Degrees of Freedom

Degrees of freedom is an important value to report when describing the results of a statistical analysis. Dependent upon the statistical procedure being performed, the degrees of freedom may be easy or complex to calculate. Most statistical analysis software packages will include the degrees of freedom within the results of a statistical procedure.

The number of **degrees of freedom** is the number of values in the final calculation of a statistic that are free to vary. Determination of the degrees of freedom is based on the statistical procedure being used. For a correlation analysis between two variables the *df* = # of data points within a group - the number of groups. Thus, if determining the correlation between two groups of 20 participants, the *df* = 20 - 2 = 18. For a means comparison using a *t* test to compare two independent-samples with the same sample size in each group the *df* would be the total number of subjects from both groups minus the number of groups (2n-2). Thus, if comparing two groups, each with 4 subjects, the degrees of freedom would be = (4+4) - 2 = 6. These are only examples, the exact method of calculating *df* is dependent upon the statistical procedure being used and the make up of the data, and some can involve rather complex operations (although its easy for our statistical analysis software :-)).

Making Sense of Degrees of Freedom
quoted exerts from Hellen Walker, Degrees of Freedom. Journal of Educational Psychology. 31(4) (1940) 253-269

"If you are asked to choose a pair of numbers (x, y) at random, you have complete freedom of choice with regard to each of the two numbers, have two degrees of freedom. The number pair may be represented by the coordinates of a point located in the x, y plane, which is a two-dimensional space. The point is free to move anywhere in the horizontal direction parallel to the x axis, and is also free to move anywhere in the vertical direction, parallel to the y axis. There are two independent variables and the point has two degrees of freedom.

Now suppose you are asked to choose a pair of numbers whose sum is 7. It is readily apparent that only one number can be chosen freely, the second being fixed as soon as the first is chosen. Although there are two variables in the situation, there is only one independent variable. The number of degrees of freedom is reduced from two to one by the imposition of the condition $x + y = 7$. The point is not now free to move anywhere in the xy plane but is constrained to remain on the line whose graph is $x + y = 7$, and this line is a one--dimensional space lying in the original two-dimensional space.

Suppose you are asked to choose a pair of numbers such that the sum of their squares is 25. Again it is apparent that only one number can be chosen arbitrarily, the second being fixed as soon as the first is chosen. The point represented by a pair of numbers must lie on a circle with center at the origin and radius 5. This circle is a one-dimensional space lying in the original two-dimensional plane. The point can move only forward or backward along this circle, and has one degree of freedom only. There were two numbers to be chosen (N = 2) subject to one limiting relationship (r = 1) and the resultant number of degrees of freedom is .

Suppose we simultaneously impose the two conditions $x + y = 7$ and $x^2 + y^2 = 25.$ If we solve these equations algebraically we get only two possible solutions, $x = 3$, $y = 4$, or $x = 4$, $y = 3$. Neither variable can be chosen at will. The point, once free to move in two directions, is now constrained by the equation $x + y = 7$ to move only along a straight line, and is constrained by the equation $x^2 + y^2 = 25$ to move only along the circumference of a circle, and by the two together is confined to the intersection of that line and circle. There is no freedom of motion for the point. $N = 2$ and $r = 2$. The number of degrees of freedom is $N - r = 2 - 2 = 0$."