Credit Risk: Intro and Merton Model

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Outline

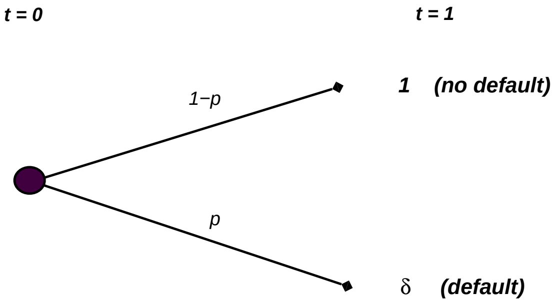
A Gentle Introduction

The Merton Model

- Pricing credit risk
- Predicting credit risk

Basic Idea

- Consider a zero-coupon corporate bond with maturity $T = 1$.



$\delta \leq 1$: recovery rate
p: 1-year default probability

- How to price the bond given p, δ?
- Where do p, δ come from?

A First Attempt

- Historical data on ratings, defaults, and recovery rates.
  - 10-year default rate for Baa-rated firms: ~ 5%
  - average recovery rate of defaulted bonds: ~ 50%
- Suppose one-year risk-free rate is r.
  - Default-free bond:

$$B = e^{-r}$$

  - Baa-bond:

$$P = e^{-r}((1-p)\cdot 1 + p\cdot\delta) = B(1-p\cdot(1-\delta))$$

- Yield-to-maturity and credit spread:

$$P = e^{-y} \quad \Longrightarrow \quad y - r = -\ln(1-p\cdot(1-\delta)) \approx p\cdot(1-\delta)$$

  - Baa-bond:

$$y - r \approx 5\%/10\cdot(1-50\%) = 25\,bps$$

QUESTIONS

- How reliable are the estimates of p and δ based on historical data?
  - ↪ Small sample + "rare event" (more on this later)
  - ↪ Average vs. conditional value

- What about risk adjustments?

$$P = E[\pi_T P_T] = (1-p)\cdot\pi^{ND}\cdot 1 + p\cdot\pi^{D}\cdot\delta$$

$$= e^{-r}\left(\frac{(1-p)\pi^{ND}}{(1-p)\pi^{ND}+p\pi^{D}}\cdot 1 + \frac{p\pi^{D}}{(1-p)\pi^{ND}+p\pi^{D}}\cdot\delta\right)$$

$$= e^{-r}\left((1-\widehat{p})\cdot 1 + \widehat{p}\cdot\delta\right) = e^{-r}E^{\mathbb{Q}}[P_T]$$

  - ↪ What's the intuition for $p \to \widehat{p}$?

- Other factors: taxes, liquidity

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The Merton Model

- The fundamental challenge of limited data remains for reduced-form approach (especially for aggregate components of default risk, and for the Chinese market).

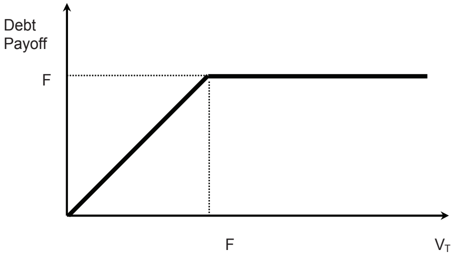
- Structural models: impose structural assumptions to model default (and capital structure) decisions.

- A firm finances its operation by issuing both equity and debt. Its total asset value is V_t. Assume the firm issues a zero coupon bond with face value F and maturity T.

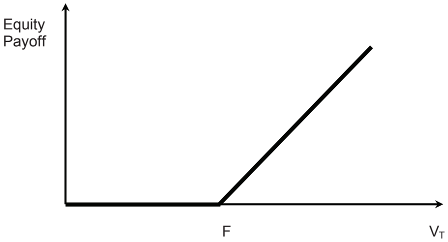
- Possible outcomes for debt holders at maturity T:
  1. $V_T > F \Longrightarrow$ the firm sells some assets and pay the debt holders
  2. $V_T < F \Longrightarrow$ the firm is unable to pay debt holders in full

The Merton Model

Debt holders Payoff at T



Equity holders Payoff at T



A Structural Credit Risk Model

- Probability of default at T (between $[0, T]$) = $\Pr(V_T < F)$
- Need a model for V_t
- Merton (1974): Assume the firm's return on (market) assets between 0 and T is log-normally distributed:

$$dV_t = \mu V_t dt + \sigma V_t dZ_t$$

- This implies a log-normally distributed V_T, from which we can easily compute $\Pr(V_T < F)$.

$$V_T = V_0 \times e^{\left(\mu - \frac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}\epsilon}$$

- Using the B-S analogy, we can also price the bond (and equity) and derive the credit spread.

