STRAND: FINANCE

Unit 2 *Simple and Compound Interest*

TEXT

Contents

2 Simple and Compound Interest

2.1 Simple Interest

When money is deposited in a bank or building society account, it commonly attracts interest; in a similar way, a borrower must normally pay interest on money borrowed. The rate of interest is usually (but not always) quoted as a rate per cent per year. At the time of writing a typical rate is 1.5% per annum for money deposited and 1%-2% per annum for money borrowed. Up-to-date rates are available from finance organisations.

There are two basic ways of calculating the amount of interest paid on money deposited: **simple** interest and **compound** interest.

If simple interest is paid, interest is calculated only on the principal £P, the amount deposited (the original capital sum). The interest £I payable after one year years at rate r% per annum is given by the formula

$$I = \frac{r}{100} \times P$$

and the total amount owing can then be calculated by adding I to P.

Worked Example 1

Natasha invests £250 in a building society account. At the end of the year her account is credited with 2% interest.   How much interest had her £250 earned in the year?

Solution

$$\begin{aligned} \text{Interest} &= 2\% \text{ of } £250 \\ &= \frac{2}{100} \times £250 \\ &= £5 \end{aligned}$$

Worked Example 2

Alan invests £140 in an account that pays r% interest. After the first year he receives £4.20 interest.

What is the value of r, the rate of interest?

Solution

After one year, the amount of interest is given by

$$\frac{r}{100} \times £140 = £4.20$$

$$\begin{aligned} r &= \frac{100 \times 4.20}{140} \\ &= \frac{420}{140} \\ &= 3 \end{aligned}$$

So the interest rate is 3%.

Exercises

1. Calculate (a) the interest payable and (b) the total amount owing on the following deposits at simple interest.

   (i) £300 borrowed for 5 years at 8% p.a.

   (ii) £1000 invested for 4 years at 9.5% p.a.

   (iii) £50 borrowed for 2 years at 18% p.a.

   (iv) £2500 invested for 6 months at 8.75% p.a. ($T = 0.5$ years)

   (v) £45 000 borrowed for 2 weeks at 15.5% p.a.

The following questions relate to simple interest.

2. What is the actual rate of interest if £4000 deposited for 3 years attracts interest of £1440?

3. For how long would £500 have to be left in an account paying 4% interest p.a. to give a balance of £600 ?

4. A school's rich benefactor wants to deposit a certain sum in an account paying interest at 10.5% so that it will produce interest of £1200 per year, to pay for scholarships. How much should she deposit?

5. A boy borrows £1.00 from his sister and promises to pay back £1.10 a week later. What is this as an annual rate of interest?

6. For how long should a depositor leave a sum in a 6.25% p.a. savings account in order to earn the same amount in interest, assuming the interest is withdrawn each year?

2.2 Compound Interest

Simple interest is very rarely used in real life: almost all banks and other financial institutions use **compound interest**.

This is when interest is added (or compounded) to the principal sum so that interest is paid on the whole amount. Under this method, if the interest for the first year is left in the account, the interest for the second year is calculated on the whole amount so far accumulated.

Worked Example 1

I deposit £250 in a high-earning account paying 9% compound interest and leave it for three years. What will be the balance on the account at the end of that time?

Solution

Balance after 0 years		£250.00
Interest: 9% of £250.00	=	£ 22.50
Balance after 1 year:		£272.50
Interest: 9% of 272.50	=	£ 24.52
Balance after 2 years		£297.02

Interest: 9% of £297.02	=	£ 26.73
Balance after 3 years	=	£323.75

(Note that, for simplicity, all results here are rounded to the nearest penny; computer calculations are often made to several decimal places.)

Worked Example 2

Jodie invests £1200 in a bank account which pays interest at the rate of 4% per annum. Calculate the value of her investment after 4 years.

Solution

At an interest rate of 4% per annum, the value of her investment after one year is

$$\begin{aligned} &\pounds 1200 + \frac{4}{100} \times \pounds 1200 \\ &= 1.04 \times \pounds 1200 \\ &= \pounds 1248 \end{aligned}$$

After two years, the investment is worth

$$1.04 \times \pounds 1248 = \pounds 1297.92$$

and after three years,

$$1.04 \times \pounds 1297.92 = \pounds 1349.84$$

At the end of four years, the value of Jodie's investment will be

$$1.04 \times \pounds 1349.84 = \pounds 1403.83$$

Exercises

By working from year to year as in the worked example above, calculate the amount accumulated after three years at compound interest in the following cases.

1. £500 deposited at 10% p.a.
2. £1000 borrowed at 15% p.a.
3. £150 deposited at 6% p.a.
4. £1200 borrowed at 8.5% p.a.
5. £25 000 borrowed at 13.75% p.a.

2.3 Compound Interest Formula

It should already be clear that for long periods, the year-on-year method of calculating compound interest is somewhat cumbersome, but fortunately there is a formula.

