CHAPTER 5

Time Value of Money

Learning Objectives

1. Construct cash flow timelines to organize your analysis of problems involving the time value of money.
2. Understand compounding and calculate the future value of cash flows using mathematical formulas and a financial calculator.
3. Understand discounting and calculate the present value of cash flows using mathematical formulas and a financial calculator.
4. Understand how interest rates are quoted and know how to make them comparable.

Principals Applied in this Chapter

- Principle 1: Money Has a Time Value.

3 Rules of Financial Time Travel

- **Rule 1**: Only values at the same point in time can be compared

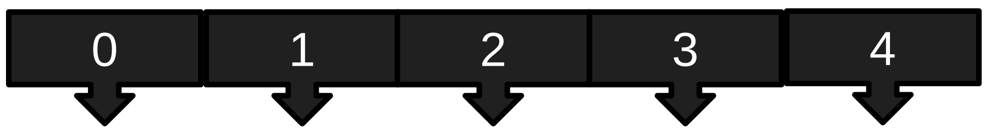
- **Rule 2**: To move a cash flow forward in time, you must compound it
  - $FV = PV(1 + i)n$

- **Rule 3**: To move a cash flow backward in time, you must discount it
  - $PV = FV/(1 + i)^n$

Using Timelines to Visualize Cashflows

- A **timeline** identifies the timing and amount of a stream of payments – both cash received and cash spent - along with the interest rate earned.

- i= 10%
-



Years

Cash flow -$100 $30 $20 -$10 $50

The 4-year timeline illustrates the following:

- The interest rate is 10%.
- A cash outflow of $100 occurs at the beginning of the first year (at time 0), followed by cash inflows of $30 and $20 in years 1 and 2, a cash outflow of $10 in year 3 and cash inflow of $50 in year 4.

Compounding and Future Value

Time value of money calculations involve

Present value (what a cash flow would be worth to you today) ***Future value*** (what a cash flow will be worth in the future).

$$FV = PV(1 + i)^{n}$$

Compound Interest and Time

Example: Suppose that you deposited $500 in your savings account that earns 5% annual interest.

How much will you have in your account after two years?

After five years?

- $FV_2 = PV(1+i)^n = 500(1.05)^2$ = **$551.25**
- $FV_5 = PV(1+i)^n = 500(1.05)^5$ = **$638.14**