Value of Equity – Analogy to Black-Scholes

- The payoff to equity holders is just like a call option on the stock:

$$\max\left(V_T - F, 0\right)$$

- While B-S models stock price as lognormal, we have firm value as lognormal.
- We can simply apply Black and Scholes formula and obtain

$$E_0 = \mathrm{Call}\left(V_0, F, r, T, \sigma\right)$$

where $\mathrm{Call}\left(V_0, F, r, T, \sigma\right)$ is given by the Black-Scholes formula

Value of Equity

$$\mathrm{Call}\left(V_0, F, r, T, \sigma\right) = V_0\, N(d_1) - F\, e^{-rT}\, N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + \left(r + \sigma^2/2\right) T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

The Value of Debt

- The payoff to debt holders is

$$\min(V_T, F) = V_T - \max(V_T - F, 0)$$

- The value today of this payoff is then

$$D_0 = V_0 - E_0 = V_0 - Call(V_0, F, r, T, \sigma) \qquad (\star)$$

Accounting identity:

$$\text{Total Asset Value of a Firm} = \text{Debt} + \text{Equity}$$

- An alternative (more intuitive) expression for the value of debt:

$$D_0 = Fe^{-r\times T} - Put(V_0, F, r, T, \sigma) \qquad (\dagger)$$

$$\text{Value of risky debt} = \text{Value of risk-free debt} - \text{Put}$$

↪ Put option: the risk-adjusted expected losses due to default.

CREDIT SPREADS

$$\text{Credit Spread} = \text{YTM on corporate bond} - \text{YTM on Tresuary}$$

- From the definition of yield to maturity y for a corporate bond, we have:

$$D_0 = e^{-y\times T} \times F \quad \Longrightarrow \quad Fe^{-r\times T} - Put(V_0, F, r, T, \sigma) = e^{-y\times T} F$$

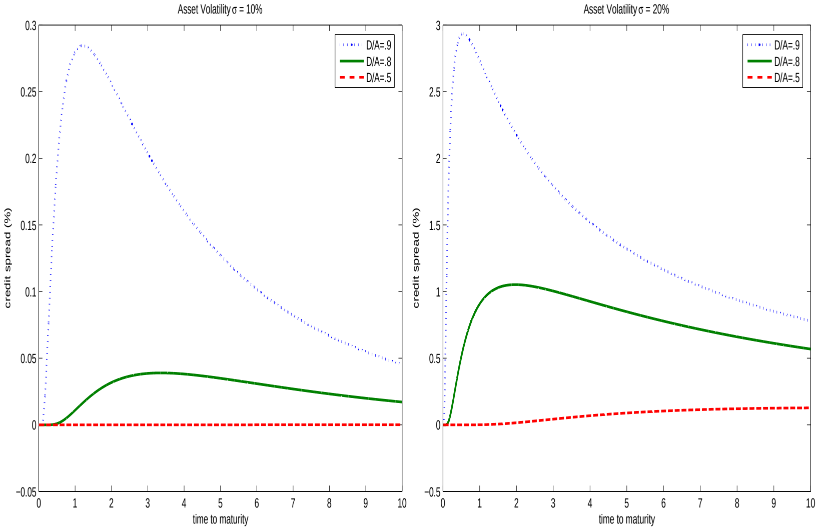
which implies

$$e^{-r\times T} - Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right) = e^{-y\times T}$$

$$1 - e^{r\times T} \times Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right) = e^{-(y-r)\times T}$$

$$\text{Credit Spread} = y - r = -\frac{1}{T} log\left[1 - e^{r\times T} Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right)\right]$$

Credit Spreads under the Merton Model



- Issues: (A) They are small; (B) They converge to zero at $T \to 0$

The Volatility of Levered Equity

What is the volatility of levered equity?

$$\text{Volatility of Equity Returns} = \sigma_E = \left(\frac{VN(d_1)}{VN(d_1) - Ke^{-rT}N(d_2)} \right) \times \sigma$$

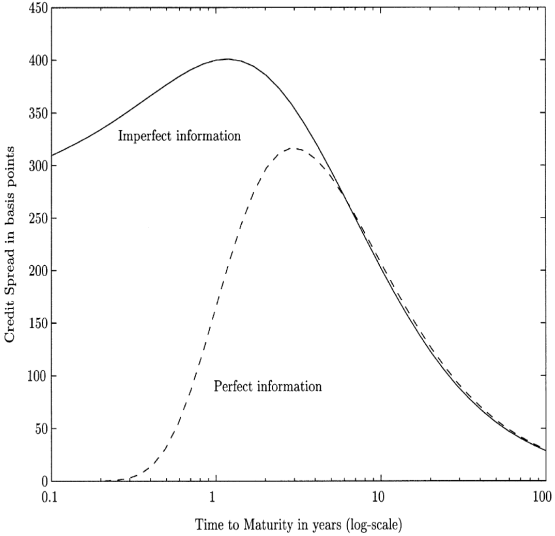
- How large can the term in parenthesis be?
- As V decreases, equity volatility increases.
- **Leverage effect**: $E = Call(V, F)$ is strictly increasing in V. Thus, the model implies that when E decreases, its volatility increases.
- The model thus features "endogenous" time-varying equity volatility that is negatively correlated with the value of equity.

