

1. **A car accelerates uniformly from rest to a velocity of 101 km/h east in 8.0 s. What is the magnitude of its acceleration?**

Solution: The given data are

$$\text{initial velocity, } v_0 = 0$$

$$\begin{aligned} \text{final velocity, } v &= 101 \text{ Km/h} \\ &= 101 \left(\frac{1000 \text{ m}}{3600 \text{ s}} \right) \\ &= 28.06 \text{ m/s} \end{aligned}$$

$$\text{time interval, } t = 8 \text{ s}$$

$$\text{acceleration, } = ?$$

The relevant kinematic equation which relates those together is $v = v_0 + at$. So

$$v = v_0 + at$$

$$28.06 = 0 + a(8)$$

$$\Rightarrow a = \frac{28.06 - 0}{8}$$

$$= \boxed{3.51 \text{ m/s}^2}$$

2. **A car slows down uniformly from 30.0 m/s to rest in 7.20 s. How far did it travel while decelerating?**

Solution: First of all, collect the given data in the interval of accelerating

$$\text{initial velocity} = 30 \text{ m/s}$$

$$\text{final velocity} = 0$$

$$\text{overall time} = 7.20 \text{ s}$$

$$\text{distance} = ?$$

One can solve this problem in two, direct and indirect, ways. In one way, first, find the acceleration of the car and then use other kinematic equations to deter-

mine the desired quantity. So, the acceleration is obtained as

$$\begin{aligned}v &= v_0 + a t \\0 &= 30 + a \quad (7.2) \\ \Rightarrow a &= -4.17 \text{ m/s}^2\end{aligned}$$

The minus sign indicates that the acceleration is in the negative x -direction.

Now substitute the acceleration in one of the kinematic equations which relate those given data and have a missing value of distance, therefore

$$\begin{aligned}v^2 - v_0^2 &= 2a(x - x_0) \\0^2 - (30)^2 &= 2(-4.17)(x - 0) \\ \Rightarrow x &\cong 108 \text{ m}\end{aligned}$$

where we can set the initial position, $x_0 = 0$.

In this kind of problems, since the acceleration is constant so we can use an special equation which is a free as following

$$\begin{aligned}x - x_0 &= \left(\frac{v + v_0}{2} \right) t \\x - 0 &= \left(\frac{0 + 30}{2} \right) (7.2) \\ &= \boxed{108 \text{ m}}\end{aligned}$$

Notice the subtle difference between these two ways. In the first approach, we have an approximate solution but the second one is exact.

To get an exact distance in the first solution, we must determine the car's acceleration with all decimal digits!

3. **An object uniformly accelerates at a rate of 1.00 m/s^2 east. While accelerating at this rate, the object is displaced 417.2 m east in 27.0 s . What is the final velocity of the object?**

Solution: The given data is

$$\begin{aligned}\text{acceleration, } a &= 1.00 \text{ m/s}^2 \\ \text{displacement, } x &= 417.2 \text{ m} \\ \text{overall time, } t &= 27 \text{ s} \\ \text{final velocity, } v &=?\end{aligned}$$

In all standard kinematic equations the initial velocity v_0 is ubiquitous. Here, the initial velocity is not given so we can use an special equation which is v_0 free i.e. $x - x_0 = vt - \frac{1}{2} a t^2$ where v is the velocity at time t . Therefore,

$$x - x_0 = vt - \frac{1}{2} a t^2$$

$$417.2 - 0 = v(27) - \frac{1}{2}(1)(27)^2$$

$$\Rightarrow v = \frac{417.2 + 364.5}{27}$$

$$= \boxed{15.0 \text{ m/s}} \quad \text{East}$$

To find the direction of vector quantities such as displacement, velocity and acceleration, one should adopt a positive direction and then compare the sign of desired quantities with that direction.

Here, we can choose the east direction as positive so the final velocity which is obtained with the positive sign is toward the east.