Figure 5.1 Future Value and Compound Interest Illustrated

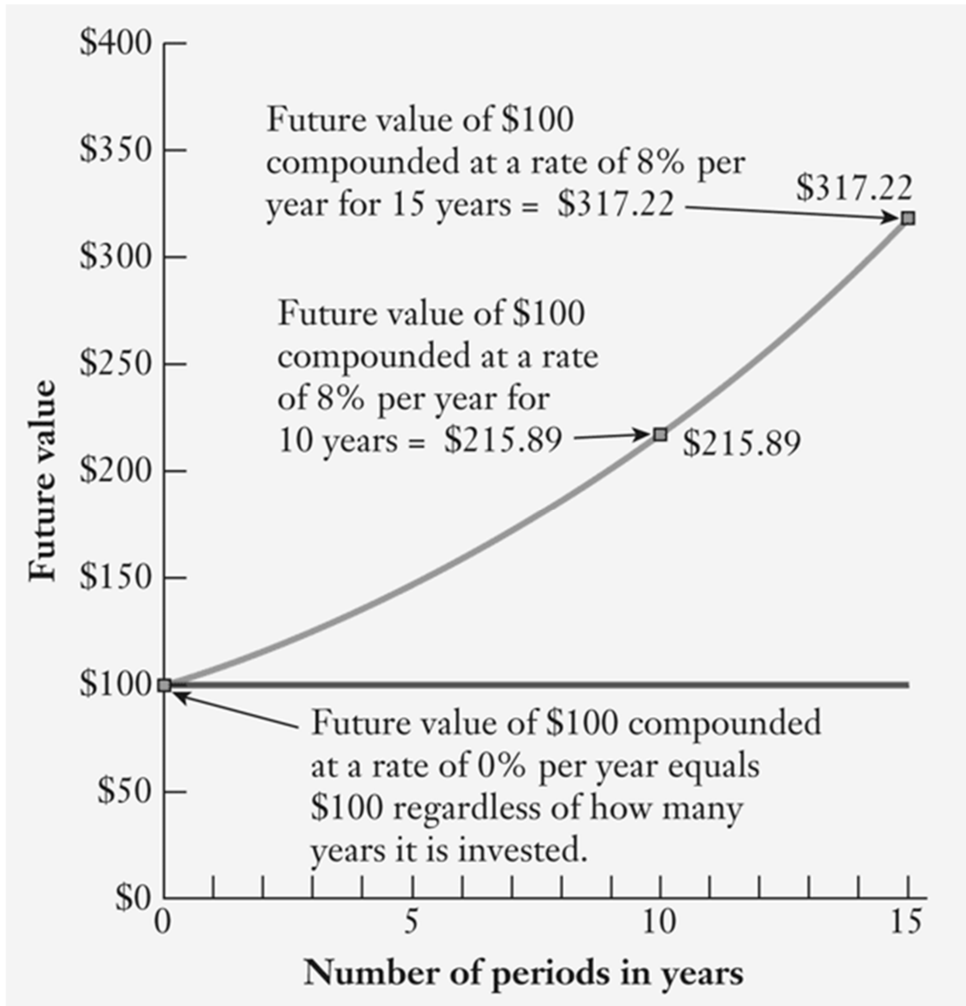
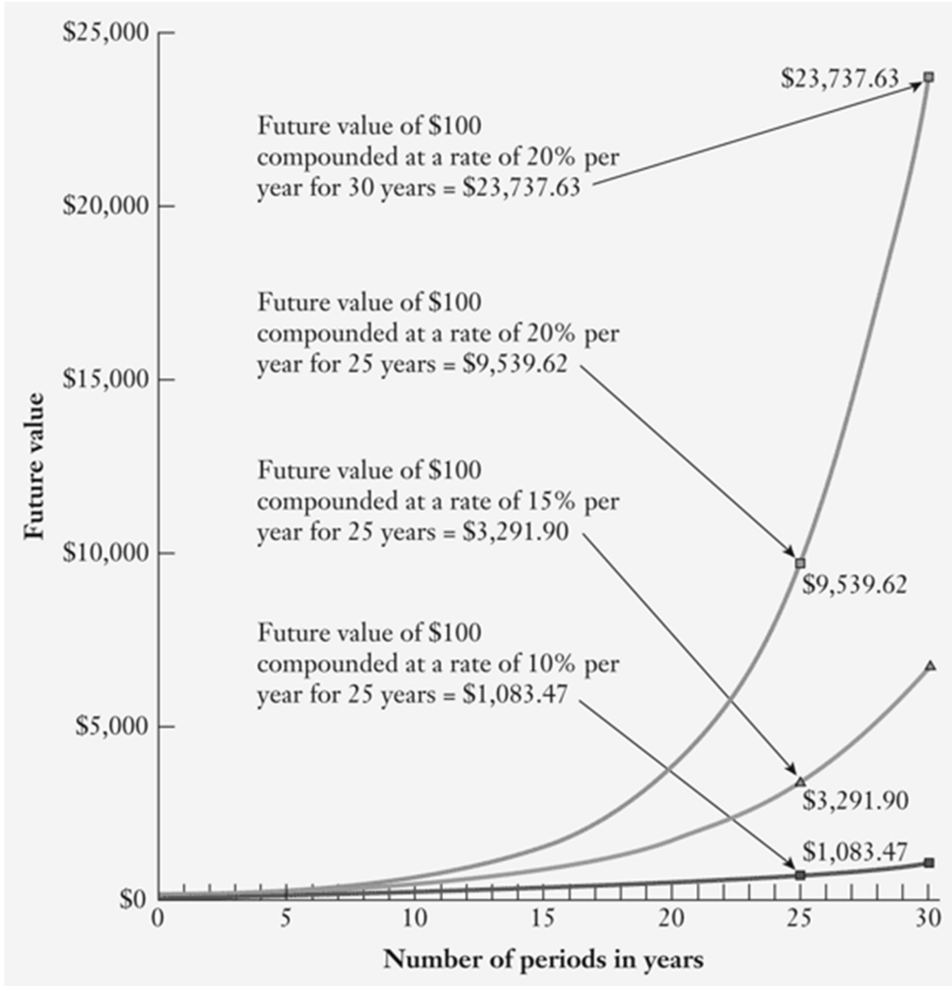


