Bond Practice Questions and Answers

1. What is the present value of the following payments?
   (a) $1000 two years from now when the effective annual interest rate is 10%.
   (b) $1000 two years from now when the bond equivalent yield is 10%.
   (c) $1000 one-half year from now when the yield on a discount basis is 10%.
   (d) Which of the above payments would you prefer?

   If the above were bonds then, under our assumption that the yield corresponds to the price, $P=PV$.
   (e) Given the prices found in (a)-(c), derive the corresponding rate.
   (f) What is the bond equivalent yield corresponding to the YTM in (a).
   (g) What is the YTM corresponding to the bond equivalent yield in (b)-(c).

2. Fill in the yields in the table for discount bonds with F=$1000

Price P	Maturity n	Yield on a discount basis	YTM
900	1 year		
950	½ year		
975	¼ year		

3. The YTM on the bond is 5% and the coupon rate is 2% with annual coupons.
   (a) What can you say about the price of the bond?
   (b) What is its price if it is a 30-year bond and has a face value of 100,000?
   (c) What is the Holding Period Return (*HPR*) on the bond after 1 year if the yield to maturity drops to 2%?

   Actual bonds are quoted at the bond equivalent yield. The quoted yield is 5% and the coupon rate is 2% with semi-annual coupons. The bond has term to maturity of 30 years. Suppose you don't know the face value.
   (d) What is its price as a percentage of face value?
   (e) What is the holding period return on the bond after 6 months if the quoted yield drops to 2%? (This drop from 5% to 2% is what happened in 2009.)

4. Consider two bonds. A consol with yield 10%. A two-year coupon bond is selling at par (i.e. face value) has yield to maturity 10%. Both bonds pay coupons yearly. At the end of the first year, the yields on all bonds fall to 5%. Which bond earns a higher holding period rate of return?

5. As part of a promotion you are offered a car loan for $30,000 at a YTM of 2% with payments made annually over 5 years.
   (a) What is the annual fixed payment?
   (b) How much would you save per year relative to taking out the same loan at a bank at 7%? How much would you save in present value terms?

   Instead of a promotion with a lower interest rate you are offered the car at a lower price but at 7%.
   (c) At what price are you better off buying the car at the higher yield?

More realistically, the car loan for $30,000 is quoted 2% (i.e. bond equivalent yield) interest with payments monthly over 5 years.

(d) What is the monthly fixed payment?
(e) How much would you save in present value terms relative to taking out the same loan at the bank's quoted rate of 7%?
(f) If the promotion wasn't offered, how much should the person be willing to pay for the lower interest rate?

Answers

1. What is the present value of the following payments:
   (a) $1000 two years from now when the effective annual interest rate is 10%.
   Given: simple loan i =.1, n =2, F = 1000.
   Find: $PV = \dfrac{F}{(1+i)^n} = \dfrac{1000}{(1+.1)^2}$ = $826.446
   (b) $1000 two years from now when the bond equivalent yield is 10%.
   Given: i_{bey} =.1, n =2, F = 1000.
   Assuming semi-annual compounding, then $i_{1/2} = i_{bey}/2 = .1/2 = .05$
   Find: $PV = \dfrac{F}{(1+i_{1/2})^{n(2)}} = \dfrac{1000}{(1+.05)^4}$ = 822.703
   (c) $1000 one-half year from now when the yield on a discount basis is 10%.
   Find: $PV = \dfrac{F}{(1+i_{1/2})^{n(2)}} = \dfrac{1000}{(1+.05)}$ = $952.381
   (d) Which of the above payments would you prefer? – (c) ; i.e. in ½ year.
   Ceteris paribus, receiving money sooner allows for reinvestment sooner. $1000 reinvested at positive interest after ½ year produces more than $1000 in two years.
   If the above were bonds then, under our assumption, $P=PV$.
   (e) Given: P in (a)-(c). Derive: interest rate given above.
   (a) $i = \left(\dfrac{F}{P}\right)^{1/n} - 1 = \left(\dfrac{1000}{826.446}\right)^{1/2} - 1 = 0.1$, similarly for (b)&(c)
   (f) Given: i =.1. Find: $i_{bey} = i_{1/2}(2) = 0.048809(2) = 0.097618$
   where $(1+i)=(1+i_{1/2})^2$ implies $i_{1/2} = (1+i)^{1/2} - 1 = 1.1^{1/2} - 1 = 0.048809$
   (g) Given: i_{bey} =.1. Find $i = (1+ i_{bey}/2)^2 - 1 = 0.1025$

