in which a group or unit is multiplied by a number or rate for a product (e.g., "Jose bought 3 boxes of crayons. Each box had 12 crayons. How many crayons did Jose buy?"); (b) comparison (i.e., scalar), in which a set is multiplied by a number of times for a product (e.g., "DeShawn picked 3 times as many apples as his sister. If DeShawn picked 21 apples, how many apples did his sister pick?"); (c) combination, in which students determine all the pairings between two or more sets (e.g., "Lola has 4 sweaters and 6 pairs of pants. How many possible outfits can Lola make?"); (d) ratios or proportions, in which students analyze relationships among quantities (e.g., "Andrew read 4 pages in 10 minutes. How many pages could he read in 30 minutes?"); and (e) products of measures, in which the product is a different unit from the factors (e.g., "A garden is 10 feet wide and 16 feet long. What is the area of the garden?").

Keyword Strategy

Despite the large research base on effective word-problem solving strategies, many general and special education teachers teach word problems by using a keyword approach (Capraro & Joffrion, 2006; Chapman, 2006; Fuchs et al., 2016; Powell et al., 2021; Seifi et al., 2012; Tan, 2011). Additionally, many students default to a keyword strategy when overwhelmed by word-problem solving (Cummins et al., 1988; Hegarty et al., 1995; Hyde et al., 2003). With the keyword strategy, students identify a single word from the word problem's text and develop a problem solution plan by tying the word to a single operation (Mevarech, 1999). For example, *in all* suggests addition, *left* suggests subtraction, and *share* suggests division (Karp et al., 2019; Lester et al.,1989).

The keyword approach might be beneficial with a careful application given that most students need assistance in understanding the semantics of word problems (Ahmad et al., 2008; Dahmus, 1970). Marshall (1995) noted the keyword approach "works moderately well for students so long as they are confronted only with very simple problems, that is, those requiring only one arithmetic operation" but then the keyword approach "breaks down, usually because problems require several operations to be carried out and because keywords occur less frequently in the problem text" (pp. 76-77). In several cases, researchers have encouraged students to look for or highlight keywords (Pennequin et al., 2010; Wadlington & Wadlington, 2008). Although the researchers did not suggest explicitly tying a keyword to a single operation in these studies, students may implicitly make a connection between a keyword and an operation (e.g., *twice* means to multiply). Thus, all educators need to be aware of the connections students make independent of high-quality mathematics instruction. Educators should explicitly teach students not to overgeneralize keywords by providing opportunities to compare different word problems with the same keyword. For example, solving a problem with *altogether* in which students can add for the correct problem solution versus solving problems with *altogether* in which subtraction, multiplication, or division are necessary.

The keyword approach attempts to simplify the comprehension process for solving a word problem by informing students of the exact operation necessary to solve the problem

(Hegarty et al., 1995). Keywords also alleviate demands on working memory (Campbell et al., 2007). However, this strategy narrowly and superficially focuses on the meaning of certain words rather than engaging students in understanding the meaning of the entire text (Verschaffel et al. 2020). Often, students only attend to a keyword and utilize an operation linked to the keyword without even reading or understanding the text. Furthermore, the keyword strategy may not work with multi-step word problems that require multiple operations to find a solution (Agostino et al., 2010; Clement & Bernhard, 2005; Powell & Fuchs, 2018).

We note that the keyword strategy differs from developing an understanding keywords. With the keyword strategy, a student identifies a keyword, ties the keyword to an operation, and performs the operation indicates by the keyword with the numbers presented in the problem. This strategy is different from understanding the meaning of a keyword. There are many keywords that can cue a student to the action or schema of the word problem. For example, in the problem "Diana and Ben have 12 pencils altogether. Diana has 9 pencils. How many pencils does Ben have?" A student using the keyword strategy would note *altogether* means to add, and the student would add 12 plus 9 for a problem solution of 21. Another student may see *altogether* as a cue that they are given the total number of pencils and have to determine one of the parts (9 + __ = 12). This second student did not tie the keyword to a specific operation so this student did not use the keyword strategy; instead, this student used a keyword (and other textual clues) to deepen the understanding of the problem. In this manuscript, we focus on the keyword strategy.