Figure 5.1 Future Value and Compound Interest Illustrated The Power of the Rate of Interest

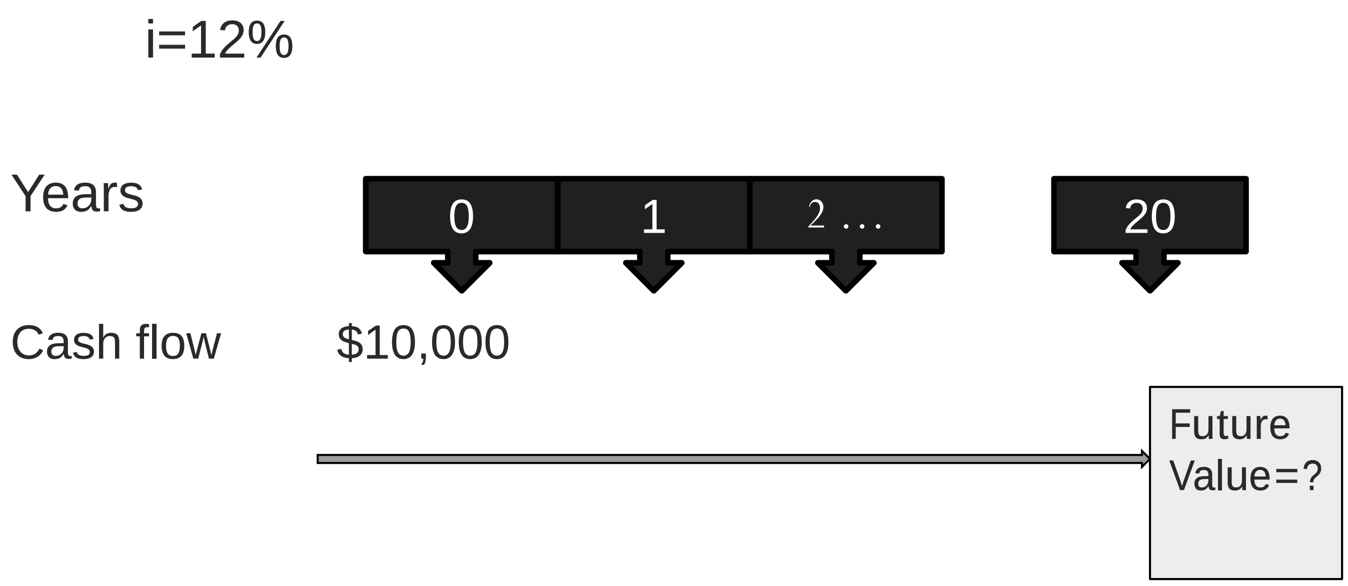


CHECKPOINT 5.2: *CHECK YOURSELF*

Calculating the FV of a Cash Flow

What is the FV of $10,000 compounded at 12% annually for 20 years?

Step 1: Picture the Problem



Step 2: Decide on a Solution Strategy

This is a simple future value problem. We can find the future value formula.

Step 3: Solve

$$FV20 = 10{,}000(1.05)^{20}$$

Step 3: Solve (cont.)

Solve Using a

Financial Calculator

N = 20

I/Y = 5%

PV = 10,000

PMT = 0

FV = **$26,532.9777**

Step 4: Analyze

If you invest $10,000 at 5%, it will grow to $26,532.98 in 20 years.

