KINEMATICS

HORIZONTAL KINEMATICS (Basic Practice)

Question 1 **(**)**

A particle passes through the point A with speed 31 ms^{-1}, moving along a straight horizontal path with constant deceleration 2.5 ms^{-2}. The particle passes through the point B, 12 s after passing through A.

Determine the speed of the particle as it passes through B.

$\boxed{v = 1 \text{ ms}^{-1}}$

$u = 31 \text{ ms}^{-1}$
$a = -2.5 \text{ ms}^{-2}$
$s =$
$t = 12 \text{s}$
$v = ?$

USING $v = u + at$
$v = 31 - 2.5 \times 12$
$v = 31 - 30$
$v = 1 \text{ ms}^{-1}$

Question 2 **(**)**

A particle passes through the point A with speed 35 ms^{-1}, moving along a straight horizontal path with constant deceleration 0.8 ms^{-2}. The particle passes through the point B, 15 s after passing through A.

Find the distance AB.

$\boxed{|AB| = 435 \text{ m}}$

$u = 35 \text{ ms}^{-1}$
$a = -0.8 \text{ ms}^{-2}$
$s = ?$
$t = 15 \text{s}$
$v =$

$s = ut + \frac{1}{2}at^2$
$s = 35 \times 15 + \frac{1}{2} \times (-0.8) \times 15^2$
$s = 525 - 90$
$s = 435 \text{ m}$

Question 3 **(**)**

A particle passes through the point A with speed 8 ms^{-1}, moving along a straight horizontal path with constant acceleration 2 ms^{-2}.

The particle passes through the point B, where $AB = 56.25 \text{ m}$.

Find the speed of the particle as it passes through B.

$\boxed{v = 17 \text{ ms}^{-1}}$

$u = 8 \text{ ms}^{-1}$
$a = 2 \text{ ms}^{-2}$
$s = 56.25 \text{ m}$
$t =$
$v = ?$

$v^2 = u^2 + 2as$
$v^2 = 8^2 + 2 \times 2 \times 56.25$
$v^2 = 64 + 225$
$v^2 = 289$
$v = 17 \text{ ms}^{-1}$

Question 4 **(**)**

A particle passes through the point A with velocity 6 ms^{-1}, moving along a straight horizontal path with constant acceleration.

The particle passes through the point B with velocity 30 ms^{-1}, 15 s after passing through A.

Find the distance AB.

$\boxed{|AB| = 270 \text{ m}}$

$u = 6 \text{ ms}^{-1}$
$a =$
$s = ?$
$t = 15 \text{ s}$
$v = 30 \text{ ms}^{-1}$

using $s = \frac{u+v}{2} \times t$
$s = \frac{6+30}{2} \times 15$
$s = \frac{36}{2} \times 15$
$s = 18 \times 15$
$s = 270 \text{ m}$

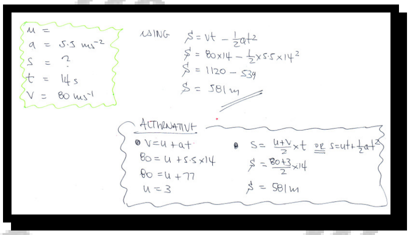
Question 5 (**)

A particle passes through the point A with velocity V ms^{-1}, $V > 0$, moving along a straight horizontal path with constant acceleration 5.5 ms^{-2}.

The particle passes through the point B with velocity 80 ms^{-1}, 14 s after passing through A.

Calculate the distance AB.

$$\boxed{|AB| = 581 \text{ m}}$$



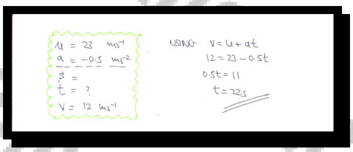
Question 6 (**)

A particle passes through the point A with velocity 23 ms^{-1}, moving along a straight horizontal path with constant deceleration 0.5 ms^{-2}.

The particle passes through the point B with velocity 12 ms^{-1}.

Calculate the time it takes the particle to travel from A to B.

$$\boxed{t = 22 \text{ s}}$$



Question 7 **(**)**

A particle passes through the point A with speed 18 ms^{-1}, moving along a straight horizontal path with constant deceleration. The particle passes through the point B, where $AB = 52 \text{ m}$, 4 s after passing through A.

Find the deceleration of the particle.

$$\boxed{|a| = 2.5 \text{ ms}^{-2}}$$

$u = 18 \text{ ms}^{-1}$
$a = ?$
$s = 52 \text{ m}$
$t = 4 \text{ s}$
$v =$

$$s = ut + \tfrac{1}{2}at^2$$
$$52 = 18 \times 4 + \tfrac{1}{2} \times a \times 4^2$$
$$52 = 72 + 8a$$
$$-20 = 8a$$
$$a = -2.5 \text{ ms}^{-2}$$

Question 8 **(**)**

A particle passes through the point A with velocity 28 ms^{-1}, moving along a straight horizontal path with constant deceleration 2.25 ms^{-2}.

The particle passes through the point B with velocity 19 ms^{-1}.

Find the distance AB.

$$\boxed{|AB| = 94 \text{ m}}$$

$u = 28 \text{ ms}^{-1}$
$a = -2.25 \text{ ms}^{-2}$
$s = ?$
$t =$
$v = 19 \text{ ms}^{-1}$

USING $v^2 = u^2 + 2as$
$$19^2 = 28^2 + 2(-2.25)s$$
$$361 = 784 - 4.5s$$
$$4.5s = 423$$
$$s = 94 \text{ m}$$

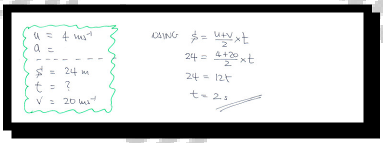
Question 9 (**)

A particle passes through the point A with velocity 4 ms^{-1}, moving along a straight horizontal path with constant acceleration.

The particle passes through the point B, where $AB = 24$ m, with velocity 20 ms^{-1}.

Calculate the time it takes the particle to travel from A to B.

$\boxed{t = 2 \text{ s}}$



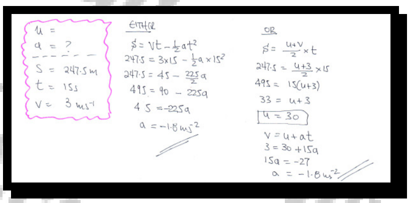
Question 10 (**+)

A particle passes through the point A with velocity $U \text{ ms}^{-1}$, $U > 0$, moving along a straight horizontal path with constant deceleration.

The particle passes through the point B, where $AB = 247.5$ m, with velocity 3 ms^{-1}, 15 s after passing through A.

Calculate deceleration of the particle.

$\boxed{a = -1.8 \text{ ms}^{-2}}$



Question 11 (**)

A particle passes through the point A with velocity V ms^{-1}, $V > 0$, moving along a straight horizontal path with constant acceleration 3.5 ms^{-2}.

The particle passes through the point B with velocity 25 ms^{-1}, 10 s after passing through A.

Find the value of V.

$\boxed{V = -10}$

$u = ?$
$a = 3.5 \text{ ms}^{-2}$
$s =$
$t = 10 \text{ s}$
$v = 25 \text{ ms}^{-1}$

USING $v = u + at$
$25 = u + 3.5 \times 10$
$25 = u + 35$
$u = -10$

$\therefore V = -10$

Question 12 (**)

A particle passes through the point A moving along a straight horizontal path with constant acceleration 2.5 ms^{-2}.

The particle passes through the point B, where $AB = 282.75$ m, 13 s after passing through A.

Determine the speed of the particle as it passes through A.

$\boxed{u = 5.5 \text{ ms}^{-1}}$

$u = ?$
$a = 2.5 \text{ ms}^{-2}$
$s = 282.75 \text{ m}$
$t = 13 \text{ s}$
$v =$

USING $s = ut + \frac{1}{2}at^2$
$282.75 = u \times 13 + \frac{1}{2} \times 2.5 \times 13^2$
$282.75 = 13u + 211.25$
$71.5 = 13u$
$u = 5.5 \text{ ms}^{-1}$

Question 13 (**)

A particle passes through the point A with speed 4 ms^{-1}, moving along a straight horizontal path with constant acceleration.

The particle passes through the point B, where $AB = 320 \text{ m}$, with speed 28 ms^{-1}.

Find the acceleration of the particle.

$\boxed{a = 1.2 \text{ ms}^{-2}}$

$u = 4 \text{ ms}^{-1}$
$a = ?$
$s = 320 \text{ m}$
$t =$
$v = 28 \text{ ms}^{-1}$

USING $v^2 = u^2 + 2as$
$28^2 = 4^2 + 2a \times 320$
$784 = 16 + 640a$
$768 = 640a$
$a = 1.2 \text{ ms}^{-2}$

Question 14 (**)

A particle passes through the point A with velocity $U \text{ ms}^{-1}$, moving along a straight horizontal path with constant deceleration.

The particle passes through the point B, where $AB = 126 \text{ m}$, with velocity 9 ms^{-1}, 12 s after passing through A.

Find the value of U.

$\boxed{U = 12}$

$u = ?$
$a =$
$s = 126 \text{ m}$
$t = 12 \text{ s}$
$v = 9 \text{ ms}^{-1}$

USING $s = \frac{u+v}{2} \times t$
$126 = \frac{u+9}{2} \times 12$
$126 = 6(u+9)$
$21 = u + 9$
$u = 12 \text{ ms}^{-1}$

ie $U = 12$

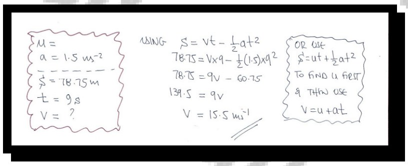
Question 15 **(**)**

A particle passes through the point A moving along a straight horizontal path with constant acceleration 1.5 ms^{-2}.

The particle passes through the point B, where $AB = 78.75 \text{ m}$, 9 s after passing through A.

Find the speed of the particle as it passes through B.

$$\boxed{v = 15.5 \text{ ms}^{-1}}$$



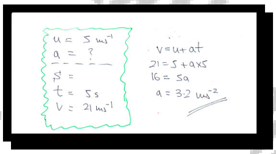
Question 16 **(**)**

A particle passes through the point A with velocity 5 ms^{-1}, moving along a straight horizontal path with constant acceleration.

The particle passes through the point B with velocity 21 ms^{-1}, 5 s after passing through A.

Calculate acceleration of the particle.

$$\boxed{a = 3.2 \text{ ms}^{-2}}$$



Question 17 (**)

A particle passes through the point A with speed 10 ms^{-1}, moving along a straight horizontal path with constant acceleration 4 ms^{-2}.

The particle passes through the point B, where $AB = 59.5 \text{ m}$.

Calculate the time it takes the particle to travel from A to B.