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4. **An object accelerates uniformly from rest at a rate of 1.9 m/s^2 west for 5.0 s. Find:**

- (a) **the displacement**
- (b) **the final velocity**
- (c) **the distance traveled**
- (d) **the final speed**

Solution: The given data,

$$\text{Initial velocity, } v_0 = 0$$

$$\text{Acceleration, } a = 1.9 \text{ m/s}^2$$

$$\text{Time interval, } t = 5 \text{ s}$$

(a) Use the following equation,

$$x - x_0 = \frac{1}{2}at^2 + v_0t$$

$$x - 0 = \frac{1}{2}(1.9)(5)^2 + 0(5)$$

$$x = \boxed{23.75 \text{ m/s}}$$

In the second line, for convenience, we simply adopt the initial position (x_0) at time $t = 0$ as 0.

(b) the equation below gives the velocity at the end of time interval

$$v = v_0 + at$$

$$v = 0 + (1.9)(5)$$

$$= +9.5 \text{ m/s} \quad \text{West}$$

(c) In a straight-line motion, if the velocity and acceleration have the same signs, the speed of the moving object increases. Here, by establishing a coordinate system and choosing west as a positive direction, we can see the acceleration and velocity are in the same direction.

Therefore, the object moves west without any changing direction. In this type of motion, in which the object does not change its direction, the displacement (vector) and distance traveled are the same.

Thus, as calculated in part (a), the total distance is approximately $\boxed{24 \text{ m}}$.

(d) As reasoning of (c), since the direction of the motion does not change so the magnitude of vector quantities shows the value of scalar ones. Here, the magnitude of the final velocity ($v = 9.5 \text{ m/s}$) is equal to the final speed.

5. **A ball is thrown upwards with a speed of 24 m/s. Take the acceleration due to gravity to be 10 m/s^2 .**

(a) **When is the velocity of the ball 12.0 m/s?**

(b) **When is the velocity of the ball -12.0 m/s ?**

(c) **What is the displacement of the ball at those times?**

(d) **What is the velocity of the ball 1.50 s after launch?**

(e) **What is the maximum height reached by the ball?**

Solution: The kinematic equations of freely falling motions are the same as the horizontal straight-line motion but with some modifications. Here, the motion is in the vertical direction (the y direction) and the acceleration is always downward with the magnitude of $a_y = -g = -10 \text{ m/s}^2$.

Now, applying the above changes to the following kinematic equation in the horizontal direction, we obtain

$$\begin{aligned}v_x &= v_{0x} + a_y t \\v_y &= v_{0y} + (-g)t \\12 &= 24 + (-10)t \\&\Rightarrow \boxed{t = 1.2 \text{ s}}\end{aligned}$$

(b) Recall that velocity is a vector, so in these equations its sign is important. Therefore,

$$\begin{aligned}v_y &= v_{0y} + (-g)t \\-12 &= 24 + (-10)t \\&\Rightarrow \boxed{t = 3.6 \text{ s}}\end{aligned}$$

(c) The only equation which involves a relation between displacement and time is $y_1 - y_0 = \frac{1}{2} a_y t^2 + v_{0y} t$. To solve the kinematic problems, we should first establish a coordinate system. Here, we place the origin of that coordinate system at the ground where the thrower is located. Using $v_{0y} = 24 \text{ m/s}$ and $y_0 = 0$, we have

$$y_1 - y_0 = \frac{1}{2} a_y t^2 + v_{0y} t$$

$$y_1 - y_0 = \frac{1}{2} (-g) t^2 + v_{0y} t$$

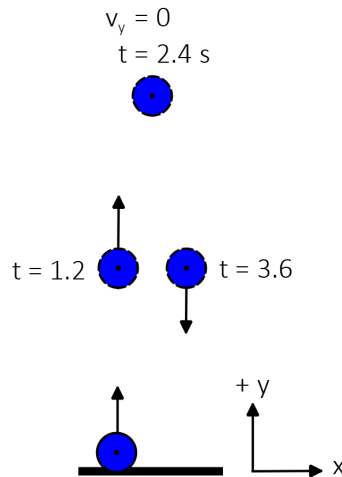
$$y_1 - 0 = \frac{1}{2} (-10)(1.2)^2 + (24)(1.2) \quad \text{at } t = 1.2 \text{ s}$$

$$\Rightarrow \boxed{y_1 = 21.6 \text{ m}}$$

$$y_2 - 0 = \frac{1}{2} (-10)(3.6)^2 + (24)(3.6) \quad \text{at } t = 3.6 \text{ s}$$

$$\Rightarrow \boxed{y_2 = 21.6 \text{ m}}$$

The amount of displacement in the two cases is equal! This shows that the ball is at the same height relative to the ground at times 1.2 s and 3.6 s. Such a thing is possible when the object has the same velocity in different directions, as shown in the figure below.