Many Extensions

- Early bankruptcy (Black and Cox 92)
  - ↪ American put option: there is a lower bound V_b to assets so that as soon as $V_t < V_b$ the firm is bankrupt
- Coupon bond: a compound option problem (Geske 92)
- Stochastic interest rates (Longstaff and Schwartz 92)
- Stationary leverage (Collin-Dufresne and Goldstein 00)
  - ↪ Merton model indicates decline in leverage over time
- Unobservable firm value (Duffie and Lando 01)
  - ↪ Investors can only rely on noisy accounting information to estimate V_t: the default barrier could be closer than you think
- Optimal capital structure and default: with perpetual debt (Leland 94); “finite” maturity (Leland and Toft 96); dynamic adjustment (Goldstein, Ju, Leland 01)

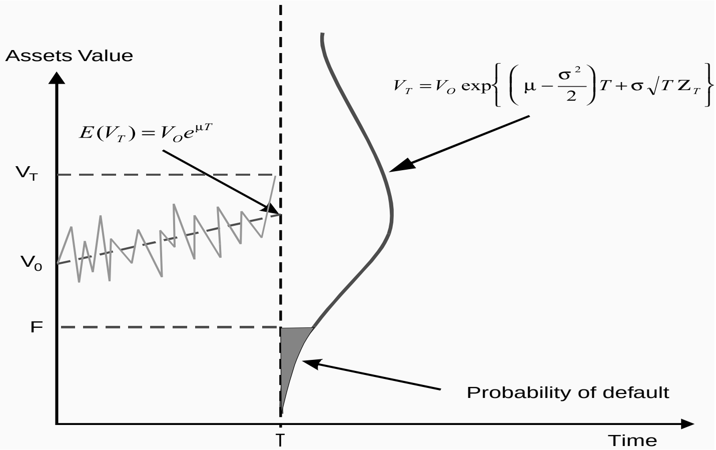
Credit Spreads under Imperfect Information

Duffie and Lando (2001)



Credit Risk Measurement: KMV

- KMV argues that credit ratings did not tell the whole story
  - ↪ e.g., Bonds with same rating show different risks of default
- They use Merton model to compute the probabilities of default:



Distribution of asset value at the maturity of debt

Credit Risk Measurement: KMV

- More specifically, they obtain

$$\text{Expected Default Frequency} = p_T = \Pr\left[V_T < F | V_0\right] = N\left(-d_2\right)$$

$$\text{Distance to Default} = d_2 = \frac{\ln\left(\frac{V_0}{F}\right) + \left(\mu - \sigma^2/2\right) T}{\sigma\sqrt{T}}$$

- What are the unknowns?
  1. V_0: book values of assets are unreliable
  2. μ: expected growth rate of assets
  3. σ: the volatility of assets
  4. F: the default point

- They set F = Short Term Debt + 1/2 Long Term Debt.

WHERE TO FIND V_0 AND σ?

- What can we observe about a public firm? Equity value and volatility.
- Recall what Merton model implies about equity value:

$$E_0 = \text{Call}(V_0, F, T, r, \delta, \sigma) = N(d_1)\, V_0 - Fe^{-rT} N(d_2)$$

- Equity volatility:

$$\sigma_E = N(d_1)\left(\frac{V_0}{E_0}\right)\sigma$$

- Therefore, we get to use the two equations to solve for two unknown V_0 and σ:

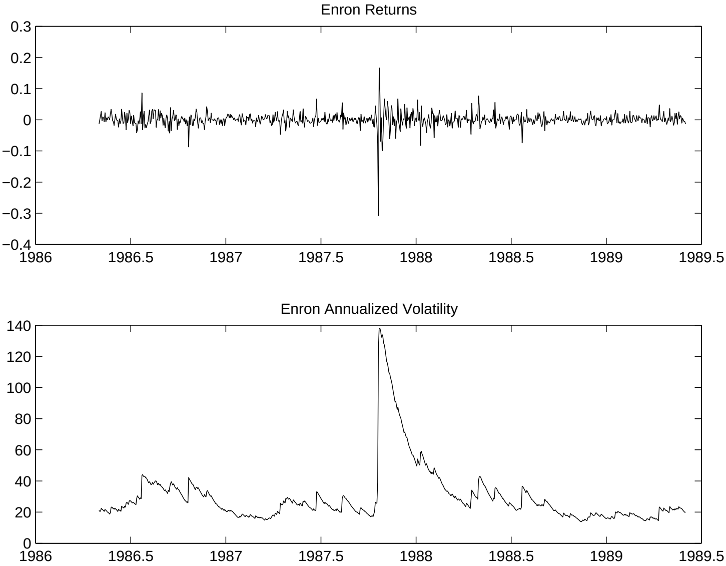
$$E_0 = \text{Market Value of Equity}; \quad \sigma_E = \text{Volatility of Equity}.$$