$\boxed{t = 3.5 \text{ s}}$

$u = 10 \text{ ms}^{-1}$
$a = 4 \text{ ms}^{-2}$
$s = 59.5 \text{ m}$
$t = ?$
$v =$

$s = ut + \frac{1}{2}at^2$
$59.5 = 10t + \frac{1}{2} \times 4 \times t^2$
$59.5 = 10t + 2t^2$
$119 = 20t + 4t^2$
$4t^2 + 20t - 119 = 0$
$(2t+17)(2t-7) = 0$
$t = \frac{7}{2}$ or $\cancel{-\frac{17}{2}}$
$t = 3.5 \text{ s}$

ALTERNATIVE
- $v^2 = u^2 + 2as$
  $v^2 = 10^2 + 2 \times 4 \times 59.5$
  $v^2 = 576$
  $v = 24$
- $v = u + at$
  $24 = 10 + 4t$
  $14 = 4t$
  $t = 3.5$

Question 18 (**)

A particle passes through the point A with velocity $V \text{ ms}^{-1}$, where $V > 0$, moving along a straight horizontal path with constant deceleration 6 ms^{-2}.

The particle passes through the point B, where $AB = 40 \text{ m}$, with velocity 14 ms^{-1}.

Find the value of V.

$\boxed{V = 26}$

$u = ?$
$a = -6 \text{ ms}^{-2}$
$s = 40 \text{ m}$
$t =$
$v = 14 \text{ ms}^{-1}$

USING $v^2 = u^2 + 2as$
$14^2 = u^2 + 2(-6) \times 40$
$196 = u^2 - 480$
$676 = u^2$
$u = 26 \text{ ms}^{-1}$

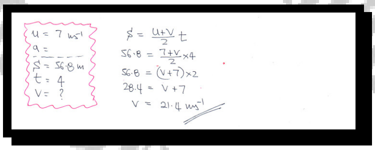
ie $V = 26$, $V > 0$

Question 19 (**)

A particle passes through the point A with speed 7 ms^{-1}, moving along a straight horizontal path with constant acceleration. The particle passes through the point B, where $AB = 56.8 \text{ m}$, 4 s after passing through A.

Determine the speed of the particle as it passes through B.

$\boxed{v = 21.4 \text{ ms}^{-1}}$



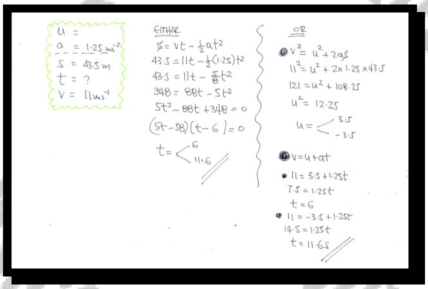
Question 20 (**)

A particle passes through the point A moving along a straight horizontal path with constant acceleration 1.25 ms^{-2}.

The particle passes through the point B, where $AB = 43.5 \text{ m}$, with speed 11 ms^{-1}.

Calculate the **times** it takes the particle to travel from A to B.

$\boxed{t = 6 \text{ s}, 11.6 \text{ s}}$



Question 21 (**)

A particle passes through the point A with speed 7 ms^{-1}, moving along a straight horizontal path with constant acceleration 2 ms^{-2}.

The particle passes through the point B, where $AB = 44 \text{ m}$.

a) Find the speed of the particle as it passes through B.

b) Calculate the time it takes the particle to travel from A to B.

$\boxed{v = 15 \text{ ms}^{-1}}$, $\boxed{t = 4 \text{ s}}$

LOOKING AT AB

$u = 7 \text{ ms}^{-1}$
$a = 2 \text{ ms}^{-2}$
$s = 44 \text{ m}$
$t = ?$
$v = ?$

a) $v^2 = u^2 + 2as$
$v^2 = 7^2 + 2 \times 2 \times 44$
$v^2 = 225$
$v = 15 \text{ ms}^{-1}$

b) $v = u + at$
$15 = 7 + 2t$
$8 = 2t$
$t = 4 \text{ s}$

Question 22 (**)

A particle passes through the point A with speed 11 ms^{-1}, moving along a straight horizontal path with constant acceleration. The particle passes through the point B, where $AB = 111 \text{ m}$, 6 s after passing through A.

a) Find the acceleration of the particle.

b) Determine the speed of the particle as it passes through B.

$\boxed{a = 2.5 \text{ ms}^{-2}}$, $\boxed{v = 26 \text{ ms}^{-1}}$

LOOKING AT AB

$u = 11 \text{ ms}^{-1}$
$a = ?$
$s = 111 \text{ m}$
$t = 6 \text{ s}$
$v = ?$

a) $s = ut + \frac{1}{2}at^2$
$111 = 11 \times 6 + \frac{1}{2} \times a \times 6^2$
$111 = 66 + 18a$
$45 = 18a$
$a = 2.5 \text{ ms}^{-2}$

b) $v = u + at$
$v = 11 + 2.5 \times 6$
$v = 11 + 15$
$v = 26 \text{ ms}^{-1}$

Question 23 (**)

A particle passes through the point A with speed 41 ms^{-1}, moving along a straight horizontal path with constant deceleration 3.5 ms^{-2}. The particle passes through the point B, 8 s after passing through A.

a) Find the distance AB.

b) Determine the speed of the particle as it passes through B.

$\boxed{|AB| = 216 \text{ m}}$, $\boxed{v = 13 \text{ ms}^{-1}}$

LOOKING AT AB

$u = 41 \text{ ms}^{-1}$
$a = -3.5 \text{ ms}^{-2}$
$s = ?$
$t = 8 \text{ s}$
$v = ?$

a) $s = ut + \frac{1}{2}at^2$
$s = 41 \times 8 + \frac{1}{2}(-3.5) \times 8^2$
$s = 328 - 112$
$s = 216 \text{ m}$

b) $v = u + at$
$v = 41 - 3.5 \times 8$
$v = 41 - 28$
$v = 13 \text{ ms}^{-1}$

Question 24 (**)

A particle passes through the point A moving along a straight horizontal path with constant acceleration 1.5 ms^{-2}.

The particle passes through the point B, where $AB = 162.25 \text{ m}$, 11 s after passing through A.

a) Determine the speed of the particle as it passes through A.

b) Find the speed of the particle as it passes through B.

$\boxed{u = 6.5 \text{ ms}^{-1}}$, $\boxed{v = 23 \text{ ms}^{-1}}$

LOOKING AT AB

$u = ?$
$a = 1.5 \text{ ms}^{-2}$
$s = 162.25 \text{ m}$
$t = 11 \text{ s}$
$v = ?$

a) $s = ut + \frac{1}{2}at^2$
$162.25 = u \times 11 + \frac{1}{2}(1.5) \times 11^2$
$162.25 = 11u + 90.75$
$71.5 = 11u$
$u = 6.5 \text{ ms}^{-1}$

b) $v = u + at$
$v = 6.5 + 1.5 \times 11$
$v = 6.5 + 16.5$
$v = 23 \text{ ms}^{-1}$

Question 25 (**)

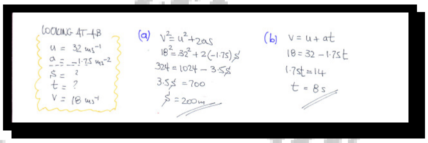
A particle passes through the point A with velocity 32 ms^{-1}, moving along a straight horizontal path with constant deceleration 1.75 ms^{-2}.

The particle passes through the point B with velocity 18 ms^{-1}.

a) Find the distance AB.

b) Calculate the time it takes the particle to travel from A to B.

$\boxed{|AB| = 200 \text{ m}}$, $\boxed{t = 8 \text{ s}}$



Question 26 (**)

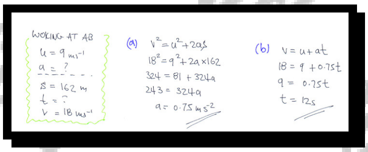
A particle passes through the point A with speed 9 ms^{-1}, moving along a straight horizontal path with constant acceleration.

The particle passes through the point B, where $AB = 162 \text{ m}$, with speed 18 ms^{-1}.

a) Find the acceleration of the particle.

b) Calculate the time it takes the particle to travel from A to B.

$\boxed{a = 0.75 \text{ ms}^{-2}}$, $\boxed{t = 12 \text{ s}}$



Question 27 (**)

A particle passes through the point A with speed V ms^{-1}, moving along a straight horizontal path with constant deceleration 4 ms^{-2}.

The particle passes through the point B, where $AB = 21$ m, with speed 11 ms^{-1}.

a) Find the value of V.

b) Calculate the time it takes the particle to travel from A to B.

$\boxed{V = 17}$, $\boxed{t = 1.5 \text{ s}}$

LOOKING AT AB
$u = ?$
$a = -4 \text{ ms}^{-2}$
$s = 21$ m
$t = ?$
$v = 11 \text{ ms}^{-1}$

(a) $v^2 = u^2 + 2as$
$11^2 = u^2 + 2(-4)(21)$
$121 = u^2 - 168$
$u^2 = 289$
$u = 17 \text{ ms}^{-1}$
i.e. $V = 17$

(b) $v = u + at$
$11 = 17 - 4t$
$4t = 6$
$t = 1.5$ s

Question 28 (**)

A particle passes through the point A with velocity 5 ms^{-1}, moving along a straight horizontal path with constant acceleration.

The particle passes through the point B with velocity 5.8 ms^{-1}, 2.5 s after passing through A.

a) Find the distance AB.

b) Calculate acceleration of the particle.

$\boxed{|AB| = 13.5 \text{ m}}$, $\boxed{a = 0.32 \text{ ms}^{-2}}$

LOOKING AT AB
$u = 5 \text{ ms}^{-1}$
$a = ?$
$s = ?$
$t = 2.5$ s
$v = 5.8 \text{ ms}^{-1}$

(a) $s = \frac{u+v}{2} \times t$
$s = \frac{5+5.8}{2} \times 2.5$
$s = 5.4 \times 2.5$
$s = 13.5$ m

(b) $v = u + at$
$5.8 = 5 + a \times 2.5$
$0.8 = 2.5a$
$a = 0.32 \text{ ms}^{-2}$

Question 29 (**)

A particle passes through the point A with velocity U ms^{-1}, $U > 0$, moving along a straight horizontal path with constant deceleration.

The particle passes through the point B, where $AB = 27$ m, with velocity 9.6 ms^{-1}, 2.5 s after passing through A.

a) Find the value of U.

b) Calculate deceleration of the particle.

$\boxed{U = 12}$, $\boxed{a = -0.96 \text{ ms}^{-2}}$

LOOKING AT AB
$u = ?$
$a = ?$
$s = 27$ m
$t = 2.5$ s
$v = 9.6$ ms^{-1}

(a) $s = \frac{u+v}{2}t$
$27 = \frac{u+9.6}{2} \times 2.5$
$54 = 2.5(u+9.6)$
$21.6 = u + 9.6$
$u = 12$ ms^{-1}
$\therefore U = 12$

(b) $v = u + at$
$9.6 = 12 + a \times 2.5$
$-2.4 = 2.5a$
$a = -0.96$ ms^{-2}

Question 30 (**)

A particle passes through the point A with velocity V ms^{-1}, $V > 0$, moving along a straight horizontal path with constant acceleration 3.5 ms^{-2}.