Purpose and Research Questions

Many researchers oppose the keyword strategy (Clement & Bernhard, 2005; Powell & Fuchs, 2018; Sowder, 1989; Stigler et al., 1990), whereas others continue to document educator use of a keywords approach (Capraro & Joffrion, 2006; Seifi et al., 2012; Tan, 2011). Even with opposition to the keyword strategy, keyword posters are prevalent in mathematics classrooms across the United States (U.S.) at the elementary and middle school levels, and visits to teacher websites and teacher resource stores unveil a variety of keyword posters for printing or purchase. Even select core mathematics curricula specifically teach the keyword strategy, as shown in Figure 1 (Larson & Boswell, n.d.).

<<Figure 1 here>>

In this study, we analyzed word problems from high-stakes assessments to quantify the types of word problems students solve, to determine whether word problems feature a keyword(s), and to identify whether the keyword(s) leads to a correct solution. We asked the following research questions:

1. What are the types of word problems (i.e., directive, routine, non-routine) on high-stakes assessments?
2. On single-step word problems, which schemas are used? How often do keywords appear? How often does a keyword match the correct operation used to solve a problem?
3. On multi-step word problems, which schemas are used? How often do keywords appear? How often does a keyword match the correct operation used to solve a problem?

Method

We used all items from the Partnership for Assessment of Readiness for College and Careers (PARCC) mathematics released items from 2 different years (2014, 2016) at grades 3, 4, 5, 6, 7, and 8 to create a database of released items from different high-stakes mathematics assessments. We also used all items from the Smarter Balanced mathematics released items from

3 different years (2013, 2014, 2016) at grades 3, 4, 5, 6, 7, and 8. We selected these two assessments because of their use across the U.S. (Gewertz, 2019). For example, for the 2016-2017 school year, eight states used the PARCC (CO, IL, LA, MD, MA, NJ, NM, and RI) and 15 states and the District of Columbia used the Smarted Balanced (CA, CT, DE, HI, ID, MI, MT, NV, NH, ND, OR, SD, VT, WA, WV).

<<Figure 2 here>>

Figure 2 shows the process of identifying items for analysis. After deleting 150 duplicate items, we had 747 distinct items for analysis. Of the 747 items, 57 items did not involve comprehension of text. That is, students merely solved a computation problem (e.g., 3950 + 405 = __ or 6.3 ÷ 0.1 = __). The remaining 690 items required the reading of a prompt to answer the item. The first and second authors separately coded all 690 items. We did not identify any non-routine word problems within our collection of high-stakes test items.

Table 1 shows the coding process and provides operational definitions for each step within our coding scheme. For each word problem we coded: (Step 1) whether the problem was directive or routine; (Step 2) whether the problem involved single or multiple steps; (Step 3) the schema(s) of the word problem; (Step 4) all keywords in the word problem; and (Step 5) whether the keyword(s) matched to an operation (see Table 2 for keywords and their assumed operations). For the schemas, we focused on seven schemas (i.e., total, difference, change, equal groups, comparison, combinations, ratios or proportions, and product of measures), all of which are commonly identified in the literature (e.g., Carpenter et al., 1981; Fuchs et al., 2014; Jitendra et al., 2017; Van de Walle et al., 2019; Xin & Zhang, 2009). We did not identify any combinations schemas and, therefore, did not include the combinations schema in our analysis.

<<Table 1 here>>

For keywords, we compiled a list of keywords from educator classroom resources. Table 2 displays the keywords searched within the word problems. Note, this table should not be used as a classroom resource.

<<Table 2 here>>

After the first and second authors coded 100% of the data independently, we checked for discrepancies between the two databases. We calculated inter-rater reliability by using the following formula: agreements / (agreements + disagreements) × 100. Overall, inter-rater reliability was 96.8%. Reliability by the five steps to coding was as follows: (Step 1) whether the problem was directive, routine, non-routine, or only involved computation (n = 747 items with reliability of 99.8%); (Step 2) whether the routine word problem involved single or multiple steps (n = 214 routine items with reliability of 100.0%); (Step 3) the schema(s) of the word problem (7 schemas and 4 operations multiplied by 214 routine items; n = 2,354 items with reliability of 96.0%); (Step 4) all keywords in the word problem (n = 690 directive and routine items with reliability of 98.1%); and (Step 5) whether the keyword(s) matched to an operation (n = 214 routine items with reliability of 88.6%). We discussed and resolved all discrepancies for 100% of items.