Suppose the compound interest rate is 9%. The amount at the start of each year is treated as 100%, and adding 9% to 100% gives 109%. So adding 9% to any amount of money is equivalent to multiplying that amount by 1.09. Check that

$$£250.00 \times 1.09 = £272.50$$

$$£272.50 \times 1.09 = £297.02, \text{ and}$$

$$£297.02 \times 1.09 = £323.75$$

More generally, adding $r\%$ to a sum of money corresponds to multiplying by $\left(1 + \frac{r}{100}\right)$.

If the money is left untouched for T years, then the original amount $£P$ will be multiplied by $\left(1 + \frac{r}{100}\right)$, so that $£A$ is the total amount at the end of that time,

$$\boxed{A = P\left(1 + \frac{r}{100}\right)^{T}}$$

(This formula actually works for fractional values of T as well as for whole numbers. The amount of interest, if it is needed, is calculated by subtracting the principal, $£P$, from the total amount.)

Worked Example 1

You borrow £500 for four years and agree to pay $6\frac{1}{2}\%$ compound interest for this period. What amount will you have to pay back?

Solution

Using the formula,

$$\begin{aligned} A &= 500 \times 1.065^4 \\ &= 500 \times 1.28646... \\ &= 643.233... \end{aligned}$$

So you will have to pay back £643.23, to the nearest penny.

The same formula can be used to calculate the principal sum, the interest rate, or the length of time, as the following examples show.

Worked Example 2

How much must Sam deposit in a 6% savings account if he wants it to amount to £120 after two years?

Solution

Using the formula

$$120 = P \times 1.06^2$$

giving

$$P = 120 \div 1.06^2$$
$$= 106.799$$

He must deposit £106.80.

Worked Example 3

The value of a computer depreciates at a rate of 20% per annum. A new computer costs £1200. What will be its value after

(a) 2 years (b) 6 years (c) 10 years?

Solution

(a) $\text{Value} = £1200\left(1 - \frac{20}{100}\right)^2$

$= £1200 \times \left(\frac{4}{5}\right)^2$

$= £768$

(b) $\text{Value} = £1200 \times \left(\frac{4}{5}\right)^6$

$= £314.57$

(c) $\text{Value} = £1200 \times \left(\frac{4}{5}\right)^{10}$

$= £128.85$

Worked Example 4

What rate of interest will allow £350 to grow to £500 in five years?

Solution

From the formula,

$$500 = 350 \times \left(1 + \frac{r}{100}\right)^5$$

$$\Rightarrow \quad \left(1 + \frac{r}{100}\right)^5 = 500 \div 350$$

$$\Rightarrow \quad = 1.42857...$$

$$\Rightarrow \quad 1 + \frac{r}{100} = \sqrt[5]{1.42857...} \quad \left(\text{or } 1.42857^{\frac{1}{5}}\right)$$

$$= 1.0739$$

$$\Rightarrow \quad r = 7.39...$$

The interest rate is approximately 7.4%

Worked Example 5

For how long must a sum be deposited in an account paying 14% compound interest in order to double in value?

Solution

$$2P = P \times 1.14^T$$

$$\Rightarrow \quad 1.14^T = 2$$

$$\Rightarrow \quad T \log 1.14 = \log 2$$

$$\Rightarrow \quad T = \log 2 \div \log 1.14$$

$$= (0.3010\ldots) \div (0.0569\ldots)$$

$$= 5.29$$

The deposit must be left for 5.3 years but as interest is paid yearly, it would have to be left for 6 years.

(Note that after 5 years, the multiplier would be $(1.14)^5 \approx 1.925$ but after 6 years $(1.14)^6 \approx 2.195$.)

Exercises

1. Use the formula to calculate the total amount accumulated at compound interest in the following cases.
   (a) £2000 deposited for 5 years at 7% p.a.
   (b) £600 borrowed for 8 years at 12% p.a.
   (c) £500 deposited for 20 years at 8.25% p.a.
   (d) £10 000 borrowed for 6 months at 14% p.a.
   (e) £100 deposited for 18 months at 9.5% p.a.

2. Calculate the principal sum which, if deposited at 9.5% compound interest, will grow to £400 after three years.

3. Calculate the annual rate of compound interest that will allow a principal sum, to double in value after five years.

4. How long would it take for £1000 to grow to £1500 if deposited at 8% p.a. compound interest?

5. I borrow £5000 and agree to pay back £6000 after 18 months. What is the annual rate of compound interest?

6. For how long must you leave an initial deposit of £100 in a 12% savings account to see it grow to £1000 ?

7. A car costs £9000 and depreciates at a rate of 20% per annum. Find the value of the car after 3 years.

8. John invests £500 in a building society with interest of 8.4% per annum. Karen invests £200 at the same rate.

   (a) How many years does it take for the value of Karen's investment to become greater than £300?

   (b) How many years does it take for the value of John's investment to become greater than

   (i) £700 (ii) £900?