The Value of $100 Compounded at Various Non-Annual Periods and Various Rates

For 1 Year at *i* Percent	*i* = 2%	5%	10%	15%
Compounded annually	$102.00	$105.00	$110.00	**$115.00**
Compounded semiannually	102.01	105.06	110.25	115.56
Compounded quarterly	102.02	105.09	110.38	115.87
Compounded monthly	102.02	105.12	110.47	116.08
Compounded weekly (52)	102.02	105.12	110.51	116.16
Compounded daily (365)	102.02	105.13	110.52	**116.18**
For 10 Years at *i* Percent	***i* = 2%**	**5%**	**10%**	**15%**
Compounded annually	$121.90	$162.89	$259.37	**$404.56**
Compounded semiannually	122.02	163.86	265.33	424.79
Compounded quarterly	122.08	164.36	268.51	436.04
Compounded monthly	122.12	164.70	270.70	444.02
Compounded weekly (52)	122.14	164.83	271.57	447.20
Compounded daily (365)	122.14	164.87	271.79	**448.03**

$1.18

$43.47

CHECKPOINT 5.3: *CHECK YOURSELF*

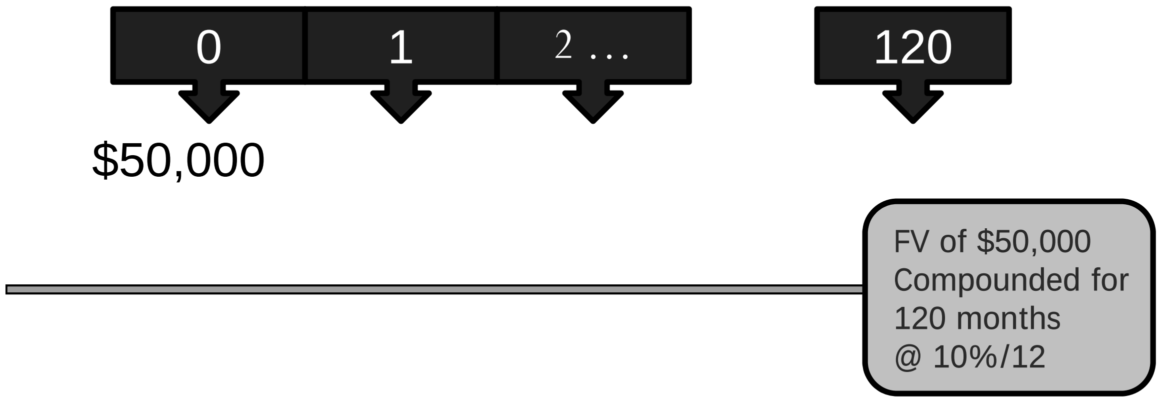
Calculating Future Values Using
Non-Annual Compounding Periods

If you deposit $50,000 in an account that pays an annual interest rate of 10% compounded monthly, what will your account balance be in 10 years?

Step 1: Picture the Problem

i=10%

Months



Cash flow

$50,000

Step 2: Decide on a Solution Strategy

This involves solving for future value of $50,000. Since the interest is compounded monthly, we will use equation 5-1b.

Step 3: Solve

Monthly interest rate is .10/12 = .0083, for 120 months

Using a Mathematical Formula

$$FV = PV\,(1+i)^T \quad = \$50{,}000\,(1+0.10/12)^{120}$$

$$= \$50{,}000\,(2.7070)$$

$$= \mathbf{\$135{,}352.07}$$

Step 4: Analyze

More frequent compounding leads to a higher FV as you are earning interest more often on interest you have previously earned.

Present Value: The Key Question

- **What is value today of cash flow to be received in the future?**

- The answer to this question requires computing the **present value (PV)** i.e. the value **today** of a future cash flow,
- The process of **discounting -** determining the present value of an expected future cash flow.