We formulated a "match rate" for keywords by determining the problems in which the keyword strategy (e.g., *less than* means to subtract; *times* means to multiply) would lead to a correct problem solution using the keyword-cued operation (e.g., subtract, multiply). We calculated keyword match rates by dividing the number of word problems in which a keyword led to a correct problem solution by the total number of word problems within each category.

Results

As described, of the 747 items, 57 items (7.6%) did not involve comprehension of text, but instead required students to solve for an unknown within a computation problem. Of the 690 items with text, we identified 476 of the 690 (69.0%) as directive word problems and 214 (31.0%) as routine word problems. Of the 214 routine word problems, we identified 130 (60.7%) as single-step word problems and 84 (39.3%) as multi-step word problems.

Directive Word Problems

We categorized over two-thirds of the word-problem items as directive. These word problems required students to read and comprehend text. We noted directive word problems included a wide variety of mathematical vocabulary. That is, items included technical vocabulary exclusive to mathematics (e.g., *diameter*, *hundredth*, *parallelogram*), subtechnical vocabulary with meanings in mathematics and other topics (e.g., *digit*, *expression*, *regular polygon*, *whole number*, *volume*), and general vocabulary (e.g., *complete*, *missing values*, *shade*, *true*; Monroe & Panchyshyn, 1995). We identified keywords in 14.7% (n = 70) of directive word problems.

Single-Step Routine Word Problems

We identified 130 items (60.7% of routine word problems) as single-step routine word problems (see Table 3 with keyword matches across and by grade level). Of these 130 problems, we noted the total, equal groups, and ratios or proportions schemas appeared most often. Within each schema, we calculated a keyword appearing in an item almost 90% of the time, with exceptions for the ratios or proportions schema (79%, n = 23) and change schema (70%, n = 7). When a keyword appeared in an item, over 80% of the time at least one keyword in the prompt could be specific to the schema (e.g., *times* for the comparison schema; see the Schema-specific keywords column of Table 3). Exceptions to this trend included the change schema, in which only 71% (n = 5) of change items with a keyword used a change-specific keyword, and the ratios or proportions schema, in which only 39% (n = 9) of items with a keyword used a keyword about ratios or proportions.

<<Table 3 here>>

Across the 130 single-step word problems, 89% (*n* = 116) featured at least one keyword. Of the problems with a keyword, 34% (*n* = 44) included multiple keywords. For example, “The *total* number of students who chose the zoo is how many *times* as great as the total number of students who chose the planetarium?” featured a keyword often associated with total problems (i.e., *total*) and a keyword often associated with comparison problems (i.e., *times*).

We determined how often a keyword within a word problem could be used to lead to a correct problem solution by matching the keyword to its assumed operation. Even when a word problem featured multiple keywords, if a single keyword in a word problem led to a correct solution, we scored the keyword as leading to a correct problem solution. We calculated keyword match rates by dividing the number of word problems in which a keyword led to a correct problem solution by the total number of word problems within a category (e.g., single-step word problems). If a student used a keyword and tied the keyword to an operation, the keyword matched in 63% (*n* = 46) of word problems with only one keyword (*n* = 73); however, for problems with multiple keywords (*n* = 44), the match rate of keywords was 29% (*n* = 13).

We noted the match rate of using a keyword strategy differed by the type of schemas. On total problems (*n* = 27), 81% (*n* = 21) with a keyword could be solved correctly when tied to the addition operation. Difference problems with a keyword (*n* = 16) could be solved correctly using subtraction with a match rate of 69%. For the other schemas, the match rate of keywords was substantially lower. Students could use keywords to lead to a correct solution within comparison and product of measures schemas approximately 56% of the time. For change, equal groups, and ratios or proportions schemas, the match rate was less than 30%. Table 4 provides examples of word problems in which keywords match or do not match an implied operation.

<<Table 4 here>>

Multi-Step Routine Word Problems

Overall, we identified 84 (39.3% of routine word problems) of the routine word problems as multi-step word problems (see Table 5 for data across and by grade level). Many of these word problems involved the combination of multiple schemas, although a few problems involved the application of the same schema twice (e.g., using equal groups schema, then using equal groups schema again). Of the multi-step problems, the total schema ($n = 40$) and equal groups schema ($n = 49$) appeared most often, followed by the change schema ($n = 21$) and ratios or proportions schema ($n = 22$). Difference ($n = 11$), comparison ($n = 7$), and product of measures ($n = 7$) schemas occurred with little frequency within multi-step word problems.