2. Fill in the yields in the table for discount bonds with F=$1000

Price P	Maturity n	Yield on discount basis	YTM
900	1 year	(1000/900)-1 = .111	.111
950	½ year	$2i_{1/2}$= 2[(1000/950)-1] =.1052632	$(1+i_{1/2})^2 - 1$ =.1080332
975	¼ year	$4i_{1/4}$=4[(1000/975)-1]= .1025641	$(1+i_{1/4})^4 - 1$ =.1065767

3. Given: i = .05 , *CouponRate* =.02 = C/F , n = ?, F = ?

(a) Find: $P < F$ *iff* i =.05 > .02= *CouponRate*; i.e., price is is less than face value.

Given: n =30 and F=100000

(b). Find: $P = \frac{C}{i}\left(1 - \frac{1}{(1+i)^n}\right) + \frac{F}{(1+i)^n}$

$$= \frac{.02(100000)}{.05}\left(1 - \frac{1}{(1+.05)^{30}}\right) + \frac{100000}{(1+.05)^{30}} = 53883.0$$

Given: i drops to i =.02 at the end of the year.

(c) Find: $HPR = \frac{C}{P} + \frac{P_1 - P}{P} = \frac{2000}{53883} + \frac{100000 - 53883}{53883} = 0.89299$

where $P_1 = F$ as i= *CouponRate*.

Given: *CouponRate* =.02 = $2C/F$, i_{bey} = .05 (quote is bey), n = 30, F = ?

(d) Find: P as percentage of F

$$\text{Find}: P = \frac{C}{i_{1/2}}\left(1 - \frac{1}{(1+i_{1/2})^{n(2)}}\right) + \frac{F}{(1+i_{1/2})^{n(2)}}$$

$$= \frac{.02F/2}{.05/2}\left(1 - \frac{1}{(1+.05/2)^{60}}\right) + \frac{F}{(1+.05/2)^{60}} = 0.5363702F$$

Quotes are often made as a percentage of face value, 53.64%.

Given: i_{bey} drops to i_{bey} =.02 at the end of the year.

(e) Find: $HPR = \frac{P_{1/2} + C - P_0}{P_0} = \frac{F + (0.02F/2) - .5363702F}{.5363702F} = .883$ or 88.3%

$$\text{where}: P_{1/2} = \frac{.02F/2}{.02/2}\left(1 - \frac{1}{(1+.02/2)^{(29.5)2}}\right) + \frac{F}{(1+.02/2)^{29.5)2}} = F$$

(Advanced: It turns out that *CouponRate* = i_{bey} *iff* $P = F$ for semi-annual compounding.)

4. Consider two bonds. A consol yield to maturity 10%. A two-year coupon bond is selling at par (i.e. face value) has yield to maturity 10%. Both bonds pay coupons yearly. At the end of the first year, the yields on all bonds fall to 5%. Which bond earns a higher holding period rate of return (*HPR*)?

Consol: $HPR = \frac{P_1 + C - P_0}{P_0} = \frac{C/.05 + C}{C/.1} - 1 = \frac{1/.05 + 1}{1/.1} - 1 = 1.1$,

where $P_1 = C/i$ at time 1 i=.05.

Coupon Bond: Given $P_0 = F$, i =0.1.

As $P_0 = F$ *iff* i = *CouponRate* implies i_c =0.1= C/F

$$HPR = \frac{P_1 + C - P_0}{P_0} = \frac{[(C+F)/1.05] + C}{P_0} - 1 = (\frac{C}{F} + 1)/1.05 + \frac{C}{F} - 1$$

$$= \frac{(.1+1)}{1.05} + .1 - 1 = .147619$$

where $P_1 = (C+F)/1.05$ as there is one year to go for the bond, and (C/F)=.1 is the coupon rate when coupons are paid annually. The consol has a much higher *HPR*. This is because the term to maturity is much greater.