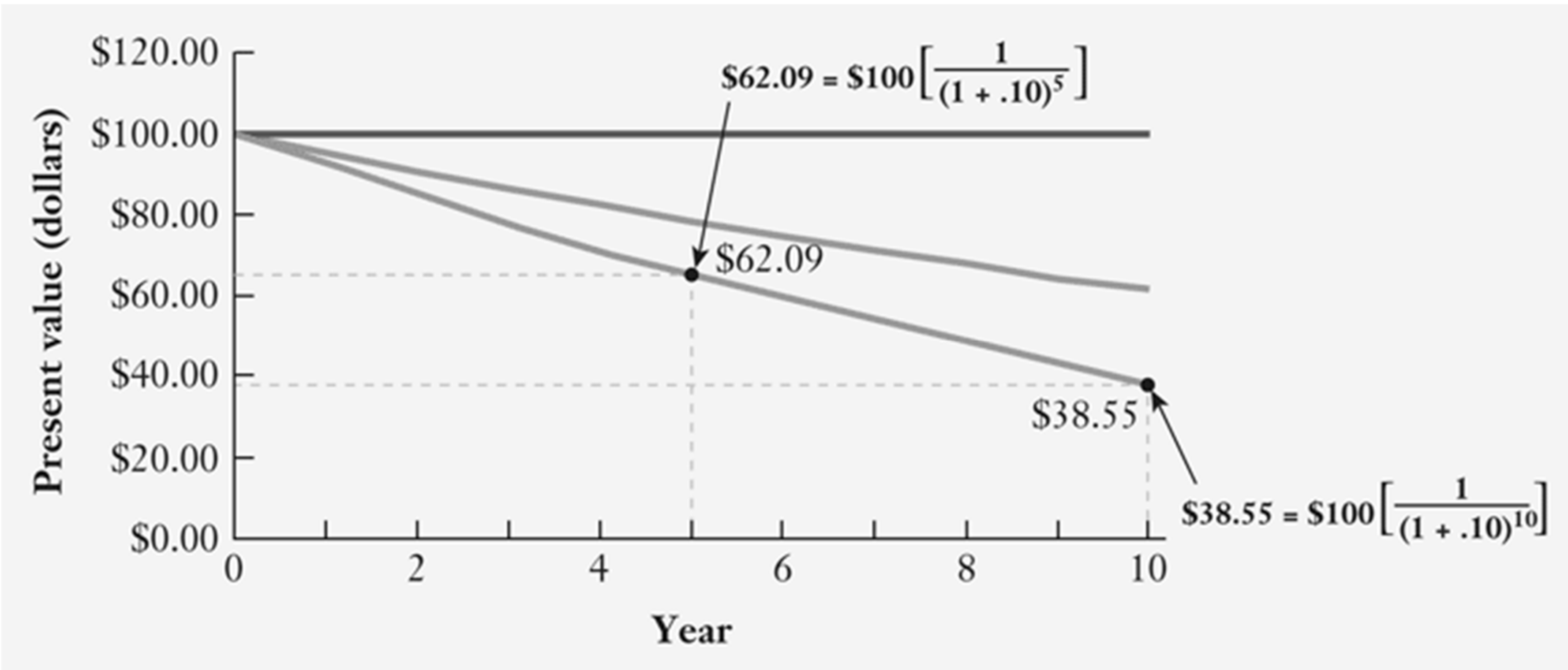
The Mechanics of Discounting Future Cash Flows

$$PV = FV_n[1/(1+i)^n]$$

- The term in the bracket is known as the Present Value Interest Factor (PVIF).

