

Uncertainties and significant figures for PH3 and PH6

DEFINITIONS

The usage of various terms is not always consistent between the examination boards. The following definitions reflect the wording in the booklet “The Language of Measurement” produced by the ASE and Nuffield Foundation in consultation with the examination boards.

Resolution

This is the smallest change in the quantity being measured by a measuring instrument which gives a perceptible change in the output.

With a digital instrument this is usually 1 in the least significant figure of the output; e.g. a digital ammeter which gives an output in mA with two decimal places has a resolution of 0.01 mA. With analogue instruments there is no such clear cut answer. For practical purposes, WJEC takes the resolution of an analogue instrument to be the interval between the smallest scale divisions, e.g. a metre rule has a resolution of 1 mm (however we allow full marks for candidates taking half the smallest scale division or similar).

Uncertainty

This estimates the range within which the true value is expected to lie, and is usually expressed as an absolute value, but can be given as a percentage. In more advanced work this can be expressed in terms of a percentage confidence, e.g. “the temperature is $20\text{ }^{\circ}\text{C} \pm 2\text{ }^{\circ}\text{C}$ at a level of confidence of 95%” expresses the idea that if the determination were made a large number of times, it would be between 18 and $22\text{ }^{\circ}\text{C}$, 19 times out of 20. At WJEC A level we do not use confidence limits.

For a single **static** measurement, the uncertainty in the reading can be taken as the resolution of the instrument. If this is impractical [as with some dynamic measurements] then candidates will be given the uncertainty in the question. Alternatively they may be asked to estimate the uncertainty from several readings.

Accuracy

A measurement result is considered to be accurate if it is judged to be close to the true value. It is a quality denoting the closeness of agreement between a measured value and the true value of that quantity. Accuracy normally refers to an individual result. It can be influenced by random and systematic errors. It is also possible to refer to the accuracy of the mean value of a set of results. If there are no systematic errors, the greater the number of measurements the more accurate the mean value is, i.e. the closer it is expected to be to the true value. If there is a systematic error in the readings, e.g. because of a zero error, additional measurements will not improve the accuracy of the mean.

Appropriate significant figures for raw data

In all cases we expect candidates to quote any raw data obtained to a number of significant figures consistent with the resolution of the instrument used.

For example, the width of a microscope slide could be measured using a metre rule, a pair of vernier callipers or a micrometer:

The resolution of a metre ruler is ± 1 mm and so the width could be correctly written as 25 ± 1 mm. However, if we use a pair of vernier callipers of resolution ± 0.1 mm then a (possible) correct value could be 25.2 ± 0.1 mm. Similarly 25.0 ± 0.1 mm would be correct; however writing this as 25 ± 0.1 mm would be incorrect.

Finally, if we use a micrometer of resolution ± 0.01 mm, then a possible value would be written as 25.21 ± 0.01 mm.

Note that it is not the number of significant figures which is the issue here; the place of the least significant figure should reflect the resolution of the instrument. It is expected that the number of significant figures increases as an instrument with a greater resolution is used i.e. 2 significant figures for the metre ruler, 3 for the vernier callipers and 4 for the micrometer [unless it is used for determining a very small dimension, e.g. the diameter of a human hair]. It is probably worth noting that it is very rare for measured values, using school laboratory equipment to be quoted to more than 3 or 4 significant figures.

Processing raw data

When processing raw data it is usual for the processed data to be given the same number of significant figures as the raw data e.g. when determining averages, or square or square root values.

One example is finding the period of oscillation from the timings of 20 oscillations. For a stopwatch of resolution 0.1 s the time taken for 20 oscillations was:

Time for 20 oscillations				Period
Reading 1	Reading 2	Reading 3	Mean	
(s)	(s)	(s)	(s)	(s)
20.3	20.4	20.6	20.4	1.02

In this case all raw data and processed data are quoted to 3 significant figures. For a series of measurements, possibly for pendulums of different lengths, all the readings should be quoted to the resolution of the stopwatch (and not necessarily the same number of significant figures). E.g. for a pendulum with a shorter length:

Time for 20 oscillations				Period
Reading 1	Reading 2	Reading 3	Mean	
(s)	(s)	(s)	(s)	(s)
8.7	8.4	8.6	8.6	0.43

The whole row is now quoted to the same number, 2, significant figures. This is different from the first set of readings. It would be incorrect to give this second set of readings to 3 significant figures as it would imply a precision that wasn't there with readings of 8.70, 8.40 implying an uncertainty of ± 0.01 s.

It is important to remember that when manipulating a value its uncertainty needs to be manipulated in the same way, e.g. when dividing a value by 10, the uncertainty needs to be divided by 10 also.

Note that significant figures will only be assessed in the practical modules (PH3 and PH6) and not in any of the theory modules.

