Module 3 – Significant Figures

Pretest: Do you know how to use significant figures correctly? If you think you do, take the following pretest to be sure. Check your answers at bottom of this page.

If you do all of the pretest correctly, you may skip Module 3.

1. Add, then write the answer using proper significant figures: 1.008 + 1.008 + 16.0 =
2. Multiply using a calculator, then express your answer in proper sig figs.

   3.14159 x 2.32 =
3. How many significant figures are in each of these?

   a. 0.002030 b. 670.0 c. 2 (exactly)
4. Round these numbers as indicated.

   a. 62.75 to the tenths place. b. 0.090852 to 3 sig figs.

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Lesson 3A: Rules for Significant Figures

Nearly all measurements have *uncertainty*. In science, we need to express

- how much uncertainty exists in measurements, and
- the uncertainty in calculations based on measurements.

The *differentials* studied in calculus provide one method to find a precise range of the uncertainty in calculations based on measurements, but differentials can be time-consuming.

The easier method for expressing uncertainty is **significant figures**, also known as *sig figs*, abbreviated *sf***.** Significant figures provide only an approximation of uncertainty, but for all but the most precise needs, significant figures is the method of choice for measurements and calculations in science.

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Pretest Answers: Your answers must match these exactly.

1. 18.0 2. 7.29 3a. 4 3b. 4 3c. Infinite sig figs 4a. 62.8 4b. 0.0909

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Significant Figures: Fundamentals

Use these rules when recording measurements and rounding calculations in chemistry.

1. **When Recording a Measurement**

   Write all the digits you are sure of, plus the *first* digit that you must *estimate* in the measurement – the first **doubtful digit** (the first **uncertain digit**). Then *stop*.

> When writing a measurement in significant figures, the *last* digit is the first *doubtful* digit.

2. **Adding and Subtracting**

   a. First, add or subtract as you normally would.

   b. Next, search the numbers for the doubtful digit in the *highest place*. The answer's *doubtful* digit must be **in** that *place*. *Round* the answer to that *place*.

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Example:     23.1     ←
           + 16.01
           +  1.008
             40.118   =  40.1
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   This answer must be rounded to **40.1** because the tenths place has doubt.

   The tenths is the highest *place* with doubt among the numbers added.

   Recall that the tenths place is *higher* than the hundredths place, which is higher than the thousandths place.

   c. The logic: If you add a number with doubt in the tenths place to a number with doubt in the hundredths place, the answer has doubt in the tenths place.

   In a measurement, if the number in the tenths place is doubtful, numbers after the tenths place are garbage. We allow one *doubtful* digit in answers, but no garbage.

   d. Another way to state this rule: When adding or subtracting, round your answer back to the last *full column* on the right. This will be the first column of numbers, moving right to left (←), with no *blanks* above.

   The blank space *after* a doubtful digit indicates that we have no idea what that number is, so we cannot add a blank space and get a significant number in the answer in that column.

> **Summary:** When **adding or subtracting**, round your answer back to
> - the highest *place* with doubt, which is also
> - the *leftmost place* with doubt, which is also
> - the last full column on the right, which is also
> - the last column to the right with*out* a blank space.

3. Multiplying and Dividing

This is the rule you will use most often.

a. First multiply or divide as you normally would.

b. Then *count* the *number* of sig figs in *each* of the numbers you are multiplying or dividing. Count sure, certain digits *plus* the doubtful digit.

c. Your answer can have *no more* sig figs than the measurement with the *least* sig figs that you multiplied or divided by. *Round back* to that *number* of sig figs.

Example: 3.1865 cm x **8.8** cm = 28.041 = **28** cm^2 (must *round* to **2** sig figs)
^5 sf ^2 sf ^2 sf

> **Summary: Multiplying and Dividing**
>
> If you **multiply** and/or **divide** a 10-sig fig number and a 9-sig fig number and a 2-sig fig number, you must round your answer to 2 sig figs.

4. Doing Calculations With Steps or Parts

The rules for sig figs should be applied at the **end** of a calculation.

In problems that have several parts, and earlier answers are used for later parts, it is a generally accepted practice to carry one extra sig fig until the end of a calculation, then round to proper sig figs at the final step. This practice minimizes changes in the final answer due to rounding in the steps.

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Practice: First memorize the rules above. *Then* do the problems. (Problems should be a practice *test* that tell you how well you have learned the material.) When finished, check your answers at the end of the lesson.