- $PV = FV_n \times PVIF$

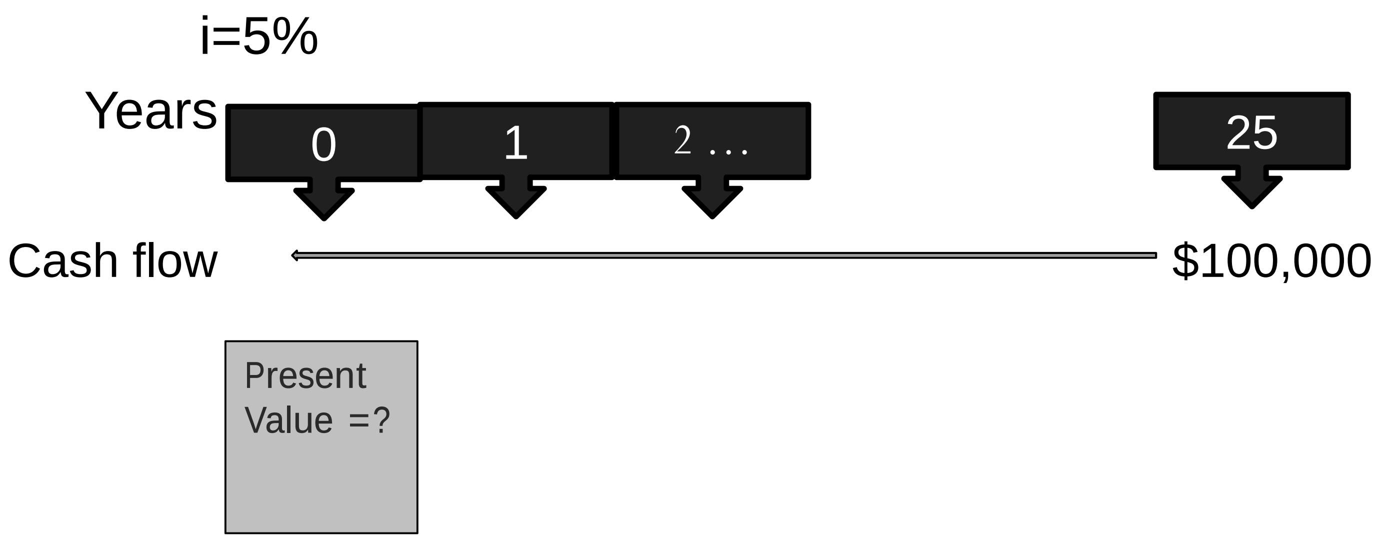
The Present Value of $100 Compounded at Different Rates and for Different Time Periods



Checkpoint 5.4
Step 1: Picture the Problem

Solving for the PV of a Future Cash Flow

What is the present value of $100,000 to be received at the end of 25 years given a 5% discount rate?



Step 2: Decide on a Solution Strategy

Here we are solving for the present value (PV) of $100,000 to be received at the end of 25 years using a 5% interest rate.

We can solve using equation 5-2.

Step 3: Solve

Using a Mathematical Formula

$$PV = \$100,000 \ [1/(1.05)^{25}]$$

$$= \$100,000 \ [0.2953]$$

$$= \mathbf{\$29,530}$$

Step 4: Analyze

Once you've found the present value, it can be compared to other present values.

Present value computation makes cash flows that occur in different time periods comparable so that we can make good decisions.

Two Additional Types of Discounting Problems

Solving for: (1) Number of Periods; and

(2) Rate of Interest

(1): How long will it take to accumulate a specific amount in the future?

- It is easier to solve for "n" using the financial calculator or Excel rather than mathematical formula.

The Rule of 72

- It determine the number of years it will take to double the value of your investment.