Quoting uncertainties

The uncertainty, and also the resolution, should be quoted to a maximum of 2 significant figures, with generally 1 significant figure being sufficient, especially if the significant figure is greater than 2. E.g. writing ± 0.15 or $\pm 18\%$ is acceptable because both values are less than 2 however it would be advisable to write ± 5.5 as ± 6 . For marking purposes we will accept 1 or 2 significant figures but **no** more than 2.

Derived data

When more than one result is being used to determine an overall value by using a formula or equation, the number of significant figures used is determined by the overall, or absolute, uncertainty. Candidates will be asked to calculate a percentage uncertainty and from this find the absolute uncertainty. The value quoted should then be consistent with this value e.g. for a solution of $50.3578 \pm 5\%$, this would give an absolute uncertainty of ± 0.251789 . This should then be quoted to 1 [or 2] significant figures i.e. ± 0.3 [or ± 0.25] so the final result should be written as 50.4 ± 0.3 [or, just, 50.36 ± 0.25].

How many significant figures / decimal places in derived data?

The result from a calculation should be given to no more significant figures than can be justified. This is usually straightforward when a value for the estimated uncertainty is known. For example, if a resistance value is calculated as 359.27Ω and the estimated uncertainty is $\pm 5 \Omega$, the result should be quoted as $359 \pm 5 \Omega$. Note again that we normally give the uncertainty to 1 significant figure.

Very often however, no information is given about the uncertainty in the data which are used in the calculation; e.g. the length of a rectangular strip is 48.3 cm and the width is 8.9 cm. What is its area?

Simple multiplication of the two numbers gives the area as $48.1 \times 8.9 \text{ cm}^2 = 428.09 \text{ cm}^2$. The basic **safe** rule of thumb is to quote the answer to the smallest number of significant figures in the data. In this case 8.9 cm has 2 significant figures, so the answer should be given as 430 cm^2 [2 significant figures]. On this occasion the rule is a little pessimistic. Suppose we knew that the original data were $\pm 0.1 \text{ cm}$ then:

$$\text{Percentage uncertainty in length} = \frac{0.1}{48.1} \times 100 = 0.2 \%$$

$$\text{Percentage uncertainty in width} = \frac{0.1}{8.9} \times 100 = 1.1 \%$$

So the total percentage uncertainty is 1.3 %. Now 1.3% of $428.09 \text{ cm}^2 = 5.6 \text{ cm}^2$, so if we knew the original uncertainties we'd express the final value for the area as $428 \pm 6 \text{ cm}^2$. For this reason either 2 or 3 significant figures in the answer is acceptable. We just need to remember, if we quote the answer to 3 significant figures, that we are less sure of the last significant figure than we were in the data.

The same rule of thumb holds when we are raising to a power, e.g. finding the volume of a sphere of radius 5.35 mm.

$$\text{Volume} = \frac{4}{3}\pi r^3 = 641.43 \text{ mm}^3 \text{ [by calculation]}$$

We should express this to 3 significant figures, in line with the data, so volume = 641 mm³.

Exercise: Assume that the uncertainty in the radius is ± 0.01 mm. What is the uncertainty in the volume? Are 3 significant figures justified?

Can we ever justify giving more significant figures in the result than in the data?

What is the circumference of a cylinder which has a diameter of 3.5 m? [i.e. 2 significant figures]

$$\text{Circumference} = \pi D = 10.996 \text{ m [by calculation].}$$

If we assume that the original diameter was ± 0.1 m that gives a percentage uncertainty of $\sim 3\%$. Using the calculated figure for circumference with 3% uncertainty gives ± 0.33 m so it would be reasonable to quote the circumference as 11.0 m, i.e. 3 significant figures. This happens when the answer has a low first significant figure [1] but the data had a higher first significant figure [3].

Should we ever give fewer significant figures in the result than in the data?

2 microscope slides [assumed to be identical], placed end to end have a total length of 19.5 cm [3 significant figures]. What is the length of a single microscope slide?

Again, assume an uncertainty of ± 0.1 cm in the total length. This represents 0.5%.

By calculation: $0.5 \times 19.5 \text{ cm} = 9.75 \text{ cm}$ and 0.5% of 9.75 cm = 0.05 cm.

This suggests that it would be reasonable to quote the length of a single slide as 9.75 cm but to bear in mind that the uncertainty in this figure is much more than ± 1 in the last place. On the other hand, it would not be unreasonable to quote the length as 9.8 cm.

So, the answer to how many significant figures are justified in derived data is that it is usually safe to give the same number of significant figures as the least accurate piece of data, but that it is often justified to go up by 1 significant figure in the answer, especially if the first significant figure in the answer is lower than in the data.

Another example: Finding $1/T$ e.g. when wanting to plot $1/T$ versus \sqrt{m} for a spring oscillating:

$$T = 1.01 \text{ s} \quad \text{hence } 1/T = 0.99 \text{ (3s.f. to 2s.f.)}$$

$$T = 0.99 \text{ s} \quad \text{hence } 1/T = 1.01 \text{ (2s.f. to 3s.f.)}$$