(d) Use the following equation to find the ball's velocity 1.5 s after throwing

$$v_y = v_{0y} + a_y t$$

$$v_y = 24 + (-10)(1.5)$$

$$\Rightarrow \boxed{v_y = +9 \text{ m/s}}$$

(e) Choose the initial and final points at the beginning and the end of the upward path. First, find the time at which the ball reaches its maximum height,

$$v_{yf} = v_{0y} + a_y t$$

$$0 = 24 + (-10)t_{\text{max}}$$

$$\Rightarrow \boxed{t_{\text{max}} = 2.4 \text{ s}}$$

where v_{yf} is the ball's velocity at the end of the upgoing path where it is zero. Now that the maximum time is found, substitute it into the following equation

to find the corresponding maximum height.

$$y_1 - y_0 = \frac{1}{2}a_y t^2 + v_{0y}t$$

$$y_{\max} - y_0 = \frac{1}{2}(-g)t_{\max}^2 + v_{0y}t_{\max}$$

$$y_{\max} - 0 = \frac{1}{2}(-10)(2.4) + 24(2.4)$$

$$\Rightarrow \boxed{y_{\max} = 28.8 \text{ m}}$$

For more problems on freely falling motion, refer to here.

6. **A stone is thrown vertically upwards with an initial speed of 10.0 m/s^2 from a cliff that is 50.0 m high.**
- When does it reach the bottom of the cliff?**
 - What speed does it have just before hitting the ground?**
 - What is the total distance traveled by the stone? Take the acceleration due to gravity to be 10 m/s^2 .**

Solution: Place the origin of the coordinate system where the stone is thrown, so $y_0 = 0$. In kinematic problems, one should specify two points and apply the kinematic equation of motion to those.

(a) Label the bottom of the cliff as \textcircled{C} . Therefore, given the initial velocity and the height of the cliff, one can use the following kinematic equation which relates those to the fall time.

$$y - y_0 = \frac{1}{2} a_y t^2 + v_{0y}t$$

$$y_{\textcircled{C}} - y_0 = \frac{1}{2} (-g)t^2 + v_{0y}t$$

$$(-50) - 0 = \frac{1}{2} (-10)t^2 + 10t$$

Since the landing point is 50 m below the origin so its coordinate is -50 m . Rearranging above, we get a quadratic equation, $t^2 - 2t - 10 = 0$, whose solution gives the fall time.

Note : for a quadratic equation $ax^2 + bx + c = 0$, the values of x which are the solution of it are given by the following relation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Therefore, using above relation we can get the fall time as

$$t^2 - 2t - 10 = 0$$

$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-10)}}{2(1)}$$

$$\Rightarrow \boxed{t = 4.31 \text{ s}}$$

(b) Substituting the fall time, computed in part (a), in the equation $v = v_0 + a_y t$
OR using the equation $v^2 - v_0^2 = 2a_y(y - y_0)$, we can obtain the velocity at the moment of hitting to the ground.

$$v = v_0 + a_y t$$

$$v_{\text{ⓐ}} = v_{0y} + (-g)t$$

$$v_{\text{ⓐ}} = 10 + (-10)(4.31)$$

$$v_{\text{ⓐ}} = \boxed{-33.1 \text{ m/s}}$$

OR

$$v^2 - v_0^2 = 2a_y(y - y_0)$$

$$v_{\text{ⓐ}}^2 - v_{0y}^2 = 2(-g)(y_{\text{ⓐ}} - y_0)$$

$$v_{\text{ⓐ}}^2 - (10)^2 = 2(-10)(-50 - 0)$$

$$v_{\text{ⓐ}} = -33.1 \text{ m/s}$$

(c) Applying the equation $v^2 - v_0^2 = 2(-g)(y - y_0)$ to find the distance traveled during climbing, then twice that value yields the total distance to the thrown point. Now add the cliff's height to find the total distance traveled by the object.