$$N = 72/\text{interest rate}$$

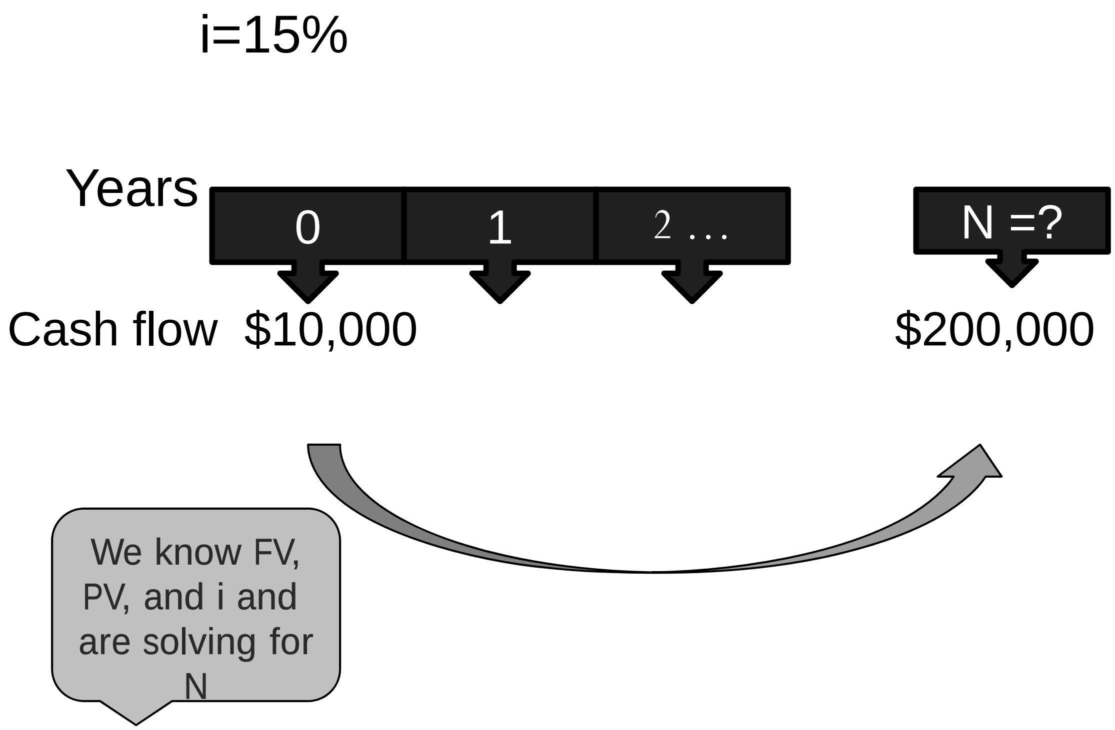
For example, if you are able to generate an annual return of 9%, it will take 8 years (=72/9) to double the value of investment.

CHECKPOINT 5.5:
CHECK YOURSELF

Solving for Number of Periods, n

How many years will it take for $10,000 to grow to $200,000 given a 15% compound growth rate?

Step 1: Picture the Problem



Step 2: Decide on a Solution Strategy

In this problem, we are solving for "n". We know the interest rate, the present value and the future value. We can calculate "n" using a financial calculator or an Excel spreadsheet.

Step 3: Solve

- Using a Financial Calculator

  I/Y = 15, PMT = 0, PV = -10,000, FV = 200,000

  N = **21.4 years**

- **Step 4 : Analyze**

It will take 21.4 years for $10,000 to grow to $200,000 at an annual interest rate of 15%.

Solving for the Rate of Interest

(2): What rate of interest will allow your investment to grow to a desired future value?

We can determine the rate of interest using mathematical equation, the financial calculator or the Excel spread sheet.