The particle passes through the point B with velocity 37 ms^{-1}, 10 s after passing through A.

a) Find the value of V.

b) Calculate the distance AB.

$\boxed{V = 2}$, $\boxed{|AB| = 195 \text{ m}}$

LOOKING AT AB
$u = ?$
$a = 3.5$ ms^{-2}
$s = ?$
$t = 10$ s
$v = 37$ ms^{-1}

(a) $v = u + at$
$37 = u + 3.5 \times 10$
$37 = u + 35$
$u = 2$ ms^{-1}
ie $V = 2$

(b) $s = \frac{u+v}{2} \times t$
$s = \frac{2+37}{2} \times 10$
$s = 195$ m

HORIZONTAL KINEMATICS (Standard Problems)

Question 1 (**)

A car of mass 1300 kg is travelling at a speed of 30 ms^{-1} along a straight horizontal motorway when the driver sees a traffic jam ahead, and applies the brakes for 15 s. The car covers a distance of 270 m while the driver is braking.

The car is modelled as a particle, further assuming that the braking force is the only **constant** force acting on the car for those 15 s.

a) Find the speed of the car at the end of the 15 s braking interval.

b) Determine the magnitude of the braking force.

$\boxed{v = 6 \text{ ms}^{-1}}$, $\boxed{F = 2080 \text{ N}}$

(a)
$u = 30 \text{ ms}^{-1}$
a
$s = 270 \text{ m}$
$t = 15 \text{ s}$
$v = ?$

$s = \frac{u+v}{2}t$
$270 = \frac{30+v}{2} \times 15$
$540 = 15(30+v)$
$36 = 30 + v$
$v = 6 \text{ ms}^{-1}$

(b) NEED THE ACCELARATION
$v = u + at$
$6 = 30 + a \times 15$
$-24 = 15a$
$a = -1.6 \text{ ms}^{-2}$

NOW $F = ma$
$F = 1300 \times (-1.6)$
$F = -2080 \text{ N}$

$\therefore$ MAGNITUDE OF 2080 N

Question 2 (**)

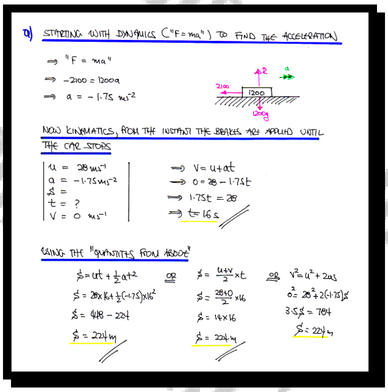
A car of mass 1200 kg is travelling at a speed of 28 ms^{-1} along a straight horizontal road when the driver applies the brakes and a constant braking force of 2100 N acts on the car until it comes to rest.

The car is modelled as a particle without any other external forces acting on it.

a) Find the time taken to bring the car to rest.

b) Determine the distance the car covers from the instant the brakes were first applied until the car is brought to rest.

$\boxed{}$, $\boxed{t = 16 \text{ s}}$, $\boxed{s = 224 \text{ m}}$



Question 3 (**+)

The points A, B and C lie on a straight horizontal road with B between A and C, so that $|AB| = 300$ m and $|BC| = 200$ m.

A car travelling with constant acceleration a ms^{-2} passes A with speed 5 ms^{-1} and travels directly to C in 20 seconds.

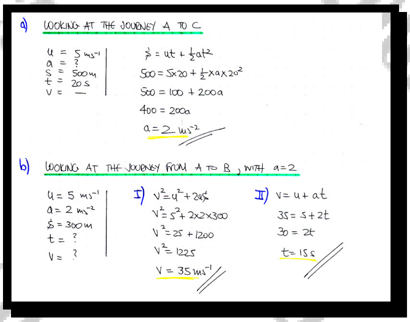
a) Find the value of a.

b) Calculate …

i. … the speed of the car at B.

ii. … the time it takes the car to travel from A to B.

$\boxed{\text{MMS-B}}$, $\boxed{a = 2}$, $\boxed{v = 35 \text{ ms}^{-1}}$, $\boxed{t = 15 \text{ s}}$



a) LOOKING AT THE JOURNEY A TO C

$u = 5$ ms^{-1}
$a = ?$
$s = 500$ m
$t = 20$ s
$v = $ —

$s = ut + \frac{1}{2}at^2$
$500 = 5 \times 20 + \frac{1}{2} \times a \times 20^2$
$500 = 100 + 200a$
$400 = 200a$
$a = 2$ ms^{-2}

b) LOOKING AT THE JOURNEY FROM A TO B, WITH $a = 2$

$u = 5$ ms^{-1}
$a = 2$ ms^{-2}
$s = 300$ m
$t = ?$
$v = ?$

I) $v^2 = u^2 + 2as$
$v^2 = 5^2 + 2 \times 2 \times 300$
$v^2 = 25 + 1200$
$v^2 = 1225$
$v = 35$ ms^{-1}

II) $v = u + at$
$35 = 5 + 2t$
$30 = 2t$
$t = 15$ s

Question 4 (***)

A car is travelling along a straight horizontal road with constant acceleration a ms^{-2}.

The points A, B and C lie in that order on this road.

The car is passing through A with speed 11 ms^{-1}, through B with speed 17 ms^{-1}, and through C with speed 29 ms^{-1}.

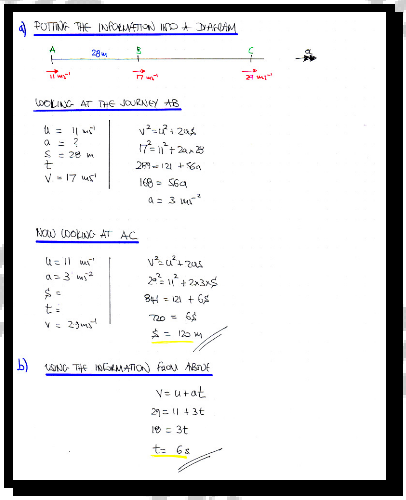
The distance $AB = 28$ m.

By modelling the car as a particle calculate in any order …

a) … the distance AC

b) … the time it takes the car to travel from A to C.

$\boxed{}$, $\boxed{|AC| = 120 \text{ m}}$, $\boxed{t = 6 \text{ s}}$



a) PUTTING THE INFORMATION INTO A DIAGRAM

A — 28 m — B — C, a →

11 ms^{-1}, 17 ms^{-1}, 29 ms^{-1}

LOOKING AT THE JOURNEY AB

$u = 11$ ms^{-1}
$a = ?$
$s = 28$ m
t
$v = 17$ ms^{-1}

$v^2 = u^2 + 2as$
$17^2 = 11^2 + 2a \times 28$
$289 = 121 + 56a$
$168 = 56a$
$a = 3$ ms^{-2}

NOW LOOKING AT AC

$u = 11$ ms^{-1}
$a = 3$ ms^{-2}
$s =$
$t =$
$v = 29$ ms^{-1}

$v^2 = u^2 + 2as$
$29^2 = 11^2 + 2 \times 3 \times s$
$841 = 121 + 6s$
$720 = 6s$
$s = 120$ m

b) USING THE INFORMATION FROM ABOVE

$v = u + at$
$29 = 11 + 3t$
$18 = 3t$
$t = 6$ s

Question 5 (***)

The points A, B and C lie in that order on a road, where the distance $AB = 476$ m and the distance $BC = 855$ m.

The car passes through A with speed 24 ms^{-1} decelerating uniformly until it passes through B with speed 10 ms^{-1}.

a) Find the deceleration of the car as it travels from A to B.

b) Calculate the time it took the car to travel from A to B.

As the car passes through B it begins to accelerate uniformly until it passes through C, 45 s after passing through B.

c) Find the acceleration of the car as it travels from B to C.

d) Determine the speed of the car as it passes through C.

e) Find the **average** speed for the journey from A to C.

$\boxed{}$, $\boxed{t = 28 \text{ s}}$, $\boxed{|a| = 0.5 \text{ms}^{-2}}$, $\boxed{a = 0.4 \text{ms}^{-2}}$, $\boxed{v = 28 \text{ ms}^{-1}}$, $\boxed{v \approx 18.23 \text{ ms}^{-1}}$

a) b) LOOKING AT THE JOURNEY A TO B

$u = 24 \text{ ms}^{-1}$
$a = ?$
$s = 476 \text{ m}$
$t = ?$
$v = 10 \text{ ms}^{-1}$

"$v^2 = u^2 + 2as$"
$10^2 = 24^2 + 2a(476)$
$100 = 576 + 952a$
$-952a = 476$
$a = -0.5$

I.E DECELERATION 0.5 ms^{-2}

"$v = u + at$"
$10 = 24 + (-0.5)t$
$0.5t = 14$
$t = 28 \text{ s}$

c) d) LOOKING AT THE JOURNEY FROM B TO C

$u = 10 \text{ ms}^{-1}$
$a = ?$
$s = 855 \text{ m}$
$t = 45 \text{ s}$
$v = ?$

$s = ut + \frac{1}{2}at^2$
$855 = 10 \times 45 + \frac{1}{2} \times a \times 45^2$
$855 = 450 + \frac{2025}{2}a$
$405 = 1012.5a$
$a = 0.4 \text{ ms}^{-2}$

"$v = u + at$"
$v = 10 + 0.4 \times 45$
$v = 28 \text{ ms}^{-1}$

e) TO FIND THE AVERAGE SPEED FOR THE ENTIRE JOURNEY

$$\text{AVERAGE SPEED} = \frac{\text{TOTAL DISTANCE}}{\text{TOTAL TIME}} = \frac{476 + 855}{28 + 45} = 18.23 \text{ ms}^{-1}$$

Question 6 (*+)**

A car is travelling along a straight horizontal road with constant acceleration.

The car passes a point A with speed u ms^{-1}, where $u < 18$ and 12 seconds later passes a point B with speed 18 ms^{-1}.

The distance AB is 180 m.

a) Find the value of u.

b) Calculate, correct to two decimal places, the time taken for the car to move from A to the midpoint of AB.

$\boxed{}$, $\boxed{u = 12}$, $\boxed{t \approx 6.59 \text{ s}}$

a) LOOKING AT THE JOURNEY A TO B

$u = ?$
$a =$
$s = 180\text{m}$
$t = 12$
$v = 18 \text{ ms}^{-1}$

USING $s = \frac{1}{2}(u+v)t$

$180 = \frac{1}{2}(u+18) \times 12$

$180 = 6(u+18)$

$30 = u + 18$

$u = 12 \text{ ms}^{-1}$

b) FIRSTLY FIND THE ACCELERATION FROM PART (a)

$v = u + at$

$18 = 12 + a \times 12$

$6 = 12a$

$a = 0.5 \text{ ms}^{-2}$

NOW THE JOURNEY FROM A TO THE MIDPOINT OF AB

$u = 12 \text{ ms}^{-1}$
$a = 0.5 \text{ ms}^{-2}$
$s = 90\text{m}$ ← HALF WAY
$t = ?$
$v =$

USING $s = ut + \frac{1}{2}at^2$

$90 = 12t + \frac{1}{2} \times 0.5 t^2$

$90 = 12t + \frac{1}{4}t^2$

$360 = 48t + t^2$

$t^2 + 48t - 360 = 0$

$(t+24)^2 - 24^2 - 360 = 0$

$(t+24)^2 = 936$

$t + 24 = \sqrt{936}$ or $-\sqrt{936}$

$t = -24 + \sqrt{936} \approx 6.59 \text{ s}$

$t = -24 - \sqrt{936} \approx \cancel{-54.6}$

Question 7 (***+)

A particle is travelling along a straight line with constant acceleration a ms^{-2}.