1. Add and subtract using sig figs.

a.
```
   23.1
 + 23.1
   16.01
```

b. 2.016 + 32.18 + 64.5

c.
```
   19.76
 –  7.3
```

2. Multiply and divide using a calculator. Write the calculator result, then re-write the answer in proper sig fig notation.

a. 3.42 cm x 2.3 cm^2 =

b. 74.3 divided by 12.4 =

3. a. 9.76573 **x** 1.3 = A =

b. A/2.5 =

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ANSWERS: Your answers must match these exactly.

1. (a)

	23.1
+	23.1
	16.01
	62.21 Round to **62.2**

(b) 2.016 + 32.18 + 64.5 = 98.696 -- round to **98.7**

(c)

	19.76
–	7.3
	12.46
	Round to **12.5**

2. a. **7.9 cm^3 (2 sf)** 2b. **5.99 (3 sf)**

3a. **12.7** If this answer were not used in part b, the proper answer would be 13.(2 sf), but since we need the answer in part b, carry an extra sig fig. 3b. 12.7/2.5 = **5.1**

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Lesson 3B: Sig Figs -- Special Cases

There are special sig fig rules for rounding off a 5, zeros, and exact numbers.

1. **Rounding.** If the number *beyond* the place you are rounding to is
   a. *Less* than 5: Drop it (round *down*). Example: 1.342 rounded to *tenths* = 1.3
   b. *Greater* than 5: Round *up*. Example: 1.48 = 1.5
   c. A **5** followed by *other* digits: Round *up*. Example: 1.252 = 1.3

2. **Rounding a lone 5** (A 5 without following digits).

   Some instructors prefer the simple "round 5 up" rule. Others prefer a slightly more precise "engineer's rule" described as follows.

   a. If the number in *front* of the 5 is *even*, round *down* by dropping the 5.

      Example: 1.45 = 1.4

   b. If the number in *front* of the five is *odd*, round it *up*.

      Example: 1.35 = 1.4

   Rounding a lone 5, the rule is "even in front of 5, leave it. Odd? Round up."

   Why not always round 5 up? On a number line, a 5 is exactly halfway between 0 and 10. If you always round 5 up in a large number of calculations, your average will be slightly high. When sending a rover on a 300 million mile trajectory to Mars, if you calculate *slightly* high, you may miss your target by hundreds of miles.

   The "even leave it, odd up" rule rounds a 5 down half the time and up half the time. This keeps the average of rounding 5 in the middle, where it should be.

When rounding off a lone 5, these lessons will use the more precise "engineer's rule," but you should use the rule preferred by *your* instructor.

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Practice A

Round these to the underlined place. Check your answers at the end of this lesson.

1. 23.25 2. 0.0655 3. 0.075 4. 2.659

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3. **Zeros.** When do zeros *count* as sig figs? There are four cases.

   a. Zeros in *front* of *all* other digits (leading zeros) are *never* significant.

      Example: 0.0006 has one sig fig.

   b. Zeros embedded *between* other digits *are* always significant.

      Example: 300.07 has 5 sig figs. (Zeros *sandwiched* by sig figs *count*.)

   c. Zeros *after all other* digits *as well as after* the decimal point *are* significant.

      Example: 565.0 has 4 sig figs. You would not need to include that zero if it were not significant.

   d. Zeros *after* all other digits but *before* the decimal point are assumed to be *not* significant.

      Example: 300 is assumed to have 1 sig fig, meaning "give or take at least 100."

   When a number is written as 300, or 250, it is not *clear* whether the zeros are significant. Many textbooks address this problem by using this rule:

   - "500 meters" means *one* sig fig, but
   - "500**.** meters," with an *unneeded decimal point* added after a zero that is not at the end of a sentence, means 3 sig figs.

   These modules will use that convention as well.

   However, the best way to avoid this ambiguity in the number of significant figures is to use scientific notation.

   4 x 10^2 has *one* sig fig; **4.00** x 10^2 has *3* sig figs.

   In *exponential* notation, only the significand determines the significant figures.

   In *scientific* notation, all of the digits in the significand are significant.

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Why are zeros complicated? Zero has multiple uses in our numbering system.