$$v_{\text{ⓑ}}^2 - v_{0y}^2 = 2(-g)(y_{\text{ⓑ}} - y_0)$$

$$0 - (10)^2 = 2(-10)(y_{\text{ⓑ}} - 0)$$

$$\Rightarrow y_{\text{ⓑ}} = 5 \text{ m}$$

$$\begin{aligned} \text{total distance traveled} &= 2y_{\text{ⓑ}} + \text{cliff's height} \\ &= 2(5) + 50 \\ &= \boxed{60 \text{ m}} \end{aligned}$$

where ⓑ is the highest point reached by the object.

7. A rock is thrown vertically down from the roof of 25.0 m high building with a speed of 5.0 m/s.

- (a) When does the rock hit the ground?
 (b) With what speed does it hit the ground? Take the acceleration due to gravity to be 10 m/s^2 .

Solution: (a) First establish a coordinate system whose origin placed at the thrown point ($y_0 = 0$). Now, use the equation $y - y_0 = \frac{1}{2}(-g)t^2 + v_{0y}t$, which relates fall time and displacement together, to find the desired value. Note that, since the initial velocity is downward and the rock hits at a point 25 m below the origin so they come with a minus in equations.

$$y - y_0 = \frac{1}{2}(-g)t^2 + v_{0y}t$$

$$(-25) - 0 = \frac{1}{2}(-10)t^2 + (-5)t$$

In the end, we get a quadratic equation, $t^2 + t - 5 = 0$, whose solutions give the fall time. Using the standard way of solution of quadratic equations, we have

$$t^2 + t - 5 = 0$$

$$t = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-5)}}{2(1)}$$

$$\Rightarrow \boxed{t = 1.79 \text{ s}}$$

In above, for a quadratic equation $ax^2 + bx + c = 0$, we find the solutions as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(b) Apply $v = v_{0y} + (-g)t$ and substitute the time computed in (a) into it OR use $v_y^2 - v_{0y}^2 = 2(-g)(y - y_0)$. Therefore,

$$v_y = v_{0y} + (-g)t$$

$$v_y = (-5) + (-10)(1.79)$$

$$\Rightarrow v_y = \boxed{-22.9 \text{ m/s}}$$

OR

$$v_y^2 - v_{0y}^2 = 2(-g)(y - y_0)$$

$$v_y^2 - (-5)^2 = 2(-10)(-25 - 0)$$

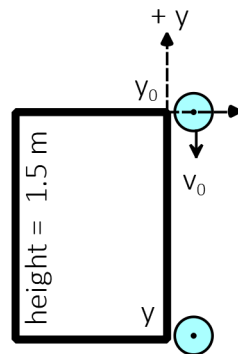
$$v_y^2 = 525$$

$$\Rightarrow v_y = \pm 22.9 \text{ m/s}$$

Note that the square roots have two roots but since the velocity vector of the rock points downward so we have to choose the negative i.e. $\boxed{v_y = -22.9 \text{ m/s}}$.

8. A window is 1.50 m high. A stone falling from above passes the top of the window with a speed of 3.00 m/s. When will it pass the bottom of the window? (Take the acceleration due to gravity to be 10 m/s².)

Solution: The stone is fallen from the upper edge of the window so place the origin of the coordinate system at this point ($y_0 = 0$). Since the vector of initial velocity is downward and the window's bottom edge is located 1.5 m below the origin, so we set $v_{0y} = -3$ m/s, $y = -1.5$ m in kinematic equations.



$$y - y_0 = \frac{1}{2}(-g)t^2 + v_{0y}t$$

$$(-1.5) - 0 = \frac{1}{2}(-10)t^2 + (-3)t$$

$$-1.5 = -5t^2 - 3t$$

After rearranging the above equation, we arrive at $t^2 + 0.6t - 0.3 = 0$ whose solution is obtained as

$$t^2 + 0.6t - 0.3 = 0$$

$$t = \frac{-0.6 \pm \sqrt{(0.6)^2 - 4(1)(-0.3)}}{2(1)}$$

$$t_1 = 0.342 \text{ s}$$

$$t_2 = -0.924 \text{ s}$$

The above quadratic equation has two roots but the physical solution is the one with positive sign. The negative one indicates a time before we dropped the stone! Thus, we choose the positive solution i.e. $t = 0.342 \text{ s}$.