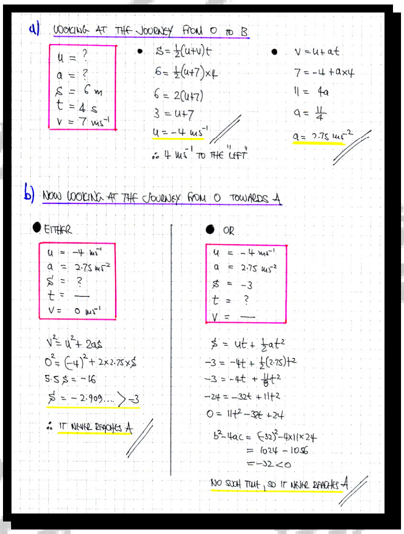
The points A, O and B lie in that order on this straight line, as shown in the figure above. The distance AO is 3 m and the distance OB is 6 m.

The particle is initially observed passing through O with speed u ms^{-1} and 4 s later is observed to be passing through B with speed 7 ms^{-1}, in the direction OB.

a) Find in any order the value of a and the value of u.

b) Prove that the particle never passes through A.

$\boxed{}$, $\boxed{u = -4}$, $\boxed{a = 2.75}$



a) LOOKING AT THE JOURNEY FROM O TO B

$u = ?$, $a = ?$, $s = 6$ m, $t = 4$ s, $v = 7$ ms^{-1}

- $s = \frac{1}{2}(u+v)t$
  $6 = \frac{1}{2}(u+7)\times 4$
  $6 = 2(u+7)$
  $3 = u+7$
  $u = -4$ ms^{-1}
  ∴ 4 ms^{-1} TO THE "LEFT"
- $v = u + at$
  $7 = -4 + a\times 4$
  $11 = 4a$
  $a = \frac{11}{4}$
  $a = 2.75$ ms^{-2}

b) NOW LOOKING AT THE JOURNEY FROM O TOWARDS A

- EITHER
  $u = -4$ ms^{-1}, $a = 2.75$ ms^{-2}, $s = ?$, $t = $ —, $v = 0$ ms^{-1}
  $v^2 = u^2 + 2as$
  $0^2 = (-4)^2 + 2\times 2.75 \times s$
  $5.5s = -16$
  $s = -2.909... > -3$
  ∴ IT NEVER REACHES A
- OR
  $u = -4$ ms^{-1}, $a = 2.75$ ms^{-2}, $s = -3$, $t = ?$, $v = $ —
  $s = ut + \frac{1}{2}at^2$
  $-3 = -4t + \frac{1}{2}(2.75)t^2$
  $-3 = -4t + \frac{11}{8}t^2$
  $-24 = -32t + 11t^2$
  $0 = 11t^2 - 32t + 24$
  $b^2 - 4ac = (-32)^2 - 4\times 11 \times 24$
  $= 1024 - 1056$
  $= -32 < 0$
  NO SUCH TIME, SO IT NEVER REACHES A

Question 8 (****)

A car is travelling along a straight horizontal road with constant acceleration a ms^{-2}.

The points A, B and C lie in that order on this road.

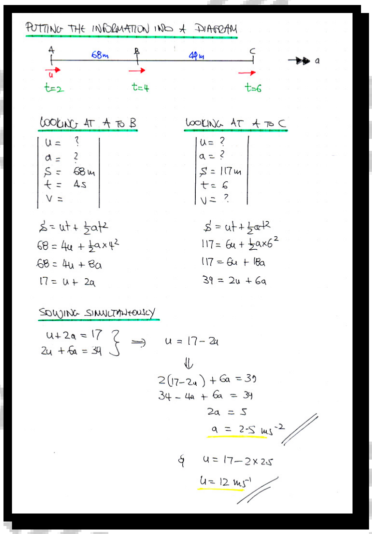
The car is passing through A with speed u ms^{-1} and 4 s later is passing through B.

The car finally passes through C, 2 s after passing through B.

The distance $AB = 68$ m and the distance $BC = 49$ m.

By modelling the car as a particle find in any order the value of a and the value of u.

$\boxed{}$, $\boxed{u = 12}$, $\boxed{a = 2.5}$



PUTTING THE INFORMATION INTO A DIAGRAM

LOOKING AT A TO B

$u = ?$
$a = ?$
$s = 68$ m
$t = 4$ s
$v =$

$s = ut + \frac{1}{2}at^2$
$68 = 4u + \frac{1}{2}a \times 4^2$
$68 = 4u + 8a$
$17 = u + 2a$

LOOKING AT A TO C

$u = ?$
$a = ?$
$s = 117$ m
$t = 6$
$v = ?$

$s = ut + \frac{1}{2}at^2$
$117 = 6u + \frac{1}{2}a \times 6^2$
$117 = 6u + 18a$
$39 = 2u + 6a$

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} u + 2a = 17 \\ 2u + 6a = 39 \end{array}\right\} \Longrightarrow u = 17 - 2a$$

$\Downarrow$

$2(17 - 2a) + 6a = 39$
$34 - 4a + 6a = 39$
$2a = 5$
$a = 2.5$ ms^{-2}

& $u = 17 - 2 \times 2.5$
$u = 12$ ms^{-1}

Question 9 (****)

A particle is travelling along a straight line with constant acceleration a ms^{-2}.

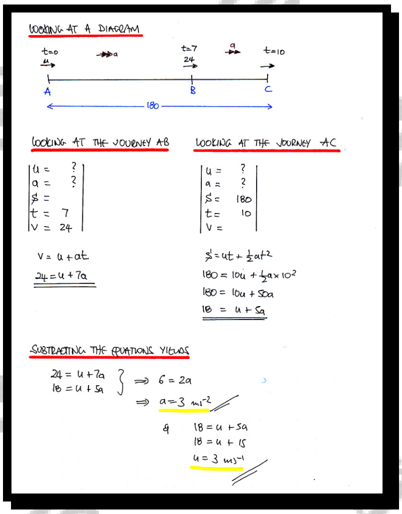
The points A, B and C lie in that order on this straight line.

The particle is initially observed passing through A with speed u ms^{-1} and 7 s later is observed to be passing through B with speed 24 ms^{-1}, in the direction AB.

Finally the particle is passing through C, 10 s after passing through A.

Given that the distance $AC = 180$ m, determine in any order the value of a and the value of u.

$\boxed{}$, $\boxed{u=3}$, $\boxed{a=3}$



LOOKING AT A DIAGRAM

$t=0$, u; a; $t=7$, 24; a; $t=10$

A B C

180

LOOKING AT THE JOURNEY AB

$u = ?$
$a = ?$
$s =$
$t = 7$
$v = 24$

$v = u + at$

$\underline{\underline{24 = u + 7a}}$

LOOKING AT THE JOURNEY AC

$u = ?$
$a = ?$
$s = 180$
$t = 10$
$v =$

$s = ut + \frac{1}{2}at^2$

$180 = 10u + \frac{1}{2}a \times 10^2$

$180 = 10u + 50a$

$\underline{\underline{18 = u + 5a}}$

SUBTRACTING THE EQUATIONS YIELDS

$\left.\begin{array}{l} 24 = u + 7a \\ 18 = u + 5a \end{array}\right\} \Rightarrow 6 = 2a$

$\Rightarrow a = 3 \text{ ms}^{-2}$

& $18 = u + 5a$

$18 = u + 15$

$u = 3 \text{ ms}^{-1}$

Question 10 (****)

The points A and B lie on a straight line, 240 m apart.

At time $t = 0$, a particle passes through A with speed 4 ms^{-1} heading towards B with constant acceleration 0.75 ms^{-2}.

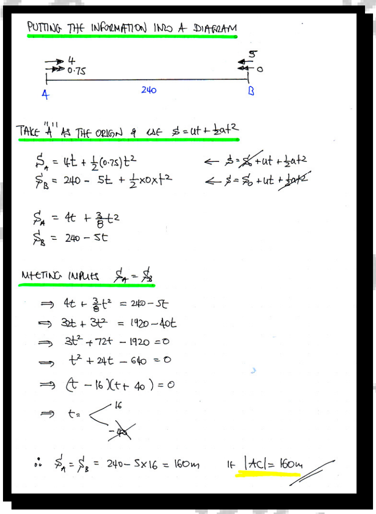
At time $t = 0$, another particle passes through B heading towards A with constant speed 5 ms^{-1}.

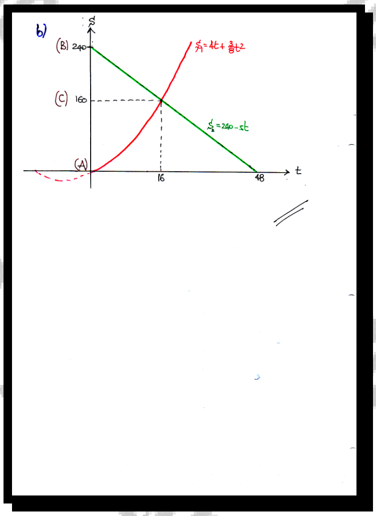
The particles meet at point C.

a) Determine the distance AC.

b) On a set of suitable axes, draw a detailed displacement time graph for both particles, using A as the origin.

$\boxed{}$, $\boxed{|AC| = 160 \text{ m}}$





Question 11 (****)

A car is travelling along a straight horizontal road with constant acceleration a ms^{-2}.

The points A, B and C lie in that order on this road.

The car is passing through A with speed u ms^{-1} and 5 s later is passing through B.

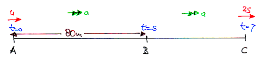
The car finally passes through C, 2 s after passing through B.

The distance $AB = 80$ m and the speed of the car at C is 25 ms^{-1}.

By modelling the car as a particle find in any order the value of a and the value of u.

$\boxed{X}$, $\boxed{u = 11}$, $\boxed{a = 2}$

PUTTING THE INFORMATION INTO A DIAGRAM



LOOKING AT THE JOURNEY AB

$u = $ "u"
$a = $ "a"
$s = 80$
$t = 5$
$v = $

$\Rightarrow s = ut + \frac{1}{2}at^2$
$\Rightarrow 80 = 5u + \frac{1}{2}a \times 5^2$
$\Rightarrow 80 = 5u + \frac{25}{2}a$ $\div 2.5$
$\Rightarrow 32 = 2u + 5a$

LOOKING AT THE JOURNEY AC

$u = $ "u"
$a = $ "a"
$s = $
$t = 7$
$v = 25$

$\Rightarrow v = u + at$
$\Rightarrow 25 = u + a \times 7$
$\Rightarrow 50 = 2u + 14a$

SUBTRACTING THE EQUATIONS "UPWARDS"

$\left.\begin{array}{l} 2u + 5a = 32 \\ 2u + 14a = 50 \end{array}\right\} \Rightarrow 9a = 18$

$\Rightarrow a = 2 \text{ ms}^{-2}$

& $u + 7a = 25$
$u + 14 = 25$
$u = 11 \text{ ms}^{-1}$

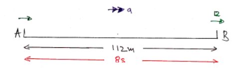
Question 12 (****)

A particle is moving in a straight line with constant acceleration, and it is first observed passing a point A.