In cases 3a and 3d above, the zeros are simply "holding the place for the decimal." In that role, they are *not* significant as measurements. In the other two cases, the zeros represent numerical values. When the zero represents "a number between a 9 and a 1 in a measurement," it *is* significant.

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Practice B

Write the number of sig figs in these.

1. 0.0075 2. 600.3 3. 178.40 4. 4640. 5. 800

6. 2.06 x 10^{-9} 7. 0.060 x 10^3

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4. **<u>Exact numbers</u>**. Measurements with *no* uncertainty have an *infinite* number of sig figs. Exact numbers do not add uncertainty to calculations.

   - If you multiply a 3 sig fig number by an *exact* number, round your answer to 3 sig figs.

   This rule means that *exact* numbers are *ignored* when deciding the sig figs in an calculated answer. In chemistry, we use this rule frequently in the following situations.

   a. Numbers in *definitions* are exact.

      <u>Example</u>: The relationship "1 km = 1000 meters," is a definition of kilo- and not a measurement with uncertainty. Both the 1 and the 1000 are exact numbers. Multiplying or dividing by those *exact* numbers will not restrict the sig figs in your answer.

   b. The number **1** in nearly all cases is *exact*.

      <u>Example</u>: The conversion "**1** km = 0.62 miles" is a legitimate equality, but it is not a *definition* and not *exactly* correct The **1** is therefore assumed to be exact, but the 0.62 has uncertainty and has 2 sig figs.

   c. *Coefficients* and *subscripts* in chemical formulas and equations are exact.

      <u>Example</u>: $\mathbf{2}\ H_2 + \mathbf{1}\ O_2 \rightarrow \mathbf{2}\ H_2O$ All of those numbers are exact.

   You will be reminded about these exact-number cases as you need them. For now, simply remember that exact numbers

   - have infinite sig figs, and
   - do not limit the sig figs in an answer.

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<u>ANSWERS</u>

<u>Practice A</u>

1. 23.<u>2</u>5 rounds to **23.2** . The number in front of the 5 is even, so leave it.
2. 0.06<u>5</u>5 rounds to **0.066** . The 5 to be rounded off follows an odd number. Round "odd up."
3. 0.0<u>7</u>5 rounds to **0.08** . When rounding a lone 5, use "even leave it, odd up."
4. 2.<u>6</u>59 rounds to **2.7** . When rounding a 5 followed by other digits, round up.

<u>Practice B</u>

1. 0.0075 has **2** sig figs. (Zeros in front never count.)
2. 600.3 has **4** sig figs. (Sandwiched zeros count.)
3. 178.40 has **5** sig figs. (Zeros after the decimal and after all the numbers count.)
4. 4640. has **4** sig figs. (Zeros after the numbers but before a written decimal count.)
5. 800 has **1** sig fig. (Zeros after all numbers but before the decimal place usually don't count.)
6. 2.06×10^{-9} has **3** sig figs. (The significand in front contains and determines the sig figs.)
7. 0.060×10^{3} has **2** sig figs. (The significand contains the sig figs. Leading zeros never count.)

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Lesson 3C: Sig Fig Summary and Practice

First, memorize the sig fig rules.

1. When expressing a measuring in significant figures, *include* the *first doubtful* digit, then stop.
2. When adding and subtracting,
   a. find the measurement that has *doubt* in the *highest place.*
   b. *Round* your answer to that *place*.
3. When multiplying and dividing,
   a. find the number in the calculation that has the least number of sig figs.
   b. Round your answer to that number of sig figs.
4. If the digit in front of a lone 5, use the rule used by your instructor. Either always round up, or use "even in front of 5, leave it. Odd? Round *up*."
5. For zeros,
   a. zeros in front of all other numbers are never significant.
   b. Sandwiched zeros are always significant.
   c. Zeros after the other numbers and after the decimal are significant.
   d. Zeros after all numbers but before the decimal place are not significant, but if an unneeded decimal point is shown after a zero, the zero is significant.
6. Exact numbers have infinite sig figs.

For reminders and reinforcement, use these flashcards. Identify the flashcards you need using the method provided in Lesson 2D. Make only the flashcards you need.