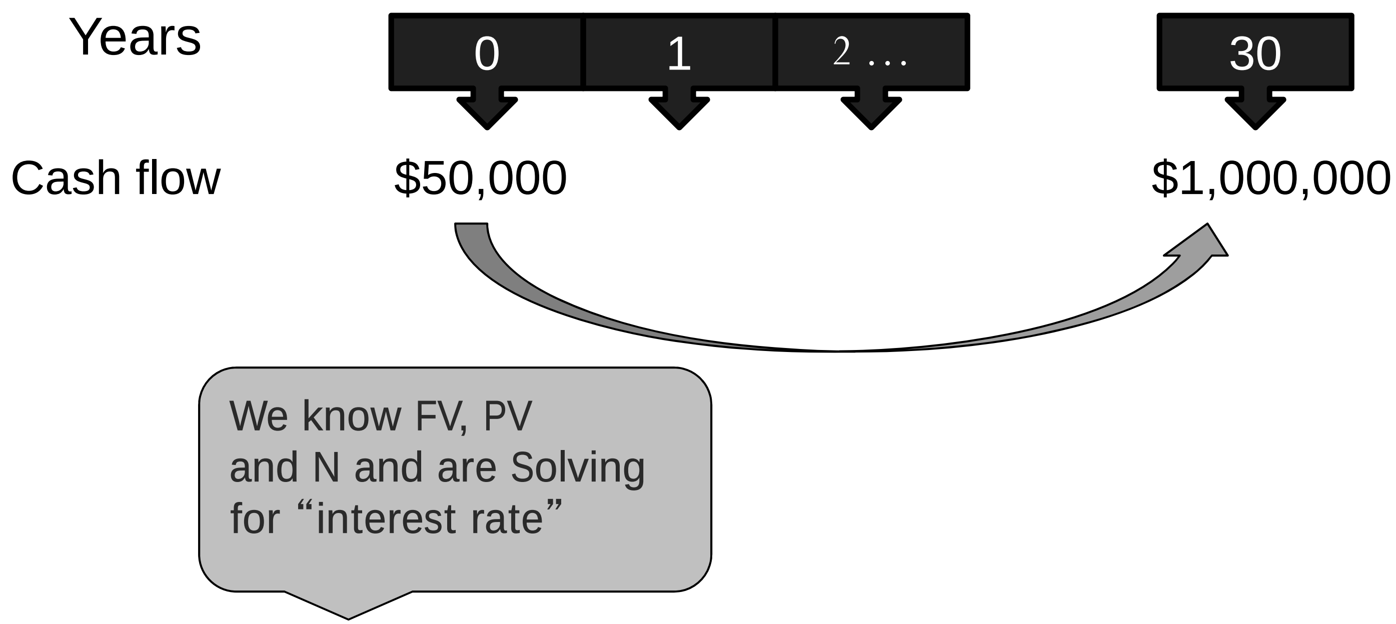
CHECKPOINT 5.6:
CHECK YOURSELF

Solving for the Interest Rate, i

At what rate will $50,000 have to grow to reach $1,000,000 in 30 years?

Step 1: Picture the Problem

i=?%



Step 2: Decide on a Solution Strategy

Here we are solving for the interest rate. The number of years, the present value, the future value are known. We can compute the interest rate using mathematical formula, a financial calculator or an Excel spreadsheet.

Step 3: Solve

<u>Using a Mathematical Formula</u>

$I = (FV/PV)^{1/n} - 1 = (1000000/50000)^{1/30} - 1 = (20)^{0.0333} - 1 =$
1.1050 – 1 = **.1050** or **10.50%**

<u>Using a calculator</u>

N = 30, PV = -50,000, FV = 1,000,000

Annual Percentage Rate (APR)

The **annual percentage rate** (APR) indicates the interest rate paid or earned in one year without compounding.

APR is also known as the **nominal or quoted (stated) interest rate**.

Rates are always stated as APR

Calculating the Interest Rate and Converting it to an EAR

We cannot compare two loans based on APR if they do not have the same compounding period.

To make them comparable, we calculate their equivalent rate using an annual compounding period. We do this by calculating the **effective annual rate (EAR)**

$$\text{Effective Annual Rate } (EAR) = \left(1 + \frac{\text{Quoted Annual Rate}}{\text{Compounding Periods per Year } (m)}\right)^{m} - 1$$

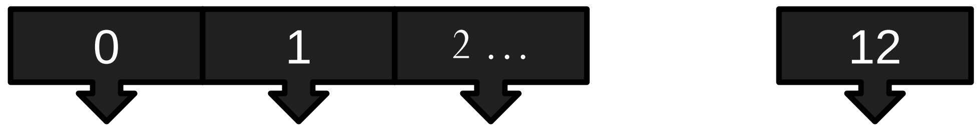
CHECKPOINT 5.7:
CHECK YOURSELF
Calculating an EAR

What is the EAR on a quoted or stated rate of 13 percent that is compounded monthly?

Step 1: Picture the Problem

i= an annual rate of 13% that is compounded monthly

Months



Compounding periods are expressed in months (i.e. m=12) and we are Solving for EAR

Step 2: Decide on a Solution Strategy

Here we need to solve for Effective Annual Rate (EAR). We can compute the EAR by using equation 5-4

Step 3: Solve

$EAR = (1 + i/m)^m – 1 = (1+ .13/12)^{12} – 1 = .1380$

Or 13.8%

Step 4: Analyze

There is a significant difference between APR and EAR (13.00% versus 13.80%).