The particle is next passing through the point B 8 s later with speed 12 ms^{-1}.

Given that the distance AB is 112 m, determine the times it takes the particle to travel from A to the midpoint of AB.

$\square$, $\boxed{T = 32 - 20\sqrt{2} \approx 3.72 \quad \cup \quad T = 32 + 20\sqrt{2} \approx 60.28}$



LOOKING AT THE JOURNEY A TO B

$u =$	?
$a =$	?
$s =$	112
$t =$	8
$v =$	12

- $s = \frac{u+v}{2} \times t$

$112 = \frac{u+12}{2} \times 8$

$112 = 4(u+12)$

$28 = u + 12$

$u = 16$

- $v = u + at$

$12 = 16 + a \times 8$

$-4 = 8a$

$a = -0.5$

NOW LOOKING AT THE JOURNEY A TO M

$u =$	16
$a =$	-0.5
$s =$	56
$t =$	?
$v =$	

$s = ut + \frac{1}{2}at^2$

$56 = 16t + \frac{1}{2}(-0.5)t^2$

$56 = 16t - \frac{1}{4}t^2$

$224 = 64t - t^2$

$t^2 - 64t + 224 = 0$

$(t-32)^2 - 1024 + 224 = 0$

$(t-32)^2 = 800$

$t - 32 = \pm\sqrt{800}$

$t = 32 \pm 20\sqrt{2}$

$\therefore t = \begin{cases} 3.72\text{ s} \\ 60.28\text{ s} \end{cases}$

Question 13 (****)

Two cars are moving on a straight road with constant speed of 18 ms^{-1}, one being 14.5 m ahead of the other.

The driver of the car in front sees a hazard and applies the brakes, which produce a constant deceleration of 6 ms^{-2}.

The driver of the other car takes 0.5 s to react and also applies his brakes, which produce a constant deceleration of 4 ms^{-2}.

The driver of the car in front sees a hazard and applies the brakes, which produce a constant deceleration of 6 ms^{-2}.

By modelling the two cars as particles, find the speed of each of the cars when a collision between them take place.

$\square$, $\boxed{8 \text{ ms}^{-1} \text{ and } 0 \text{ ms}^{-1}}$

- LET $t=0$ BE THE TIME WHEN BOTH CARS ARE TRAVELLING AT 18 ms^{-1} AND THE CAR IN FRONT ABOUT TO HIT THE BRAKES
- LET $x=0$ THE POSITION OF THE CAR BEHIND AT $t=0$

THE "DISTANCE" OF THE CAR IN FRONT AT TIME t IS GIVEN BY

$$x_1 = \underbrace{14.5}_{\text{"IN FRONT"}} + \underbrace{18t + \tfrac{1}{2}\times(-6)t^2}_{\text{"}ut + at^2\text{"}}$$

$$x_1 = 14.5 + 18t - 3t^2$$

THE DISTANCE OF THE CAR BEHIND AT TIME t IS GIVEN BY

$$x_2 = \underbrace{18\times0.5}_{\text{REACTION DISTANCE}} + \underbrace{18(t-0.5) + \tfrac{1}{2}(-4)(t-0.5)^2}_{\text{"}ut+\frac{1}{2}at^2\text{"}}$$

$$x_2 = 9 + 18(t-0.5) - 2(t-0.5)^2$$

FOR COLLISION $x_1 = x_2$

$$\Rightarrow 14.5 + 18t - 3t^2 = 9 + 18(t-0.5) - 2(t-0.5)^2$$
$$\Rightarrow 14.5 + 18t - 3t^2 = \cancel{9} + 18t - \cancel{9} - 2(t^2 - t + 0.25)$$
$$\Rightarrow 14.5 + \cancel{18t} - 3t^2 = \cancel{18t} - 2t^2 + 2t - 0.5$$
$$\Rightarrow 0 = t^2 + 2t - 15$$
$$\Rightarrow (t+5)(t-3) = 0$$
$$\Rightarrow t = \begin{cases} 3 \\ -5 \end{cases}$$

THE VELOCITY OF THE CAR IN FRONT IS

"$v = u + at$"

$v = 18 - 6\times3$

$v = 0$

THE VELOCITY OF THE CAR BEHIND IS

"$v = u + at$"

$v = 18 - 4(2.5)$

$v = 8 \text{ ms}^{-1}$

$\therefore$ THE CAR BEHIND IS TRAVELLING AT 8 ms^{-1}, AND THE CAR IN FRONT IS COMING TO REST

Question 14 (****+)

A cyclist is travelling along a straight horizontal road at constant speed 12 ms^{-1} as it passes past a set of traffic lights at time $t = 0$, where t is measured in seconds.

The cyclist continues its journey at that constant speed.

When $t = 6$ a car passes past the same set of traffic lights with speed 30 ms^{-1}, decelerating uniformly at 2 ms^{-2}.

In the consequent motion, the car overtakes the cyclist at some point A and at a later time the cyclist overtakes the car again at some point B.

Find the value of t at A and at B.

$\square$, $\boxed{t_A = 12,\ t_B = 18}$

LET THE "ORIGIN" BE THE TRAFFIC LIGHTS, AND THE TIME IS MEASURED SINCE THE CYCLIST CROSSED THEM

FOR THE CYCLIST	FOR THE CAR
$u = 12\ \text{ms}^{-1}$	$u = 30\ \text{ms}^{-1}$
$a = 0\ \text{ms}^{-2}$	$a = -2\ \text{ms}^{-2}$
$s = S$	$s = S_{CAR}$
$t = T$	$t = T-6$
$v = -$	$v = -$

"$S = ut + \frac{1}{2}at^2$" FOR EACH OF THEM YIELDS

$$S_{CYCLIST} = 12 \times T + \cancel{\tfrac{1}{2} \times 0 \times T^2} = 12T$$

$$S_{CAR} = 30 \times (T-6) + \tfrac{1}{2}(-2)(T-6)^2 = 30(T-6) - (T-6)^2$$

SOLVING SIMULTANEOUSLY $S_{CYCLIST} = S_{CAR}$

$\Rightarrow 12T = 30(T-6) - (T-6)^2$

$\Rightarrow 12T = 30T - 180 - (T^2 - 12T + 36)$

$\Rightarrow \cancel{12T} = 30T - 180 - T^2 + \cancel{12T} - 36$

$\Rightarrow T^2 - 30T + 216 = 0$

$\Rightarrow (T-18)(T-12) = 0$

$\Rightarrow T = \begin{cases} 18 \\ 12 \end{cases}$ $\quad\therefore\quad t_A = 12$, $t_B = 18$

Question 15 (****+)

Two cars, A and B, are travelling in the same direction along a straight road.

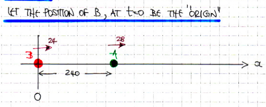
At a certain instant, A has speed 28 ms^{-1}, accelerating uniformly at 0.1 ms^{-2}.

At the same instant, B is 240 m behind A, travelling with speed 24 ms^{-1}, accelerating uniformly at 0.2 ms^{-2}.

Find the speed of B the instant it overtakes A.

, $\boxed{v = 48 \text{ ms}^{-1}}$

- LET THE TIME t BE MEASURED, FROM THE INSTANT WHEN B IS 240 METRES BEHIND A
- LET THE POSITION OF B, AT $t=0$ BE THE "ORIGIN"



USING $s = s_0 + ut + \frac{1}{2}at^2$, AT TIME t

$$\left.\begin{aligned} s_A &= 240 + 28t + \tfrac{1}{2}(0.1)t^2 \\ s_B &= 0 + 24t + \tfrac{1}{2}(0.2)t^2 \end{aligned}\right\} \Longrightarrow s_A = s_B$$

$$\Rightarrow 240 + 28t + \frac{1}{20}t^2 = 24t + \frac{1}{10}t^2$$

$$\Rightarrow 0 = \frac{1}{20}t^2 - 4t - 240$$

$$\Rightarrow t^2 - 80t - 4800 = 0$$

$$\Rightarrow (t + 40)(t - 120) = 0$$

$$\Rightarrow t = \begin{cases} 120 \\ \cancel{-40} \end{cases}$$

FINALLY USING $V = u + at$

$$\therefore V_B = 24 + 0.2 \times 120$$

$$V_B = 48 \text{ ms}^{-1}$$

Question 16 (****+)

Kodjo and Modjo are two horses running a race.

Kodjo is 250 m from the finish line and running at constant speed of 16 ms^{-1}.

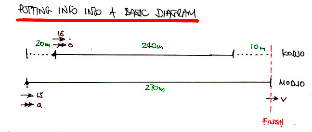
At that instant Modjo is 20 m behind Kodjo and running at 15 ms^{-1}, when his jockey demands of the horse to speed up with constant acceleration a ms^{-2}, until it crosses the finish line with speed v ms^{-1}.

a) If Modjo finishes 10 m ahead of Kodjo determine the value of a.

b) If instead there is a "dead heat" in the race between Kodjo and Modjo find the value of v.

$\boxed{}$, $\boxed{a = 0.4}$, $\boxed{v = 19.56}$

a) PUTTING INFO INTO A BASIC DIAGRAM



LOOKING AT KODJO MOVING WITH CONSTANT SPEED, COVERING 240m

"DISTANCE = SPEED × TIME"

$240 = 16 \times T$

$T = 15\text{s}$

MODJO IN THAT TIME, COVERS 270 METRES, ...

$u = 15 \text{ ms}^{-1}$
$a = ?$
$s = 270 \text{ m}$
$t = 15$
$v = —$

$s = ut + \frac{1}{2}at^2$

$270 = 15 \times 15 + \frac{1}{2} \times a \times 15^2$

$270 = 225 + \frac{225}{2}a$

$540 = 450 + 225a$

$225a = 90$

$a = 0.4 \text{ ms}^{-2}$

b) IF THEY FINISH LEVEL (DEAD HEAT) – KODJO COVERS 250m WITH CONSTANT SPEED 16 ms^{-1}

"DISTANCE = SPEED × TIME"

$250 = 16T'$

$T' = 15.625$

NOW LOOKING AT MODJO AGAIN WITH $T = 15.625$

$u = 15 \text{ ms}^{-1}$
$a = \text{N/A}$
$s = 270 \text{ m}$
$t = 15.625$
$v = ?$

"$s = \frac{1}{2}(u+v)t$"

$270 = \frac{1}{2}(15+v) \times 15.625$

$540 = (v+15) \times 15.625$

$\frac{864}{25} = v + 15$

$v = 19.56 \text{ ms}^{-1}$

VERTICAL KINEMATICS

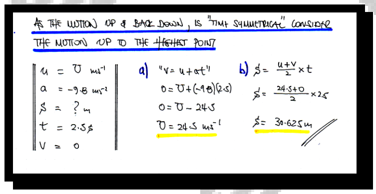
Question 1 (**)

A particle is projected vertically upwards with speed U ms^{-1}, from level horizontal ground. The particle is moving freely under gravity and returns to its starting position 5 s later.

a) Determine the value of U.

b) Calculate the greatest height the particle reaches above the ground.