9. A ball is tossed with a velocity of 10 m/s vertically upward from the window located 20 m above the ground. Knowing that the acceleration of the ball is constant and equal to 9.81 m/s^2 downward, determine:
- the velocity v and elevation y of the ball above the ground at any time t .
 - the highest elevation reached by the ball and the corresponding value of t .
 - the time when the ball will hit the ground and the corresponding velocity.

Solution: (a) First, such as in all kinematic problems, establish a coordinate system whose origin is placed at the point where the ball is tossed. In this point, we set $v_{0y} = +10 \text{ m/s}$ and $y_0 = 0$. The positive is due to the upward direction of the initial velocity's vector. The velocity of a falling object at any later time t is given by

$$v_y = v_{0y} + (-g)t$$

Where the vertical constant acceleration a_y is replaced by the always downward free-falling acceleration $-g$. Thus, by substituting the numerical values into the above equation, we get

$$v_y = 10 - 9.81t$$

The displacement at that given time interval is obtained as

$$y - y_0 = \frac{1}{2}(-g)t^2 + v_{0y}t$$

putting the values gives

$$y - y_0 = \frac{1}{2}(-g)t^2 + v_{0y}t$$

$$y - 0 = \frac{1}{2}(-9.81)t^2 + (10)t$$

$$y = -4.905t^2 + 10t$$

Note that the equation above gives the distance at any time relative to the throw's point.

(b) At the highest elevation, the vertical velocity of a falling object is always zero i.e. $v_y = 0$. Therefore, using the above equation for the velocity at any time

t , we have

$$\begin{aligned}v_y &= v_{0y} + (-g)t \\0 &= +10 - 9.81t \\ \Rightarrow & \boxed{t = 1.019 \text{ s}}\end{aligned}$$

Now, substitute this time value into the equation of distance at any time

$$y - y_0 = \frac{1}{2}(-g)t^2 + v_{0y}t$$

$$y - 0 = \frac{1}{2}(-9.81)(1.019)^2 + (10)(1.019)$$

$$\boxed{y = 5.096 \text{ m}}$$

Adding the height of windows, we can obtain the elevation from the ground at any time, i.e., the total distance becomes $20 + 5.096 = 25.09 \text{ m}$.

(c) The ball hit the ground where its coordinate is 20 m below our chosen origin so we should set, $y = -20 \text{ m}$ in the distance equation above and solve for the time t

$$y - y_0 = \frac{1}{2}(-g)t^2 + v_{0y}t$$

$$(-20) - 0 = \frac{1}{2}(-9.81)t^2 + 10t$$

Rearranging the above expression, we get a quadratic equation, $4.905t^2 - 10t - 20 = 0$, whose t solutions are obtained as

$$4.905t^2 - 10t - 20 = 0$$

$$t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4.905)(-20)}}{2(4.905)}$$

$$\Rightarrow t_1 = 3.281 \text{ s} \quad , \quad t_2 = -1.242 \text{ s}$$

The negative time refers to a time before the ball is thrown! which is obviously incorrect. Thus, we choose the correct positive fall time, $t_1 = 3.281 \text{ s}$.

The velocity at the moment of hitting to the ground is obtained by equations, $v_y^2 - v_{0y}^2 = 2(-g)(y - y_0)$ or $v_y = v_{0y} + (-g)t$. Note that in the latter you should put the time fall computed previously back into it. Therefore,

$$v_y^2 - v_{0y}^2 = 2(-g)(y - y_0)$$

$$v_y^2 - (10)^2 = 2(-9.81)(-20 - 0)$$

$$v_y^2 = 492.4$$

$$\Rightarrow \boxed{v_y = \pm 22.19 \text{ m/s}}$$

The \pm shows that there are two mathematical solutions that should be chosen by physical reasoning. Since at the moment of hitting the ground, the ball's vector velocity is downward so the correct sign is negative, and thus, $v_y = -22.19 \text{ m/s}$.