<<Table 5 here>>

The majority of multi-step word problems included a keyword or keywords, but these keywords rarely matched with an operation that would lead to a correct problem solution. In fact, we noted keywords as never matching for word problems with the comparison schema. In total, difference, change, equal groups, and ratios or proportions schemas, the match rate of keywords leading to a correct problem solution was less than 10%. Keywords only worked effectively in product of measures problems with a match rate of 29%.

Discussion

We conducted our analysis of released items from high-stakes mathematics assessments to determine the types of word problems students set up and solve, whether keywords appear in word problems, and if keywords appear, whether the keywords lead to a correct problem solution. We identified 747 unique items across Grades 3 through 8. Of these, 92% involved reading and comprehending text to answer a mathematics problem. This finding demonstrates the absolute necessity for effective word-problem instruction in classrooms at the elementary and middle school levels.

Directive Word Problems

Of the 690 problems requiring text-based comprehension, we identified two-thirds (i.e., 476) as directive word problems, which ask students to do something. The skills required to solve the directive word problems were broad from across a variety of mathematics content. For example, we noted directive word problems with whole numbers and rational numbers with both positive and negative integers. Items focused on area, bar graphs, comparison, distance, equal parts, equivalency, factors, functions, grid reading, inequalities, line plots, number lines, perimeter, picture graphs, place value, polygons, rounding numbers, scatter plots, systems of equations, three-dimensional figures, time, and volume, among others.

In terms of keywords in directive word problems, we identified keywords in 70 of the 476 directive word problems. However, the keywords used in the directive word problems were rarely associated with their typical operations. For example, in problems such as "Each model equals one whole *divided* into equal parts. Which model shows ¼ shaded?" and "Several expressions are shown. Decide if the value of the expression is *less than*, equal to, or *greater than* 15." These keywords did not lead to an operation that would help students determine a correct problem solution. Therefore, it is unlikely that students can use the keyword strategy to solve directive word problems correctly. Instead, the inclusion of keywords in directive word problems could cause students to approach directive word problems as routine word problems and solve problems incorrectly. For this reason, we consider the inclusion of keywords in directive word problems as a first strike against teaching students to tie keywords to operations as a tool for solving word problems.

Single-Step Routine Word Problems

With the single-step routine word problems, we identified several important findings. Rather than categorizing word problems by operations, which would narrowly emphasize procedural understanding, we categorized word problems by schemas. A schema describes the conceptual structure of the word problem and knowledge of word-problem schemas allows students to transfer knowledge from one word problem to another (Marshall, 1995). A keyword appeared in 63% to 100% of word problems within each word-problem schema. Interestingly, 14 word problems (11% of single-step word problems) featured no keywords. For example, in the problem, "Jo has a piece of tape that is 7/8 inch long. She cuts the tape into two pieces. One piece is 3/8 inch long. How long is the other piece of tape?," we identified no common keyword. This variability (i.e., not all word problems feature keywords) in the appearance of keywords within word problems suggests identifying a keyword to use as a cue for an operation may not be effective. We considered this finding as a second strike against teaching a keyword strategy for solving word problems.

Of the single-step word problems with keywords, 39% to 100% of problems (see Table 3 for percentages by schema) featured a schema-specific keyword, with an overall average of 77%. For example, in the problem, "A baker has 159 cups of brown sugar and 264 cups of white sugar. How many *total* cups of sugar does the baker have?," *total* could provide a clue that the problem involves an addition operation. Problematically, quite a few word problems with keywords featured multiple keywords indicating multiple operations. We calculated the range of word problems featuring multiple keywords as 11% to 71% (by schema; see Multiple keywords column of Table 3). When multiple keywords appear in a word problem (e.g., "Sean cooks 1 package of fish. He eats 3/8 pound of the fish from the package. What is the *total* weight, in pounds, of the cooked fish that is *left* after Sean eats 3/8 pound?"), students receive conflicting messages. That is (in the example about the package of fish), does the student solve the problem as an addition problem (*total*) or as a subtraction problem (*left*)? We identified the inclusion of multiple keywords within word problems as the third strike against using a keyword strategy to teach students to solve word problems.