$\boxed{}$, $\boxed{U = 24.5}$, $\boxed{H_{\text{max}} = 30.625 \text{ m}}$



AS THE MOTION UP & BACK DOWN, IS "TIME SYMMETRICAL" CONSIDER THE MOTION UP TO THE HIGHEST POINT

$u = U \text{ ms}^{-1}$
$a = -9.8 \text{ ms}^{-2}$
$s = ? \text{ m}$
$t = 2.5 \text{ s}$
$v = 0$

a) "$v = u + at$"
$0 = U + (-9.8)(2.5)$
$0 = U - 24.5$
$U = 24.5 \text{ ms}^{-1}$

b) $s = \frac{u+v}{2} \times t$
$s = \frac{24.5+0}{2} \times 2.5$
$s = 30.625 \text{ m}$

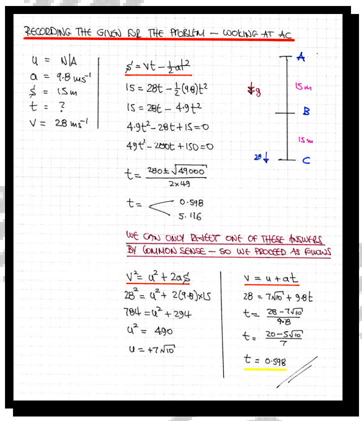
Question 2 (**+)

A particle is projected vertically downwards from a great height.

It hits the ground with speed 28 ms^{-1}.

Determine the time it took the particle to cover the last 15 m of its motion.

$\boxed{}$, $\boxed{t \approx 0.598 \text{ s}}$



RECORDING THE GIVEN FOR THE PROBLEM – LOOKING AT AC

$u = \text{N/A}$
$a = 9.8 \text{ ms}^{-1}$
$s = 15 \text{ m}$
$t = ?$
$v = 28 \text{ ms}^{-1}$

$s = vt - \frac{1}{2}at^2$
$15 = 28t - \frac{1}{2}(9.8)t^2$
$15 = 28t - 4.9t^2$
$4.9t^2 - 28t + 15 = 0$
$49t^2 - 280t + 150 = 0$

$t = \frac{280 \pm \sqrt{49000}}{2 \times 49}$

$t = 0.598$ or 5.116

WE CAN ONLY REJECT ONE OF THESE ANSWERS BY COMMON SENSE – SO WE PROCEED AS FOLLOWS

$v^2 = u^2 + 2as$
$28^2 = u^2 + 2(9.8) \times 15$
$784 = u^2 + 294$
$u^2 = 490$
$u = +7\sqrt{10}$

$v = u + at$
$28 = 7\sqrt{10} + 9.8t$
$t = \frac{28 - 7\sqrt{10}}{9.8}$
$t = \frac{20 - 5\sqrt{10}}{7}$
$t = 0.598$

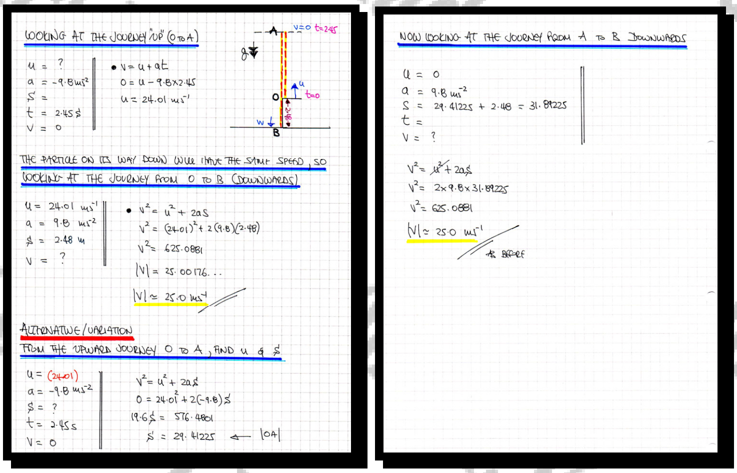
Question 3 (**+)

A particle is projected vertically upwards from a balcony which is 2.48 m above level horizontal ground.

The particle is moving freely under gravity, takes 2.45 s to reach the highest point in its path, before it strikes the ground with speed v ms^{-1}.

Calculate the value of v.

$\boxed{}$, $\boxed{v \approx 25.0}$



Question 4 (***)

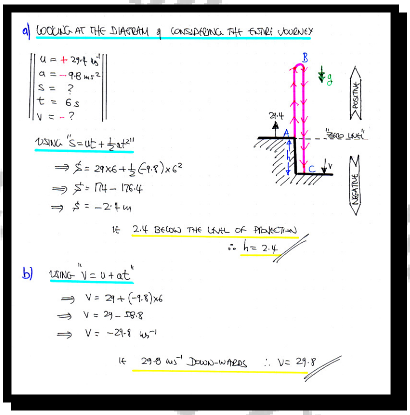
A particle is projected vertically upwards with speed 29 ms^{-1}, from a balcony which is h m above level horizontal ground.

The particle is moving freely under gravity and strikes the ground 6 s later with speed v ms^{-1}.

a) Calculate the value of h.

b) Determine the value of v.

$\boxed{\text{MS-K}}$, $\boxed{h = 2.4}$, $\boxed{v = 29.8}$



a) LOOKING AT THE DIAGRAM & CONSIDERING THE ENTIRE JOURNEY

$u = +29.4 \text{ ms}^{-1}$
$a = -9.8 \text{ ms}^{-2}$
$s = ?$
$t = 6 \text{ s}$
$v = -?$

USING "$s = ut + \frac{1}{2}at^2$"

$\Rightarrow s = 29 \times 6 + \frac{1}{2}(-9.8) \times 6^2$
$\Rightarrow s = 174 - 176.4$
$\Rightarrow s = -2.4 \text{ m}$

IE 2.4 BELOW THE LEVEL OF PROJECTION
$\therefore h = 2.4$

b) USING "$v = u + at$"

$\Rightarrow v = 29 + (-9.8) \times 6$
$\Rightarrow v = 29 - 58.8$
$\Rightarrow v = -29.8 \text{ ms}^{-1}$

IE 29.8 ms^{-1} DOWN-WARDS $\therefore v = 29.8$

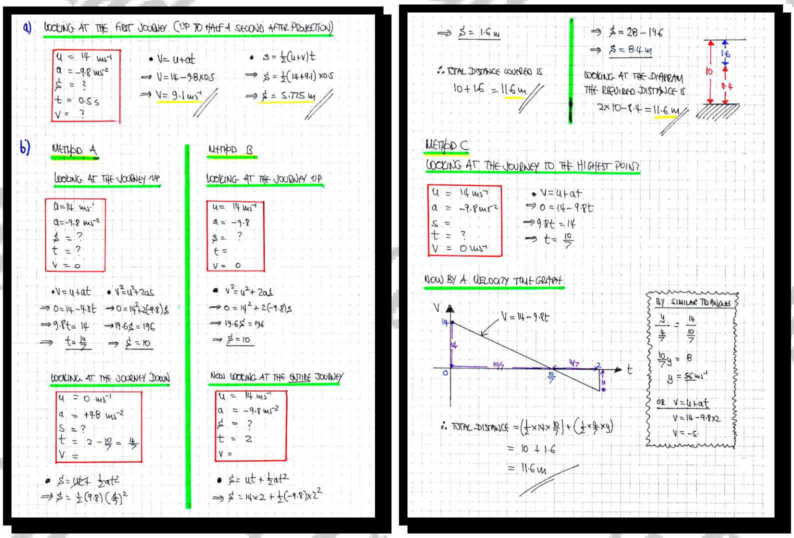
Question 5 (***)

A particle is projected vertically upwards from level ground with a speed of 14 ms^{-1}.

a) Determine the speed of the particle and the distance of the particle from the ground, 0.5 s after projection.

b) Calculate the total distance travelled by the particle during the first 2 s of its motion.

$\boxed{}$, $\boxed{v = 9.1 \text{ ms}^{-1}}$, $\boxed{d_{0.5} = 5.775 \text{ m}}$, $\boxed{d_2 = 11.6 \text{ m}}$



Question 6 (***+)

A raindrop falls freely from rest, from the top of a cliff. After it has fallen a distance of 40 m another raindrop falls freely from rest from the top of the same cliff. The height of the cliff is 80 m.

Calculate, correct to three significant figures, the distance between the two raindrops at the instant the first raindrop has reached the ground.

$\square$, $\boxed{\approx 73.1 \text{ m}}$

LOOKING AT THE JOURNEY OF THE FIRST RAINDROP

• FIRST 40 METRES

$u = 0$
$a = 9.8$
$s = 40$
$t = ?$
$v =$

$s = \cancel{ut} + \frac{1}{2}at^2$
$40 = \frac{1}{2}(9.8)t^2$
$t^2 = \frac{40}{4.9}$
$t = \frac{20}{7}$

• ENTIRE JOURNEY OF 80 METRES

$u = 0$
$a = 9.8$
$s = 80$
$t = ?$
$v =$

$s = \cancel{ut} + \frac{1}{2}at^2$
$80 = \frac{1}{2}(9.8)t^2$
$t^2 = \frac{80}{4.9}$
$t = \frac{20\sqrt{2}}{7}$

THUS THE 2ND RAINDROP WILL TRAVEL FOR $\frac{20\sqrt{2}}{7} - \frac{20}{7}$, BY THE TIME THE FIRST ONE REACHES THE GROUND

$u = 0$
$a = 9.8$
$s = ?$
$t = \frac{20\sqrt{2}}{7} - \frac{20}{7}$
$v =$

$s = \cancel{ut} + \frac{1}{2}at^2$
$s = \frac{1}{2}(9.8)\left[\frac{20\sqrt{2}-20}{7}\right]^2$
$s = 6.8629\ldots$

$\therefore$ DISTANCE BETWEEN THEM

$80 - 6.8629\ldots \approx 73.1 \text{ m}$ 3 sf

Question 7 (***+)

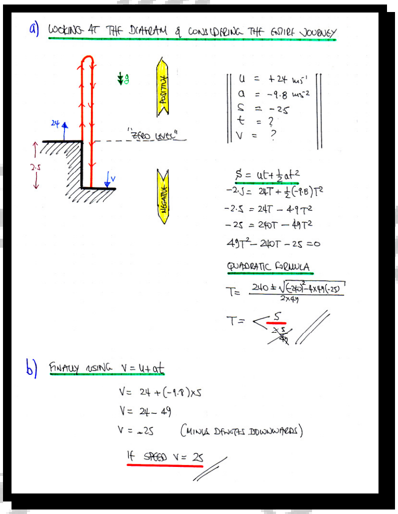
A particle is projected vertically upwards with speed 24 ms^{-1}, from a balcony which is located 2.5 m above level horizontal ground.

