Measurement Math: Problem-Solving with Increments on a Ruler

2 HOURS

Students build a conceptual understanding of fractions, focusing on halves, quarters, eighths, and sixteenths. Students then consider that understanding in the context of measurement, working with rulers.

PREP

- Teachers should cut copies of the *Photograph of ruler (for Stand and Talk)* handout in half. Teachers should make copies and cut out enough paper rulers for each student to get one. Teachers should do all of the activities, including the measurements, for themselves beforehand.

MATERIALS

- Blank sheets of paper
- *Photograph of ruler (for Stand and Talk)* handout
- *Paper Rulers* handout
- Inch rulers, tape measures (optional)
- *Measurement Chart* handout
- *Finding the Center* handout
- *Making a Round Table* handout

EXPLAIN

Introduction

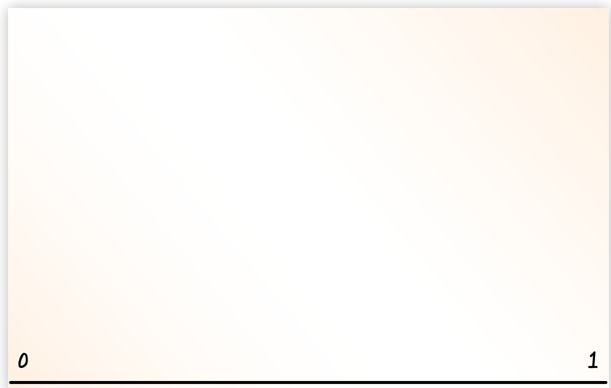
1. Measurement is at the heart of construction. And it is also something that all of us do in our everyday lives. Turn and talk with a partner about an experience with measuring—talk about things they have measured and how they did the measurement.

2. After a few minutes, ask students to report out on what they discussed. Keep a list on the board of the different situations in which measurements were taken. Ask students to discuss which of the situations required precise measurements and which ones required only rough measurements (or estimates). Explain that today's lesson focuses on situations in which precise measurements are required.

3. Ask students to consider the sector of Construction, and try to add professional situations to the list where precision in measurement would be important.

Launch: Folding Paper

1. Hand out the blank sheets of paper, giving one to each student. Each student should hold the paper in landscape view and draw a horizontal line completely across the bottom edge of the paper. Label the far left edge **0** and the far right edge **1**. This width represents one length of paper.



2. Fold the paper in half from left to right. Make a crease in the middle. Open the paper. **Ask**: How far is it from 0 to the crease?
   - *It is half the length to 1, half way across the paper.*

3. **Ask**: "How do we write one-half?"
   - *One over two.*
     $^1/_2$

4. Focus on the "$^1/_2$." Ask students what the "1" means and what the "2" means. The fraction tells us that we have something that is broken up into equal pieces. The bottom number tells us how many pieces it is broken into and the top number tells us how many of those pieces we are talking about. So "$^1/_2$" means we have something broken up into 2 pieces, in this case a sheet of paper. Since we are talking about length, starting at 0, when we reach that crease, we have moved over one of the two pieces.

5. Ask students what $^2/_2$ means and where it belongs on our line.
   - *In line with the 1*

6. Close the paper so that it is folded in half again. Then fold it in half a second time from left to right and make a crease. What do you think we will see when we open up the paper? How many sections? How many creases?

7 Open up the paper and see whether student predictions were true. You should have two new creases, one between 0 and 1/2 and one between 1/2 and 1.

- **How many equal sections do we have now?**
  Four
- **What can we call each of those four sections?**
  1/4 *or one-quarter or one-fourth*
- **How can we describe the distance from the 0 to the first crease?**
  1/4
- **How can we describe the distance from the 0 to the second crease?**
  2/4, *two-fourths, two-quarters.*

8 Have students add in 3/4 and 4/4. If they need support, have them look at the new crease on the right side. **How far is this crease from 0? How should we label it?** We should write 3/4 because it is three-fourths of the way across the paper. 3 out of 4 steps of equal size. Label the crease "3/4."