Finally, we examined whether a keyword led to a correct solution. We coded this match rate leniently. If a single keyword within multiple keywords of a problem could lead to a correct solution, we coded the keyword as a successful match. Therefore, our percentages may be inflated because anywhere from 11% (for product of measures problems) to 71% (for change problems) of routine word problems featured multiple keywords and required students to decide which keyword to use. In total and difference problems, we calculated keywords leading to a correct solution in approximately three-fourths of the problems. With change problems, however, keywords led to a correct solution in approximately one-fourth of the problems. We noted the multiplicative schemas of comparison and product of measures contained keywords leading to a correct solution in about half of the problems. We noted lower percentages for keywords assisting students to solve equal groups and ratios or proportions word problems. Across all seven schemas, keywords could be used to cue a correct operation with an average of 61% matching for the single-step word problems with a single keyword (44 of 72 items), but only 29% matching for single-step word problems with multiple keywords (13 of 44 items). We considered these low match rates as the fourth strike against the use of a keyword strategy.

In Table 3, we provided match rates by grade level for each schema. Because of the small number of items at most grade levels, with some grade levels only featuring one problem of a specific schema, we only discuss grade-level results in which two or more problems of the same schema appeared at the same grade level. For the total, difference, and product of measures problems, we noted an increase then decrease across grade levels with the match rate. For change and equal groups problems, we identified a decrease in match rates across grade levels. We did not have enough comparison problems to determine trends for the comparison schema. For ratios or proportions problems, we noted a small increase in the match rate from Grade 6 to Grade 7 but the overall match rate was below 33%. In order to reliably understand match rate patterns across grade levels, we would need to collect dozens more routine word problems from high-stakes released items.

Multi-Step Routine Word Problems

For our fifth strike against keywords, we turned our attention to multi-step routine word problems. We categorized over one-third of routine word problems as multi-step word problems. Multi-step word problems featured keywords in 76% to 100% of the problems across the schemas. Keywords led to a correct solution infrequently. In fact, with the exception of multi-step problems involving product of measures, keywords demonstrated match rates of less than 10% in total, difference, change, equal groups, and ratios or proportions problems and never (0%) in multi-step word problems featuring the comparison schema.

In Table 5, we analyzed the match rates within each grade level for each schema. While the patterns for single-step word problems varied, we noted match rates, for the most part, decreased across grade levels for the multi-step problems. This patterns shows how a strategy, such as the keyword strategy, at one grade level fades in terms of effectiveness in subsequent grade levels as the mathematics becomes more diversified and the problem solving becomes more complex. Educators need to reflect on whether a strategy they teach at a specific grade level retains its effectiveness as students move from one grade level to the next.

Limitations

Before providing implications for practice, we describe several limitations for this study. First, we only analyzed released items from two high-stakes assessments administered at five time points. We selected the PARCC and Smarted Balanced assessments because different states in the U.S. administer these assessments in Grade 3 through 8 classrooms (Gewertz, 2019). We did not analyze state-specific assessments because the word problems on such assessments may not meet the standards followed by the majority of U.S. states (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Across six grade levels of released items from the two assessments, we obtained over 700 unique word problems; therefore, we express confidence in our review of keywords within word problems in the U.S.

Second, we calculated match rates for keywords based on the assumption that a student, if tying a keyword to an operation, would solve the word problem correctly. We did not account for translation errors or computational errors that students frequently make. Computation mistakes in particular are a very common word-problem error (Haghverdi et al., 2012; Kingsdorf & Krawec, 2014), so our match rate percentages present the best case scenario for rates of word-problem success without accounting for other types of student word-problem errors.