Credit Risk Measurement: KMV

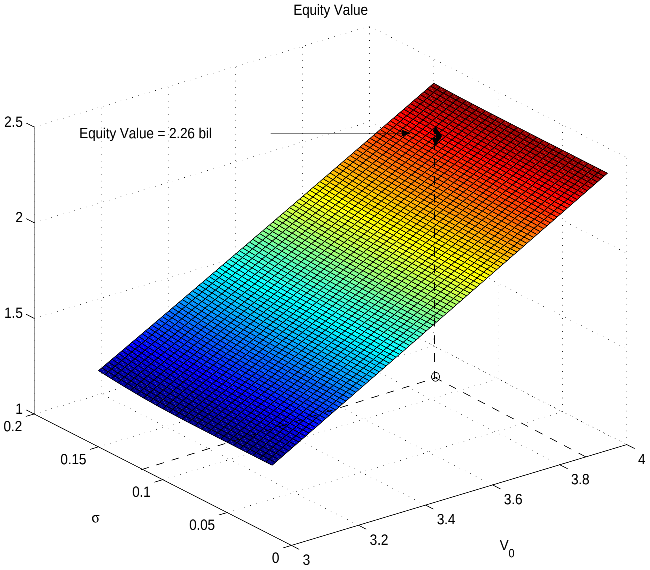
- Simple Example (KMV model is much more elaborate):
  - ↪ Enron market capitalization on May 30 1989 was 2.260 bil
  - ↪ The book value of debt = 3.249 bil (prospectus)
  - ↪ Volatility of equity return = 20%
  - ↪ The nominal one year interest rate was 8.6% (continuously compounded)
  - ↪ Assume $T = 8$ years (long term debt)

- Next two figures plot the value of equity and volatility of equity implied by the Merton model for various levels of current assets V_0 and volatility σ

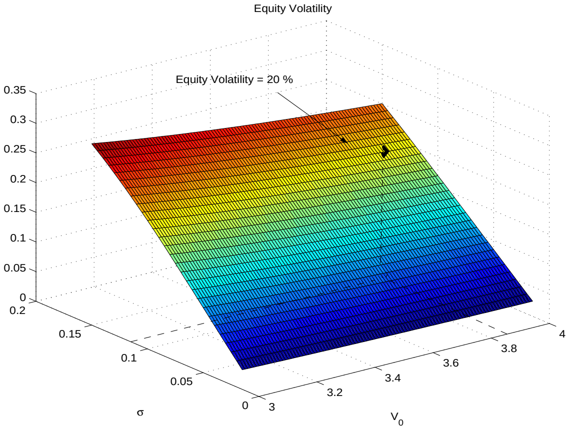
Enron Corp Returns and Volatility



Model implied value of equity



Model implied volatility of equity



- We therefore find $V_0 = 3.84$ bil and $\sigma = 12\%$
- We need one final input: the growth rate of assets μ. This must be forecasted from fundamentals.
- Assume $\mu = 15\%$. We find:

$$d_2 = 2.69 \quad \text{and} \quad p_T = 0.36\%$$

CREDIT RISK MEASUREMENT: KMV

- KMV: normal distribution imperfect, especially the thin tails.
- They estimate a new (non-parametric) mapping between distance to default and expected default frequency from data.

Distance to Default and Expected Default Frequency

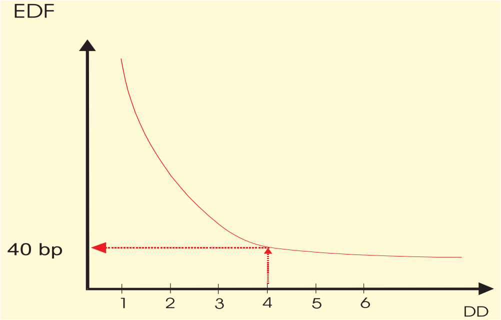


Fig. 17. Mapping of the "distance-to-default" into the "expected default frequencies", for a given time horizon.

- Barath and Shumway (08): little evidence that KMV EDF outperforms Merton model.

WHAT'S NEXT?

- How to apply Merton model to banks?
  - ↪ Merton model assumes constant volatility for asset value. Bad assumption for banks.
  - ↪ How to model asset volatility better? Big part of banks' assets are defaultable debt.
  - ↪ Use this feature to endogenously generate asset volatility. (Nagel and Purnanandam 15)
  - ↪ What about short term debt?
- How to model SOE debt?
- How to model government debt?