Advanced: Is there a general solution when we don't know the coupon rate? -Yes.

$$HPR = \frac{P_1 + C - P_0}{P_0} = \frac{[(C+F)/1.05] + C}{\frac{C}{(1+.1)} + \frac{C+F}{(1+.1)^2}} - 1$$

As *CouponRate* = *C*/*F*, then *C* = *CouponRate* (*F*).

Then substituting *C* = *CouponRate* (*F*), the *F* 's cancel leaving

$$HPR = \frac{[(CouponRate+1)/1.05] + CouponRate}{\frac{CouponRate}{(1+.1)} + \frac{CouponRate+1}{(1+.1)^2}} - 1 = \frac{(1.1)^2}{1.05}\frac{1+(2.05)CouponRate}{1+(2.1)CouponRate} - 1 ,$$

for any $i \geq 0$ the return lies in the interval: $0.1294 < HPR < 0.152$.

The consol always earns a higher rate of return.

5. Given: Fixed payment loan $LV = 30{,}000$, $i = .02$, $n = 5$, *FP* is yearly.

(a) Find: $FP = \frac{LV(i)}{1-(1+i)^{-n}} = \frac{30000(.02)}{1-(1+.02)^{-5}} = 6364.8$, Note: $LV = \frac{FP}{i}\left(1 - \frac{1}{(1+i)^n}\right)$

Given: Same except $i = .07$.

(b) Find: $FP_{i=.07} - FP_{i=.02} = 7316.7 - 6364.8 = 951.9$ per year,

where $FP_{i=.07} = \frac{30000(.07)}{1-(1+.07)^{-5}} = 7316.7$

Find: PV of savings $PV = \frac{951.9}{.07}\left(1 - \frac{1}{(1+.07)^5}\right) = 3899.3$

using payment loan formula and the bank rate as the opportunity cost of funds. Because the savings are discounted they are less than the accounting total saving $951.9(5) = 4755.45$.

(c) Find: $LV = 26{,}100.7$ at i=7 is when indifferent between the loans.

This *LV* at i=7 implies the same fixed a payment as $FP_{i=.02}$

$$FP_{i=.07} = \frac{LV(.07)}{1-(1+.07)^{-5}} = \frac{26100.7(.07)}{1-(1+.07)^{-5}} = 6365.7$$

Note: 30000 - 3899.3 = 26,100.7 .

Repeat given: $i_{bey} = .02$, $n = 5$, *FP* is monthly.

(d) Find: $FP = \frac{LV(i_{1/12})}{1-(1+i_{1/12})^{-n(12)}} = \frac{30000(.00166)}{1-(1+.00166)^{-60}} = \$525.724$,

where $i_{1/12} = (1 + i_{bey}/2)^{1/6} - 1 = (1 + .01)^{1/6} - 1 = 1.65976 \times 10^{-3}$

Note: $LV = \frac{FP}{i_{1/12}}\left(1 - \frac{1}{(1+i_{1/12})^{n(12)}}\right)$

(e) Find: $FP_{i=.07} - FP_{i=.02} = 592.622 - 525.724 = \$66.898$.

where $FP_{i=.07} = \frac{LV(i_{1/12})}{1-(1+i_{1/12})^{-n(12)}} = \frac{30000(.00575)}{1-(1+.00575)^{-60}} = \$592.622$

where $i_{1/12} = (1 + i_{bey}/2)^{1/6} - 1 = (1 + .035)^{1/6} - 1 = 5.750039 \times 10^{-3}$

Find: The present value savings is: $\frac{66.898}{.00575}\left(1 - \frac{1}{(1+.00575)^{60}}\right) = \$3386.542$;

(f) An individual should be willing to pay up to \$3386.54 for the lower interest rate. Note monthly payment lead to less savings than in part (b).