9. If the rate of inflation were to remain constant at 3%, find the price that a jar of jam, currently priced at £1.58, would be in 4 years' time.

10. The population of a third world country is 42 million and growing at 2.5% per annum.

    (a) What size will the population be in 3 years' time?

    (b) In how many years' time will the population exceed 50 million?

11. The value of a car depreciates at 15% per annum. Bill keeps a car for 4 years and then sells it.

    If the car originally cost £6000, find

    (a) its value after 4 years,

    (b) the selling price as a percentage of the original value.

2.4 Savings: Annual Equivalent Rate (AER)

The examples in the previous section are all based on interest being paid yearly but, in reality, interest can be paid biannually, monthly or even daily!

Provided the interest is compounded, this will mean a small increase in the balance of the account. This is shown in the next example.

Worked Example 1

The nominal interest rate on an account is 6% per annum. After one year, what is the total value of an initial deposit of £1000 if interest is paid

(a) annually, at the end of the year (b) twice a year, at six-monthly intervals

(c) every month?

Solution

(a) Value = £1000 × 1.06

= £1060.00

(b) If interest is paid every six months, the rate becomes 3% (6% ÷ 2) so that, after the first 6 months, the value is

£1000 × 1.03 = £1030.00

and after the second six months,

£1030 × 1.03 = £1060.90

(c) If paid monthly, the rate is $\frac{6\%}{12} = 0.5\%$ so the monthly multiplier is 1.005 and the value after 12 months is given by

$$£1000 \times (1.005)^{12} = £1061.68$$

You can see from the Worked Example above that there is a small gain from having interest paid more than once per year. Of course, the gain is more significant if the original investment is a large sum of money, and also if the interest is paid daily!

To compare savings accounts where interest is paid at regular intervals we use the term **Annual Equivalent Rate** (AER). Overall, this means that interest can be compounded more than once in a year depending on the number of times that interest payments are made. The AER is calculated as

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

where r is the AER, n the number of times in a year that interest is paid and i the 'nominal' yearly interest rate.

Worked Example 2

What is the AER for an account with nominal interest rate 6% and interest payments made each month?

Solution

Here $i = 0.06$, $n = 12$, so

$$\begin{aligned} r &= \left(1 + \frac{0.06}{12}\right)^{12} - 1 \\ &= (1 + 0.005)^{12} - 1 \\ &= (1.005)^{12} - 1 \\ &\approx 0.06168 \end{aligned}$$

That is, $r = 6.17\%$ (as shown by the calculation in part (c) of Worked Example 1).

Worked Example 3

Kate invests £5000 in a savings account that pays interest monthly at a nominal rate of 4.2%.

(a) What is the balance of the account at the end of 3 years?

(b) What is the AER for Karen's investment?

Solution

(a) The monthly rate is $\frac{4.2\%}{12} = 0.35\%$, so that after 3 years, there have been 36 payments of interest, and the balance is

$$£5000 \times (1 + 0.0035)^{36} \approx £5000 \times 1.1340$$
$$= £5670.16$$

(b) The AER formula gives the rate

$$r = \left(1 + \frac{0.042}{12}\right)^{12} - 1$$
$$= (1 + 0.0035)^{12} - 1$$
$$= 0.04282$$

So the AER $\approx 4.28\%$

Note that, if P is the initial balance of the account and A the balance after n years, then

$$A = P(1 + R)^n$$

where R is the APR.

Worked Example 4

If an account with an initial balance of £2500 grows to £3500 after 4 years, what is the AER?

Solution

If $R = \text{AER}$, then

$$3500 = 2500(1 + R)^4$$

So

$$(1 + R)^4 = \frac{3500}{2500} = 1.4$$

and

$$1 + R = (1.4)^{\frac{1}{4}}$$

So $\quad 1 + R \approx 1.087757...$, and

$$R \approx 0.087757...$$

Hence AER is 8.78%

Check: $\quad £2500 \times (1 + 0.0878)^4 = £2500 \times (1.0878)^4$
$$= £3500.55$$

Exercises

1. The nominal rate of interest for a savings account is 9% per annum. If £1000 is invested in the account what will be the balance after one year if

   (a) interest is paid once a year, at the end of the year

   (b) interest is paid bi-monthly (once every two months)?

2. What is the AER for an account with nominal interest rate 8% and interest payments made every 3 months?

3. Sara invests £4000 in a savings account that pays interest at a nominal rate of 5.4%, paid monthly.

   (a) What is the AER for this investment?

   (b) What is the balance of the account at the end of 5 years?

4. An investment of £2750 grows to £4250 in 5 years. Find the AER for this investment (as a percentage).

5. Liam invests £3000 in an account that pays interest quarterly (that is, every 3 months) at a rate of 0.6% per *month*.

   (a) What is the AER for this account?

   (b) What will be the balance of this account after one year?

   (c) What will be the balance of this account after 5 years?