Implications for Practice

With the high percentage (92%) of text-based mathematics problems, setting up and solving word problems must be a focus of mathematics instruction in the elementary and middle-school grades. Word-problem instruction cannot be an activity students participate in after they finish all other mathematics work. Word-problem instruction cannot be relegated to one day a week or the end of a textbook chapter. Because of the variable word-problem match rates of keywords, educators need to teach students more effective word-problem strategies. First and foremost, educators should help students understand the schemas of word problems. Educators can locate information about developing schema knowledge through Cognitively Guided Instruction (Carpenter et al., 2015) and other resources (Fuchs et al., 2014; Jitendra et al., 2013; Powell et al., 2015; Xin & Zhang, 2009). Second, educators should demonstrate and use metacognitive strategies to help students with the process of working through a word problem (Krawec et al., 2012; Montague et al., 2011). Metacognitive strategies help students attack different word problems in a familiar way (e.g., read the problem, make a plan, solve the problem, and check the work) and can become part of a student's internalized problem-solving process. Third, educators should use graphic organizers to help students digest and organize information from a word problem (Swanson et al., 2014; van Garderen et al., 2012). Because students often experience difficulty organizing the relevant information from a word problem, visual representations (e.g., a graphic organizer) can alleviate this difficulty.

Conclusion

The keyword strategy may appear to provide students with an easy method for solving word problems, but when educators promote keywords tied to operations or schemas, students learn a strategy that does not lead to a correct word-problem solution with a 100% match rate. In the short term, keywords can work with some frequency for simple single-step word problems, but keywords do not work with many single-step word problems and frequently fail with multi-step word problems. In many cases, keywords serve as a distractor within word problems. From the long-term perspective, the simplification of tying keywords to specific operations may cause detriment to students. Students may miss the opportunity for developing mathematical reasoning ability and may not be able to solve more complex mathematics word problems. To develop reasoning about word problems, educators must challenge themselves to not rely on the superficial word-problem strategy of using keywords; instead, educators must help students understand the meaning and representation of word problems by providing instruction on the schema of a word problem.

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Table 1

Coding Procedure and Operational Definitions

	Operational Definitions	Examples
Step 1: Determine whether the problem was directive, routine, non-routine, or only with computation		
Directive	A directive or instructional word problem is a text-based problem in which the student is given a direction to do something. Many times, it is a direct question about which or how many or solve.	Which two statements can be represented by the expression 4 × 8?
Routine	A routine word problem is a text-based problem in which numbers are presented within the problem or on a graph. The student has to do something (i.e., manipulate) those numbers in order to solve the problem.	Ana starts eating lunch at 12:15pm. She finishes lunch 40 minutes later. Which clock shows the time that Ana finishes eating lunch?
Non-routine	A non-routine problems is a text-based problems with multiple entry points and multiple correct ways to solve the problems.	At the frame shop, small frames cost $12, medium frames cost $25, and large frames cost $35. If Jamie wants to spend $90 on frames, which type and how many frames can Jamie buy? Determine three ways that Jamie can buy frames.
Only computation (not a word problem)	No text-based problem.	2/6 < ___
Step 2: Determine whether the problem involved single or multiple steps		
Step 3: Identify the word-problem schema(s)		
Total	Two or more parts put together for a total.	Lily has 11 red apples and 18 green apples. How many apples does Lily have altogether?
Difference	Two amount compared for a difference.	Taylor wrote 25 more words than Colin. If Colin wrote 85 words,

		how many words did Taylor write?
Change	An amount that increases or decreases for a new amount.	Elena had $48. Then, she spent $25 at the ballpark. How much money does Elena have now?
Equal groups	Groups with an equal number within each group.	Manuel bought 5 cartons of eggs with 12 eggs in each carton How many eggs did Manuel buy?
Comparison	A set multiplied a number of times.	Sasha picked 7 flowers. Eva picked 3 times as many flowers. How many flowers did Eva pick?
Combination	Pairings between two or more sets.	Zachary had 4 ice cream toppings and 7 flavors of ice cream. How many combinations can he make with one flavor and one topping?
Ratios or proportions	Determining relationships among sets.	Justin baked 15 cookies and 25 brownies. What is the ratio of cookies to brownies?
Product of measures	Determining the product with a different unit from the factors.	The length of the yard is 35 ft. The width of the yard is 60 ft. What is the area of the yard?
Step 4: List all keywords in the word problem Step 5: Determine whether the keyword(s) leads to a correct problem solution by matching the keyword to an operation		
Single-step keyword match	A keyword within a word problem leads to a correct problem solution by connecting the keyword to an implied operation.	There are 425 boys and 510 girls in Hank's school. How many **more** girls are there than boys?
Multi-step keyword match	When a word problem features multiple keywords, as long as a single keyword led to a correct solution.	Ryan has 1/2 pound of chocolate. He **divides** it into 4 **equal** portions. Enter the amount of chocolate, in pounds, in each portion.