Front-side (with notch at top right):	Back Side -- Answers
Writing measurements in sig figs, stop where?	At the first doubtful digit
Which digits count as sig figs?	All the sure, plus the doubtful digit
Adding and subtracting, round to where?	The *column* with doubt in highest place = last full column
Multiplying and dividing, round how?	Least # of *sf* in calculation = # *sf* allowed
In counting sig figs, zeros in front	Never count
Sandwiched zeros	count
Zeros after numbers and after decimal	count
Zeros after numbers but before decimal	Probably don't count
Zeros followed by un-needed decimal	count
Exact numbers have	Infinite sig figs

Run the flashcards once until perfect, then start the problems below.

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Practice: First try every other problem on day 1. Try the rest on day 2.

1. Write the number of sig figs in these.

   a. 107.42 b. 10.04 c. 13.40 d. 0.00640 e. 0.043×10^{-4}

   f. 1590.0 g. 320×10^{9} h. 14 (exact) i. 250. j. 4200.

2. Round to the place indicated.

   a. 5.15 cm (tenths place) b. 31.85 meters (3 sig figs)

   c. 0.819 mL (hundredths place) d. 0.0635 cm^2 (2 sig figs)

   e. 0.04070 g (2 sig figs)

3. <u>Addition and Subtraction</u>

   a.
   ```
     1.008
   + 1.008
    32.00
   ```
   b.
   ```
     17.65
   –  9.7
   ```
   c. 39.1 + 124.0 + 14.05 =

4. <u>Multiplication and Division:</u> Write the first 6 digits given by your calculator. Then write the answer with the proper number of sig figs and proper units.

   a. 13.8612 cm x 2.02 cm = b. 4.4 meters x 8.312 meters =

   c. 2.03 cm^2/1.2 cm = d. 0.5223 cm^3/0.040 cm =

   e. $(2.25 \times 10^{-2})(6.0 \times 10^{23})$ f. $(6.022 \times 10^{23}) / (1.50 \times 10^{-2})$

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ANSWERS

1. a. 107.42 **5 sig figs** (Sandwiched zeros count.)
   b. 10.04 **4** (Sandwiched zeros count.)
   c. 13.40 **4** (Zeros after numbers and after the decimal count.)
   d. 0.00640 **3** (Zeros in front never count, but after #s and after decimal count.)
   e. 0.043×10^{-4} **2** (Zeros in front never count. The significand contains and determines the sig figs.)
   f. 1590.0 **5** (The last 0 counts since after #s and after decimal. This sandwiches the first 0.)
   g. 320×10^{9} **2?** (Zeros after numbers but before the decimal usually don't count.)
   h. 14 (exact) **Infinite** (Exact numbers have infinite sig figs.)
   i. 250. **3** (The *decimal point* at the end means the zero before it counts.)
   j. 4200. **4** (The *decimal* at the end means 0 before it counts, and first 0 is sandwiched.)

2. a. 5.15 cm (tenths place) **5.2 cm** (Rounding a lone 5, if number in front is odd, round up.)
   b. 31.85 meters (3 sig figs) **31.8 meters** (Rounding a lone 5, if even in front of 5, leave it.)
   c. 0.819 mL (hundredths place) **0.82 mL** (9 rounds up.)
   d. $0.0635\ cm^2$ (2 sig figs) **$0.064\ cm^2$** (Since 3 is odd, round it up. Zeros in front don't count.)
   e. 0.04070 g (2 sig figs) **0.041 grams** (Zeros in front never count.)

3.

a.	b.	c.
1.008	17.65	39.1
+ 1.008	- 9.7	+ 124.0
32.00	7.95 = **8.0** (5-odd up)	14.05
34.016 = **34.02**		177.15 = **177.2**

4. For help with unit math, see Lesson 2B. For help with exponential math, see Module 1.
   a. 13.8612 cm x 2.02 cm = 27.9996 = **$28.0\ cm^2$ (3 sf)**
   b. 4.4 meters x 8.312 meters = 36.5728 = **$37\ meter^2$ (2 sf,** 5 plus others, round up)
   c. $2.03\ cm^2$/1. 2 cm = 1.69166 = **1.7 cm (2 sf)**
   d. $0.5223\ cm^3$/0.040 cm = 13.0575 = **$13\ cm^2$ (2 sf)**
   e. $(2.25 \times 10^{-2})(6.0 \times 10^{23}) = 14 \times 10^{21}$ **= 1.4×10^{22}** in scientific notation **(2 sf)**
   f. $(6.022 \times 10^{23}) / (1.50 \times 10^{-2})$ **= 4.01×10^{25} (3 sf)**

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