10. **A 3.0 Kg ball is thrown vertically into the air with an initial velocity of 15 m/s. What is the maximum height of the ball?**

Solution: Place the origin of the coordinate system at the ball's thrown point so $y_0 = 0$. Apply the following kinematic equation to find the maximum height where the vertical velocity is zero, $v_y = 0$,

$$v_y^2 - v_{0y}^2 = 2(-g)(y - y_0)$$

$$0 - (15)^2 = 2(-9.8)(h_{\max} - 0)$$

$$\Rightarrow \boxed{h_{\max} = 11.47 \text{ m}}$$

11. **An object starts from rest with an acceleration of 2.0 m/s^2 that lasts for 3.0 s. It then reduces its acceleration to 1.0 m/s^2 that lasts for 5.0 additional seconds. What is the velocity at the end of the 5.0 s interval?**

Solution: The motion described has two stages. In stage one, find the velocity at the end of 2, s which is considered as the initial velocity for the second stage. Therefore,

$$v = v_0 + a_x t$$

$$v_1 = 0 + 2(3)$$

$$\Rightarrow v_1 = 6 \text{ m/s}$$

where v_0 and v_1 are the initial velocity and velocity at the time $t = 2$ s later. Now, repeat this process for the second stage

$$\begin{aligned}v &= v_0 + a_x t \\v_2 &= v_1 + a_x t \\v_2 &= 6 + (1)(5) \\ \Rightarrow \quad &\boxed{v_2 = 11 \text{ m/s}}\end{aligned}$$

Thus, the object's velocity at the end of 5 seconds is 11 m/s.

12. **An object initially traveling at a velocity of 2.0 m/s west accelerates uniformly at a rate of 1.3 m/s² west. During this time of acceleration, the displacement of the object is 15 m. Find:**

(a) the final velocity

(b) the final speed

Solution: The given data is

$$\begin{aligned}\text{Initial velocity, } v_0 &= 2.0 \text{ m/s} \\ \text{Acceleration, } a &= 1.3 \text{ m/s}^2 \\ \text{Displacement, } (x - x_0) &= 15 \text{ m}\end{aligned}$$

Recall that the difference between velocity and speed is in their definitions. Velocity is a vector quantity whose magnitude appears in all kinematic equations but the speed is scalar which depends on the total distance of the moving body. Since the object moves along a straight line without any change of direction at the end of a given time interval, its speed and velocity are the same. Therefore,

$$v^2 - v_0^2 = 2a_x(x - x_0)$$

$$v^2 - (2)^2 = 2(1.3)(15)$$

$$v^2 = 43$$

$$v = \sqrt{43}$$

$$\Rightarrow \boxed{v = 6.55 \text{ m/s}} \quad \text{West}$$

Thus,

(a) final velocity is 6.55 m/s toward west.

(b) final speed is 6.55 m/s.

13. **A bungee cord is 11.0 m long. What will be the velocity of a bungee jumper just as the cord begins to stretch?**

Solution: The initial velocity of a bungee jumper is usually zero since it is at rest just before the falling. Here, the cord's unstretched length can be thought of as the vertical displacement, $y - y_0$, of the jumper. Thus, apply the following kinematic equation to the vertical direction and find the final velocity just before the cord is stretched.

$$v^2 - v_{0y}^2 = 2a_y(y - y_0)$$

$$v^2 - v_{0y}^2 = 2(-g)(y - y_0)$$

$$v^2 - 0 = 2(-9.81)(-11)$$

$$\Rightarrow \boxed{v = 14.7 \text{ m/s}}$$

The displacement is set to be negative since we placed the origin of the coordinate system at the jumper's falling point i.e. $y_0 = 0$. Therefore, the cord's end is located $y = -11$ m below the origin. The \pm indicates physically the direction of velocity. Because it is toward the falling direction, so the correct sign is minus.