The particle is moving freely under gravity and strikes the ground T s later with speed $v \text{ ms}^{-1}$.

a) Calculate the value of T.

b) Determine the value of v.

$\boxed{\text{MMS}}$, $\boxed{T = 5}$, $\boxed{v = 25}$



Question 8 (***+)

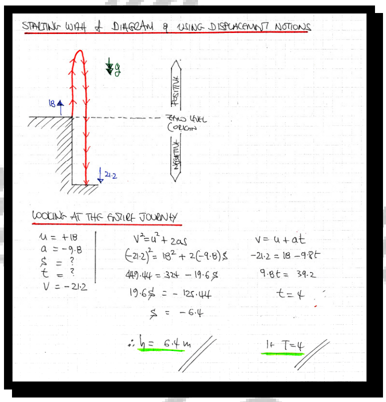
A particle is projected vertically upwards with speed 18 ms^{-1}, from a balcony which is h m above level horizontal ground.

The particle is moving freely under gravity and strikes the ground T s later with speed 21.2 ms^{-1}.

a) Calculate the value of h.

b) Determine the value of T.

$\boxed{}$, $\boxed{h = 6.4}$, $\boxed{T = 4}$



Question 9 (***+)

At time $t = 0$ s, two particles A and B are projected vertically upwards with speeds 13 ms^{-1} and 3 ms^{-1}, respectively.

The projection of A is from a point on level horizontal ground while the projection of B is from a point which is 20 m vertically above the projection point of A.

When $t = T$ s, both particles are at a height H m above ground.

a) Calculate the value of T.

b) Determine the value of H.

$\boxed{}$, $\boxed{T = 2}$, $\boxed{H = 6.4}$

a) LOOKING AT THE DIAGRAM, TAKING THE GROUND AS THE ZERO LEVEL FOR DISPLACEMENT

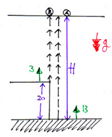
USING "$s = ut + \frac{1}{2}at^2$"

$s_A = 13t + \frac{1}{2}(-9.8)t^2$

$s_A = 13t - 4.9t^2$

$s_B = 20 + 3t - \frac{1}{2}(-9.8)t^2$

$s_B = 20 + 3t - 4.9t^2$



SAME HEIGHT ABOVE GROUND $\Longrightarrow s_A = s_B$

$13t - \cancel{4.9t^2} = 20 + 3t - \cancel{4.9t^2}$

$10t = 20$

$t = 2$

i.e. $T = 2$

b) USING $s_A = 13t - 4.9t^2$

$s_A = 13 \times 2 - 4.9 \times 2^2$

$s_A = 26 - 19.6$

$s_A = 6.4$ m

i.e. $H = 6.4$

Question 10 (***+)

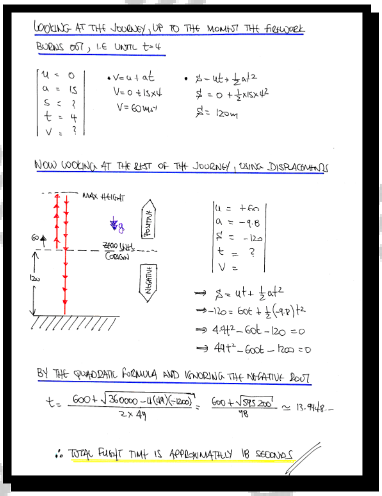
A firework is launched from rest at ground level, and moves vertically upwards.

It rises with constant acceleration of 15 ms^{-2} for 4 s. At that instant the firework has burned out and it continues to rise freely under gravity, eventually returning to the ground

The firework is modelled as a particle moving in a vertical direction.

Calculate the total flight time of the firework, from the moment of its launch until its return to the ground.

$\boxed{}$, $\boxed{\approx 18.0}$



LOOKING AT THE JOURNEY, UP TO THE MOMENT THE FIREWORK BURNS OUT, I.E UNTIL $t=4$

$u = 0$, $a = 15$, $s = ?$, $t = 4$, $v = ?$

• $v = u + at$
$v = 0 + 15 \times 4$
$v = 60 \text{ ms}^{-1}$

• $s = ut + \frac{1}{2}at^2$
$s = 0 + \frac{1}{2} \times 15 \times 4^2$
$s = 120 \text{ m}$

NOW LOOKING AT THE REST OF THE JOURNEY, USING DISPLACEMENTS

$u = +60$, $a = -9.8$, $s = -120$, $t = ?$, $v =$

$\Rightarrow s = ut + \frac{1}{2}at^2$
$\Rightarrow -120 = 60t + \frac{1}{2}(-9.8)t^2$
$\Rightarrow 4.9t^2 - 60t - 120 = 0$
$\Rightarrow 49t^2 - 600t - 1200 = 0$

BY THE QUADRATIC FORMULA AND IGNORING THE NEGATIVE ROOT

$$t = \frac{600 + \sqrt{360000 - 4(49)(-1200)}}{2 \times 49} = \frac{600 + \sqrt{595200}}{98} \simeq 13.9448...$$

∴ TOTAL FLIGHT TIME IS APPROXIMATELY 18 SECONDS

Question 11 (***+)

A particle A is released from rest from a point h m above level horizontal ground.

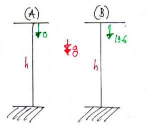
One second later, another particle B is projected vertically downwards with speed 19.6 ms^{-1} from the same point, A was released.

Given that the particles reach the ground at the same time, determine the value of h.

$\boxed{}$, $\boxed{h \approx 11.0}$

START WITH A DETAILED DIAGRAM

FOR A	FOR B
$u = 0$	$u = 19.6$
$a = 9.8$	$a = 9.8$
$s = h$	$s = h$
$t = T$	$t = T-1$
$v = \diagup$	$v = \diagup$



USING $s = ut + \frac{1}{2}at^2$ FOR BOTH

A: $s = ut + \frac{1}{2}at^2$
$h = 0(T) + \frac{1}{2}(9.8)T^2$
$h = 4.9T^2$

B: $s = ut + \frac{1}{2}at^2$
$h = 19.6(T-1) + \frac{1}{2}(9.8)(T-1)^2$
$h = 19.6T - 19.6 + 4.9(T-1)^2$

BY SUBSTITUTION

$\Rightarrow 4.9T^2 = 19.6T - 19.6 + 4.9(T-1)^2$ $\div 4.9$
$\Rightarrow T^2 = 4T - 4 + (T-1)^2$
$\Rightarrow \cancel{T^2} = 4T - 4 + \cancel{T^2} - 2T + 1$
$\Rightarrow 3 = 2T$
$\Rightarrow T = \frac{3}{2}$

FINALLY USING $h = 4.9T^2$

$h = 4.9 \times \left(\frac{3}{2}\right)^2$
$h = 11.025$
$h \simeq 11.0 \text{ m}$

Question 12 (***+)

A particle P is projected vertically upwards with speed 17.5 ms^{-1} from a point A, which is H m above level horizontal ground.

P moves freely under gravity until it hits the ground 5 s later, with speed $V \text{ ms}^{-1}$.

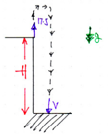
a) Determine the value of H.

A second particle Q is thrown vertically upwards with speed $U \text{ ms}^{-1}$ from A and moves freely under gravity until it hits the ground.

b) Given that Q hits the ground with speed $\frac{6}{7}V \text{ ms}^{-1}$, find the value of U.

$\boxed{}$, $\boxed{H = 35}$, $\boxed{U = \sqrt{43} \approx 6.56}$

a) LOOKING AT THE DIAGRAM, USING DISPLACEMENTS & CONSIDERING THE ENTIRE JOURNEY



$u = +17.5$
$a = -9.8$
$s = -H$
$t = 5$
$v =$ (ALSO NEEDED FOR PART (b))

USING ⇒ "$s = ut + \frac{1}{2}at^2$"

$-H = 17.5 \times 5 + \frac{1}{2}(-9.8) \times 5^2$

$-H = 87.5 - 122.5$

$-H = -35$

$H = 35$

b) V CAN FIRSTLY BE FOUND FROM PART (a)

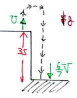
$u = 17.5$
$a = -9.8$
$s = -35$
$t = 5$
$v = ?$

"$v = u + at$" ⇒ $v = 17.5 - 9.8 \times 5$

$v = -31.5$

∴ SPEED $V = 31.5$

NEW DIAGRAM, AND CONSIDERING DISPLACEMENTS FOR THE ENTIRE JOURNEY



$u = U$
$a = -9.8$
$s = -35$
$t =$
$v = -\frac{6}{7}V$

⇒ "$v^2 = u^2 + 2as$"

⇒ $\left(-\frac{6}{7}V\right)^2 = U^2 + 2(-9.8)(-35)$

⇒ $\frac{36}{49}V^2 = U^2 + 686$

⇒ $\frac{36}{49}(31.5)^2 = U^2 + 686$

EARLIER

⇒ $729 = U^2 + 686$

⇒ $43 = U^2$

⇒ $U = +\sqrt{43}$

⇒ $U \approx 6.56 \text{ ms}^{-1}$

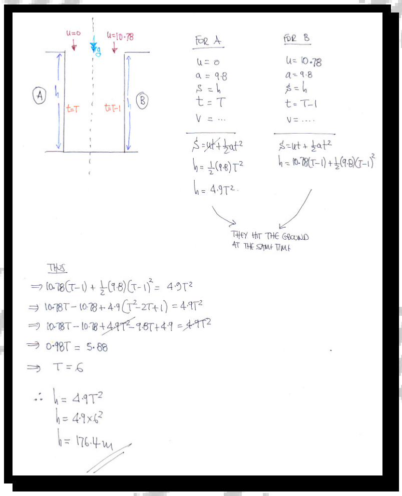
Question 13 (***+)

A particle A is released from rest from a point h m above level horizontal ground.

One second later, another particle B is projected vertically downwards with speed 10.78 ms^{-1} from the same point, A was released.

Given that the particles reach the ground at the same time, determine the value of h.

$\boxed{h = 176.4}$



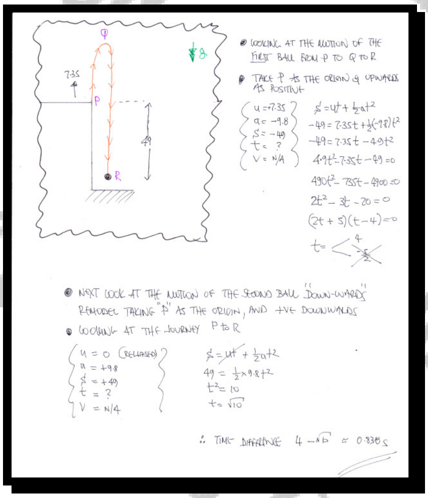
Question 14 (****)

A boy projects a small ball vertically upwards, with speed 7.35 ms^{-1}, from a point P which is located 50 m above level horizontal ground.

