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Lecture 9

Two-Degree-of-Freedom Systems

Contents

Introduction:

• The vibrating systems, which require two coordinates to describe its motion, are called two-degrees-of –freedom systems.

• These coordinates are called generalized coordinates when they are independent of each other and equal in number to the degrees of freedom of the system.

• Unlike single degree of freedom system, where only one co-ordinate and hence one equation of motion is required to express the vibration of the system, in two-dof systems minimum two co-ordinates and hence two equations of motion are required to represent the motion of the system. For a conservative natural system, these equations can be written by using mass and stiffness matrices.

• One may find a number of generalized co-ordinate systems to represent the motion of the same system. While using these co-ordinates the mass and stiffness matrices may be coupled or uncoupled. When the mass matrix is coupled, the system is said to be dynamically coupled and when the stiffness matrix is coupled, the system is known to be statically coupled.

• The set of co-ordinates for which both the mass and stiffness matrix are uncoupled, are known as principal co-ordinates. In this case both the system equations are independent and individually they can be solved as that of a singledof system.

• A two-dof system differs from the single dof system in that it has two natural frequencies, and for each of the natural frequencies there corresponds a natural state of vibration with a displacement configuration known as the normal mode. Mathematical terms associated with these quantities are eigenvalues and eigenvectors.

• Normal mode vibrations are free vibrations that depend only on the mass and stiffness of the system and how they are distributed. A normal mode oscillation is defined as one in which each mass of the system undergoes harmonic motion of same frequency and passes the equilibrium position simultaneously.

•The study of two-dof- systems is important because one may extend the same concepts used in these cases to more than 2-dof-systems. Also in these cases one can easily obtain an analytical or closed-form solutions. But for more degrees of 209 freedom systems numerical analysis using computer is required to find natural frequencies (eigenvalues) and mode shapes (eigenvectors).

Two degree-of-freedom systems require two generalized coordinates to describe the motion of every particle in the system. The system requires two (in general) coupled differential equations governing the motion of the system.

Two degree-of-freedom systems are considered before n degree-of-freedom systems because

• Many systems only require two degrees of freedom when modeling.
• While the equations are formulated in a matrix form, matrix algebra is not required to formulate a solution.
• Physical insight is gained by studying two degree-of-freedom systems.
• Viscous damping can be more easily handled.

The differential equations governing two degree-of-freedom systems are derived. A normal-mode solution for the free response for undamped systems is assumed in which both generalized coordinates are assumed to vibrate synchronously with different amplitudes. The normal-mode solution is used to obtain the natural frequencies and mode shapes, which are the relative amplitudes of vibration, for the two degree-of-freedom system. The two mode shapes are combined to formulate the free response for undamped systems. The solution is in terms of four constants of integration, which are determined through application of initial conditions.

An exponential solution is assumed for systems with viscous damping. This leads to a fourth-order algebraic equation for a parameter. The fourth-order equation includes odd powers, so it cannot be reduced to a quadratic and must be solved numerically. The modes of vibration can be underdamped, critically damped, or overdamped. The free response is obtained in terms of constants of integration. Initial conditions are applied to determine the constants. When the differential equations are written using principal coordinates as the dependent variables, they are uncoupled. However, the principal coordinates are not obvious; sometimes a principal coordinate does not represent the displacement of a particle in the system.

The forced response of systems with harmonic excitations is developed. Both undamped systems and damped systems are considered. The sinusoidal transfer functions are developed as a means of determining the harmonic response. The concept of frequency response is considered.

An application of harmonic response of two degree-of-freedom systems is the vibration absorber. A vibration absorber is an auxiliary mass-spring system that is attached to a machine that is experiencing large amplitude vibrations due to near-resonance conditions.

The addition of a vibration absorber changes a SDOF system to a two degree-of-freedom system. When the vibration absorber is properly “tuned,” the steady-state vibrations of the machine are eliminated. One problem with vibration absorbers is that the lower natural frequency of the two degree-of-freedom system is lower than the tuned speed. Thus, the lower natural frequency is passed through during start-up, which leads to large amplitude vibrations. When damping is added to the vibration absorber to control the vibrations during start-up, the ability to eliminate steady-state vibrations of the machine is lost. An optimum damped vibration absorber is determined.

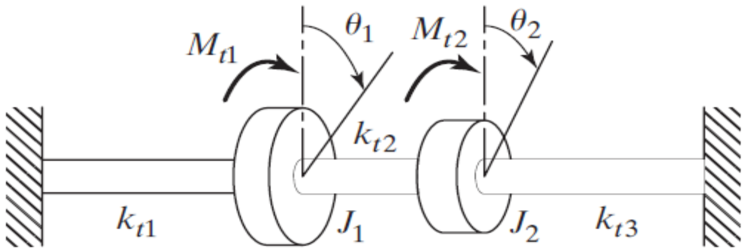
2. Derivation of the Equations of Motion

The equations of motion for a two degree-of-freedom system are derived using the free-body diagram method or an energy method. Multiple free-body diagrams or equations may be used. Newton's law (ΣF = ma) is applied to the free-body diagram of a particle. The equations ΣF and ΣM are applied to a free-body diagram of a rigid body undergoing planar motion with rotation about a fixed axis through 0. For a rigid body undergoing planar motion, D'Alembert's principle can be applied. The system of effective forces is a force equal to applied at the mass center and a moment equal to I.

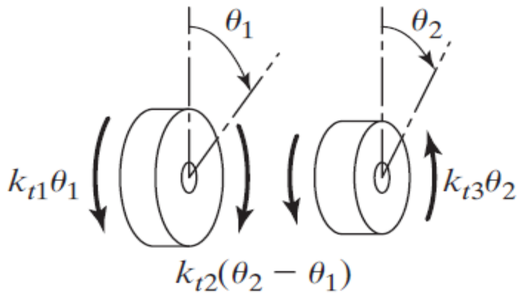
Derive the differential equations governing the motion of the two degree-of-freedom system

1. Torsional System

Consider a torsional system consisting of two discs mounted on a shaft, as shown in Fig.1.



(a)



(b)

The three segments of the shaft have rotational spring constants k_{t1} , k_{t2} and k_{t3} as indicated in the figure. Also shown are the discs of mass moments of inertia J_1 and J_2 , the applied torques M_{t1} and M_{t2}, and the rotational degrees of freedom θ_1 and θ_2. The differential equations of rotational motion for the discs J_1 and J_2 can be derived.

Example-01: The transient vibrations of the drive line developed during the application of a cone (friction) clutch lead to unpleasant noise. To reduce the noise, a flywheel having a mass moment of inertia J_2 is attached to the drive line through a torsional spring k_{t2} and a viscous torsional damper c_{t2} as shown in Figure below. If the mass moment of inertia of the cone clutch is J_1 and the stiffness and damping constant of the drive line are given by k_{t1} and c_{t1} respectively, derive the relations to be satisfied for the stable operation of the system.

Example-02: Determine the natural frequencies and normal modes of the torsional system shown in Figure below for $k_{t2} = 2k_{t1}$ and $J_2 = 2J_1$.