Note. For Step 4, we listed all the keywords (see Table 2 for a complete list) shown in the word problem.

Table 2

Overview of Keywords; This should NOT be used as a classroom resource

Addition	Subtraction	Multiplication	Division
add/addition	decrease	area of	divide
all together/altogether	difference	array	equal shares
both	fewer/fewer than	at the rate	evenly
combined	gave away	double	goes into
in all	greater/greater than	each	half
increase	how many/much more/left	groups of	how many in each
join	left/leftover	multiplied by	out of
more/more than	less/less than	of	proportion
plus	minus	product	quotient
sum	remain/remaining	times	ratio
together	subtraction	triple	share
total	take away	twice	split

Table 3

Single-Step Word Problems Data (n = 130)

Schema	Occurrence of schema		Any keyword		Schema-specific keywords[a]		Multiple keywords[a]		Keywor match operati
	n	%	*n*	%	*n*	%	*n*	%	*n*
Total	27	20.8	26	96.3	23	88.5	5	19.2	21
Grade 3[b]	9		9	100.0	8	88.9	4	44.4	6
Grade 4	11		10	90.9	10	100.0	0	0.0	10
Grade 5	5		5	100.0	4	80.0	1	20.0	4
Grade 6	2		2	100.0	1	50.0	0	0.0	1
Difference	16	12.3	16	100.0	13	81.3	2	12.5	11
Grade 3	8		8	100.0	5	62.5	0	0.0	5
Grade 4	2		2	100.0	2	100.0	0	0.0	2
Grade 5	6		6	100.0	6	100.0	2	33.3	4
Change	10	7.7	7	70.0	5	71.4	5	71.4	2
Grade 3	4		3	75.0	1	33.3	1	33.3	1
Grade 4	2		1	50.0	1	100.0	1	100.0	0
Grade 5	2		2	100.0	2	100.0	2	100.0	0
Grade 6	1		0	0.0	–	–	–	–	–
Grade 7	1		1	100.0	1	100.0	1	100.0	1
Equal groups	29	22.3	26	89.7	22	84.6	18	69.2	8
Grade 3	9		8	88.9	8	100.0	5	62.5	4
Grade 4	7		6	85.7	5	83.3	5	83.3	0
Grade 5	11		11	100.0	8	72.7	8	72.7	3
Grade 6	2		1	50.0	1	100.0	0	0.0	1
Comparison	10	7.7	9	90.0	9	100.0	4	44.4	5
Grade 4	8		8	100.0	8	100.0	3	37.5	5
Grade 5	1		0	0.0	–	–	–	–	–
Grade 7	1		1	100.0	1	100.0	1	100.0	0
Ratios or proportions	29	22.3	23	79.3	9	39.1	9	39.1	6
Grade 4	1		0	0.0	–	–	–	–	–
Grade 5	3		1	33.3	0	0.0	0	0.0	0
Grade 6	13		12	92.3	3	25.0	5	41.7	3

Grade 7	11		9	81.8	5	55.6	3	33.3	3
Grade 8	1		1	100.0	1	100.0	1	100.0	0
Product of measures	9	6.9	9	100.0	8	88.9	1	11.1	5
Grade 3	2		2	100.0	2	100.0	0	0.0	1
Grade 4	1		1	100.0	1	100.0	0	0.0	0
Grade 5	4		4	100.0	3	75.0	0	0.0	3
Grade 6	2		2	100.0	2	100.0	1	50.0	1

[a]When a problem featured a keyword.
[b]Grade level data not presented if no schema present at that grade level.