Another boy releases a second small ball from P, T s after the first ball was projected upwards.

Given that the two balls collide 1 m above the ground, determine the value of T.

$$\boxed{T = 4 - \sqrt{10} \approx 0.838}$$



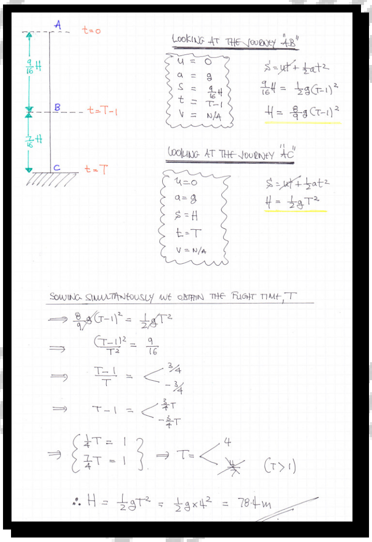
Question 15 (****)

A particle is released from rest from a point H m above level horizontal ground.

The particle covers in the last second of the flight, $\frac{7}{16}$ of the total distance.

Determine the value of H.

$\boxed{H = 78.4}$



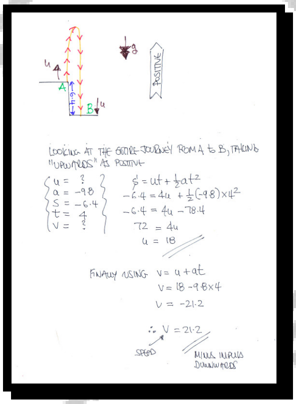
Question 16 (****)

A particle is projected vertically upwards with speed u ms^{-1}, from a balcony which lies 6.4 m above level horizontal ground.

The particle is moving freely under gravity and strikes the ground 4 s later with speed v ms^{-1}.

Calculate in any order the value of u and the value of v.

$\boxed{u = 18}$, $\boxed{v = 21.2}$



LOOKING AT THE ENTIRE JOURNEY FROM A to B, TAKING "UPWARDS" AS POSITIVE

$$\begin{cases} u = ? \\ a = -9.8 \\ s = -6.4 \\ t = 4 \\ v = ? \end{cases}$$

$s = ut + \frac{1}{2}at^2$

$-6.4 = 4u + \frac{1}{2}(-9.8) \times 4^2$

$-6.4 = 4u - 78.4$

$72 = 4u$

$u = 18$

FINALLY USING $v = u + at$

$v = 18 - 9.8 \times 4$

$v = -21.2$

$\therefore v = 21.2$

SPEED

MINUS IMPLIES DOWNWARDS

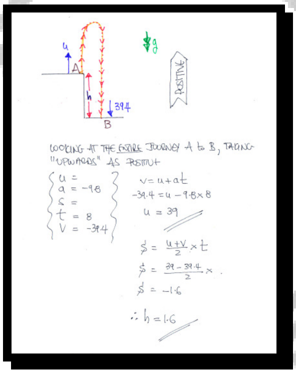
Question 17 (****)

A particle is projected vertically upwards with speed u ms^{-1}, from a balcony which lies h m above level horizontal ground.

The particle is moving freely under gravity and strikes the ground 8 s later with speed 39.4 ms^{-1}.

Calculate in any order the value of u and the value of h.

$\boxed{u = 39}$, $\boxed{h = 1.6}$



Question 18 (****)

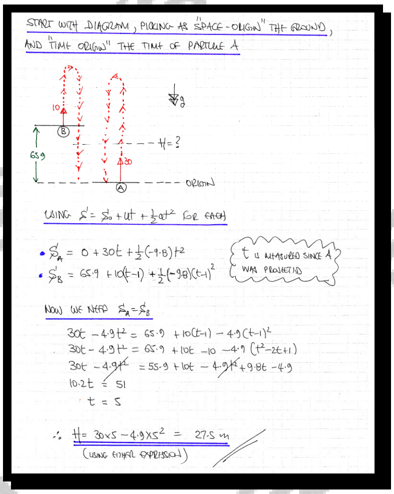
A particle A, is projected vertically upwards with speed 30 ms^{-1} from level horizontal ground.

One second later, another particle B, is projected vertically upwards with speed 10 ms^{-1} from a height of 65.9 m above the same horizontal ground

Eventually both particles the same height H m above ground.

Determine the value of H.

$\square$, $\boxed{H = 27.5}$


START WITH DIAGRAM, PICKING AS "SPACE-ORIGIN" THE GROUND, AND "TIME ORIGIN" THE TIME OF PARTICLE A

USING $s = s_0 + ut + \frac{1}{2}at^2$ FOR EACH

- $s_A = 0 + 30t + \frac{1}{2}(-9.8)t^2$
- $s_B = 65.9 + 10(t-1) + \frac{1}{2}(-9.8)(t-1)^2$

t IS MEASURED SINCE A WAS PROJECTED

NOW WE NEED $s_A = s_B$

$30t - 4.9t^2 = 65.9 + 10(t-1) - 4.9(t-1)^2$

$30t - 4.9t^2 = 65.9 + 10t - 10 - 4.9(t^2 - 2t + 1)$

$30t - \cancel{4.9t^2} = 55.9 + 10t - \cancel{4.9t^2} + 9.8t - 4.9$

$10.2t = 51$

$t = 5$

$\therefore H = 30 \times 5 - 4.9 \times 5^2 = 27.5 \text{ m}$

(USING EITHER EXPRESSION)

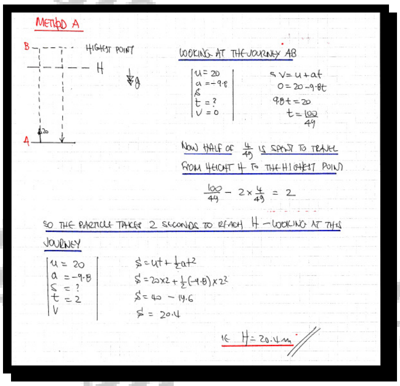
Question 19 (****)

A particle is projected vertically upwards, with speed 20 ms^{-1}, from a point O on level horizontal ground.

In the subsequent motion, the particle travels above a certain height H m for $\frac{4}{49}$ s.

Determine the value of H.

$\square$, $\boxed{H = 20.4}$



METHOD A

LOOKING AT THE JOURNEY AB

$u = 20$, $a = -9.8$, s, $t = ?$, $v = 0$

$v = u + at$

$0 = 20 - 9.8t$

$9.8t = 20$

$t = \frac{100}{49}$

NOW HALF OF $\frac{4}{49}$ IS SPENT TO TRAVEL FROM HEIGHT H TO THE HIGHEST POINT

$\frac{100}{49} - 2 \times \frac{4}{49} = 2$

SO THE PARTICLE TAKES 2 SECONDS TO REACH H – LOOKING AT THIS JOURNEY

$u = 20$, $a = -9.8$, $s = ?$, $t = 2$, v

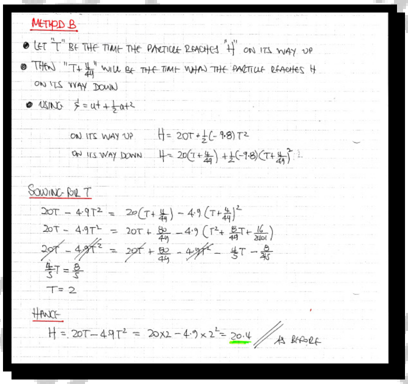
$s = ut + \frac{1}{2}at^2$

$s = 20 \times 2 + \frac{1}{2}(-9.8) \times 2^2$

$s = 40 - 19.6$

$s = 20.4$

ie $H = 20.4$ m



METHOD B

- LET "T" BE THE TIME THE PARTICLE REACHES "H" ON ITS WAY UP
- THEN "$T + \frac{4}{49}$" WILL BE THE TIME WHEN THE PARTICLE REACHES H ON ITS WAY DOWN
- USING $s = ut + \frac{1}{2}at^2$

ON ITS WAY UP $H = 20T + \frac{1}{2}(-9.8)T^2$

ON ITS WAY DOWN $H = 20(T + \frac{4}{49}) + \frac{1}{2}(-9.8)(T + \frac{4}{49})^2$

SOLVING FOR T

$20T - 4.9T^2 = 20(T + \frac{4}{49}) - 4.9(T + \frac{4}{49})^2$

$20T - 4.9T^2 = 20T + \frac{80}{49} - 4.9(T^2 + \frac{8}{49}T + \frac{16}{2401})$

$\cancel{20T} - \cancel{4.9T^2} = \cancel{20T} + \frac{80}{49} - \cancel{4.9T^2} - \frac{4}{5}T - \frac{8}{245}$

$\frac{4}{5}T = \frac{8}{5}$

$T = 2$

HENCE

$H = 20T - 4.9T^2 = 20 \times 2 - 4.9 \times 2^2 = 20.4$ AS BEFORE

Question 20 (****)

At time $t = 0$ s, a small ball is thrown vertically upwards from a point A, with a speed U ms^{-1}.

Simultaneously, another small ball is released from rest from a point B, which is 98 m vertically above A. The two balls meet T s later, at a distance D, above A.

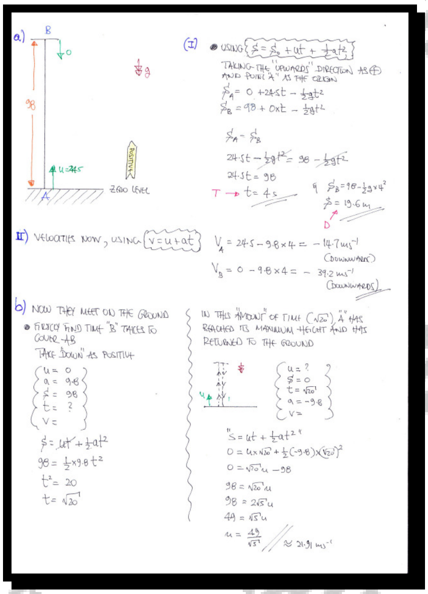
a) Given that $U = 24.5$, …

  i. … determine the value of T and the value of D.

  ii. … find the speed and direction of the two balls as they meet.

b) Given instead that $D = 0$, show that $U = \dfrac{49}{\sqrt{3}}$.

$\boxed{T = 4}$, $\boxed{D = 19.6}$, $\boxed{V_A = 14.7\,\text{ms}^{-1},\ \text{downwards}}$, $\boxed{V_B = 39.2\,\text{ms}^{-1},\ \text{downwards}}$



Question 21 (****+)

At time $t = 0$, a particle is projected vertically upwards with speed U from a point A.

The particle moves freely under gravity.

The point A is at height $8H$ above the ground, where H is the greatest height reached by the particle above A.

Find, in terms of U and g, the total time from the instant of projection to the instant when the particle hits the ground.

$$\boxed{t = \frac{4U}{g}}$$