Table 4

Single-Step Word Problem Examples

Schema	Example when keyword(s) match operation	Example when keyword(s) doe
Total	Sloan asked 117 fourth-grade students the question, "How many pets do you have?" She displayed the data she collected in the bar graph shown. Find the **total** number of pets the fourth-grade students have.	Mr. Kahn has a **total** of 148 bal white balloons and equal numb and yellow balloons. How man Mr. Kahn have?
Difference	What is the **difference** between the fraction of money Ammaar put in the bank and the fraction he spent on the book?	The table shows the number of grade classrooms. One of the te teacher, and of teachers is Sue's teacher has 26 **more** books than Tim's teacher?
Change	The storage tank holds 500 liters of water when full. During the first 5 days in January after the tank was filled, 386 liters of water was used on the farm. What is the amount of water, in liters, that **remains** in the tank after those 5 days?	Jason **begins** at the **start** of a pa 11 1/2 miles on the path. The pa long. Enter the distance, in mile reach the end of the path.
Equal groups	Jack has 24 fish. He puts them into 4 bowls. Each bowl has an equal number of fish. How many fish are in **each** bowl?	Ryan makes 6 backpacks. He us to make **each** backpack. What i cloth, in yards, Ryan uses to ma
Comparison	Mr. Soto's bicycle weighs 30 pounds. Mr. Soto's car weighs 9 **times** as much as his bicycle. What is the weight, in pounds, of Mr. Soto's car?	A basketball team scored a **tota** season. This was 9 **times** the nu in the first game. How many po during the first game?
Ratios or proportions	Rosy waxes 2/3 of her car with 1/4 bottle of car wax. At this **rate**, what fraction of the bottle of car wax will Rosy use to wax her entire car?	Chad will drive 672 **more** miles drive at the same **rate**. How ma Chad to drive to 672 miles?
Product of measures	Gina's bedroom floor is in the shape of a rectangle. It is 10 feet long and 9 feet wide. What is the **area of** Gina's bedroom floor?	Ken draws a rectangle with an **a** inches. The width of the rectang is the length, in inches, of Ken's

Table 5

Multi-Step Word Problems Data (n = 84)

Schema	Occurrence of schema[a]		Any keyword		Keyword(s) matches operation[b]	
	n	%	*n*	%	*n*	%
Total	40	47.6	39	97.5	3	7.7
Grade 3[c]	10		10	100.0	2	20.0
Grade 4	13		13	100.0	0	0.0
Grade 5	6		6	100.0	1	16.7
Grade 6	3		3	100.0	0	0.0
Grade 7	5		4	80.0	0	0.0
Grade 8	3		3	100.0	0	0.0
Difference	11	13.1	11	100.0	1	9.1
Grade 3	2		2	100.0	0	0.0
Grade 4	3		3	100.0	0	0.0
Grade 5	2		2	100.0	1	50.0
Grade 8	4		4	100.0	0	0.0
Change	21	23.8	19	95.0	1	5.3
Grade 3	7		7	100.0	1	14.3
Grade 4	3		3	100.0	0	0.0
Grade 5	3		3	100.0	0	0.0
Grade 6	3		2	66.7	0	0.0
Grade 7	5		4	80.0	0	0.0
Equal groups	49	58.3	48	98.0	1	2.1
Grade 3	12		12	100.0	1	8.3
Grade 4	10		10	100.0	0	0.0
Grade 5	11		10	90.9	0	0.0
Grade 6	7		7	100.0	0	0.0
Grade 7	7		7	100.0	0	0.0
Grade 8	2		2	100.0	0	0.0
Comparison	7	8.3	7	100.0	0	0.0
Grade 3	1		1	100.0	0	0.0
Grade 4	3		3	100.0	0	0.0
Grade 5	1		1	100.0	0	0.0
Grade 8	2		2	100.0	0	0.0

Ratios or proportions	22	25.0	16	76.2	1	6.3
Grade 5	2		1	50.0	0	0.0
Grade 6	6		4	66.7	1	25.0
Grade 7	9		6	66.7	0	0.0
Grade 8	5		5	100.0	0	0.0
Product of measures	7	8.3	7	100.0	2	28.6
Grade 3	1		1	100.0	1	100.0
Grade 5	2		2	100.0	0	0.0
Grade 6	2		2	100.0	1	50.0
Grade 7	2		2	100.0	0	0.0

[a]Sum across schemas does not equal 100 because each word problem featured more than one schema.
[b]When a problem featured a keyword.
[c]Grade level data not presented if no schema present at that grade level.

Figure 1

Image from Big Ideas Math *(Larson & Boswell, n.d.)*

Some words imply math operations.

Operation	**Addition**	**Subtraction**	**Multiplication**	**Division**
Key Words and Phrases	added to plus sum of more than increased by total of and	subtracted from minus difference of less than decreased by fewer than take away	multiplied by times product of twice of	divided by quotient of

Note. This figure should NOT be used as a classroom resource.

Figure 2
Process for Selection of Items