9 **When we say this crease is "three-fourths," what do we mean—three-fourths of what?**

› 3/4 *of the paper, starting at 0*

10 **Before we fold our paper again, what do you notice?**

› 2/4 *and* 1/2 *are on the same crease*

› 2/2, 4/4, *and 1 are all on the same crease.*

› *Our page is divided into quarters and halves.*

11 Fold the paper in half and then in half again. That brings it back to where it was before. Now fold it in half one more time. **What do you think we will see when we open up the paper? How many sections? How many creases?**

12 Open up the paper and you should have eight sections and seven creases. Have students work with a partner to label all of the creases. Walk around and listen to the conversations. If students need support, you can ask:

- **How many equal pieces did we have before?**
  Four.
- **What did we do with that four?**
  We made it the bottom number in our fraction.
- **How many equal pieces do we have now?**
  Eight

Make sure students add 8/8 with the 2/2, 4/4, and 1.

13. Repeat the procedure one final time. Fold the paper in half, then in half again, then in half again, and now add a new fold in half again. Then have students predict the number of sections and the number of creases.

14. Have students open the paper and label the creases.

15. **What does it mean that some of these fractions are on the same crease?** Students may say things like, "It means they are the same distance," or "those fractions are equal." Ask students to explain in their own words how $1/2$, $2/4$, $4/8$, and $8/16$ are all equal.

Stand and Talk: Looking at an Inch

1. The activity in this part of the lesson begins with an instructional routine called "Stand and Talk." Ask your students to stand with nothing in their hands—no pencils, no phones, no papers, etc. Once everyone is standing, tell them you are going to give them something to look at with a partner. You want them to look closely at the details and notice at least 5 things. You want to hear them asking each other things they wonder about.

2. Have your students move around and find a partner. It can be anyone in the room, just not anyone sitting next to them. They'll only be talking for 4 minutes. If there is an odd number of students, you can have one group of three.

3. Once students are standing with a partner, give them one photograph of a ruler to share between them. Remind them to look closely at the details, hold the sheet between them and to point at what they notice as they discuss it.

4. After 4 minutes, have students return to their seats and give out the remaining photographs so that now every student has their own. Ask them to call out what they noticed and record their observations. You may hear things like:

   - *There are 19 lines*
   - *Some of the lines are longer and some are shorter (There are 5 different sized lines)*
   - *This is a ruler*
   - *The ruler is divided up like our paper*
   - *This is an inch*
   - *This is an inch divided up into 16 sections*
   - *That line is* $1/2$
   - *This is a little more than an inch*
   - *This is an inch and* $1/8$

> *This is an inch and 2/16*
> *This is an inch and 3/16*

For each observation that required some reasoning, ask students to explain their thinking. In order to have students explain inferences based on what they are seeing, you might use the question, "**What do you see that makes you say that?**"

5. Eventually, you want all students to understand the following:
   - Their folded paper with labeled creases can be used as a key to help interpret this ruler.
   - The two longest lines denote the 0 and the 1, and that 1 represents 1 inch.
   - The lengths of the lines correspond to the how many equal pieces the inch is broken into—whole inch, the half, the quarters, the eighths and the sixteenths.
   - Some of the tick marks correspond to more than one fraction (i.e. the equivalent fractions)
   - How to identify each tick mark.
   - The tick marks past the 1 inch mark represent measurements of 1 and 1/16 inches, 1 and 2/16 inches. (You might hear an argument that the total width of this ruler segment is 1 3/16 inches since you can see one more tick mark on the edge.)

6. For more information on using Stand and Talk instructional routine in your class: **http://www.collectedny.org/frameworkposts/stand-and-talk/**

Paper Rulers

1. If you have access to rulers and/or tape measures, you can use them in place of, or in addition to, the paper rulers included in this lesson.

2. Give each student a paper ruler/actual ruler/tape measure. If using the paper rulers, make sure students understand that the ruler is 8 inches long.

3. Ask students to draw some lines: Try one measuring 5/16, one measuring 3 5/16, and one that measures 4 9/16. Address any issues that come up and give students additional lengths if they need or want more practice.

4. At this point, depending on time and your students you can do either or both of the handouts, the *Measurement Chart* and *Finding the Center/ Making a Round Table*. The goal is to get students to practice reading the rulers and being as precise as they can.

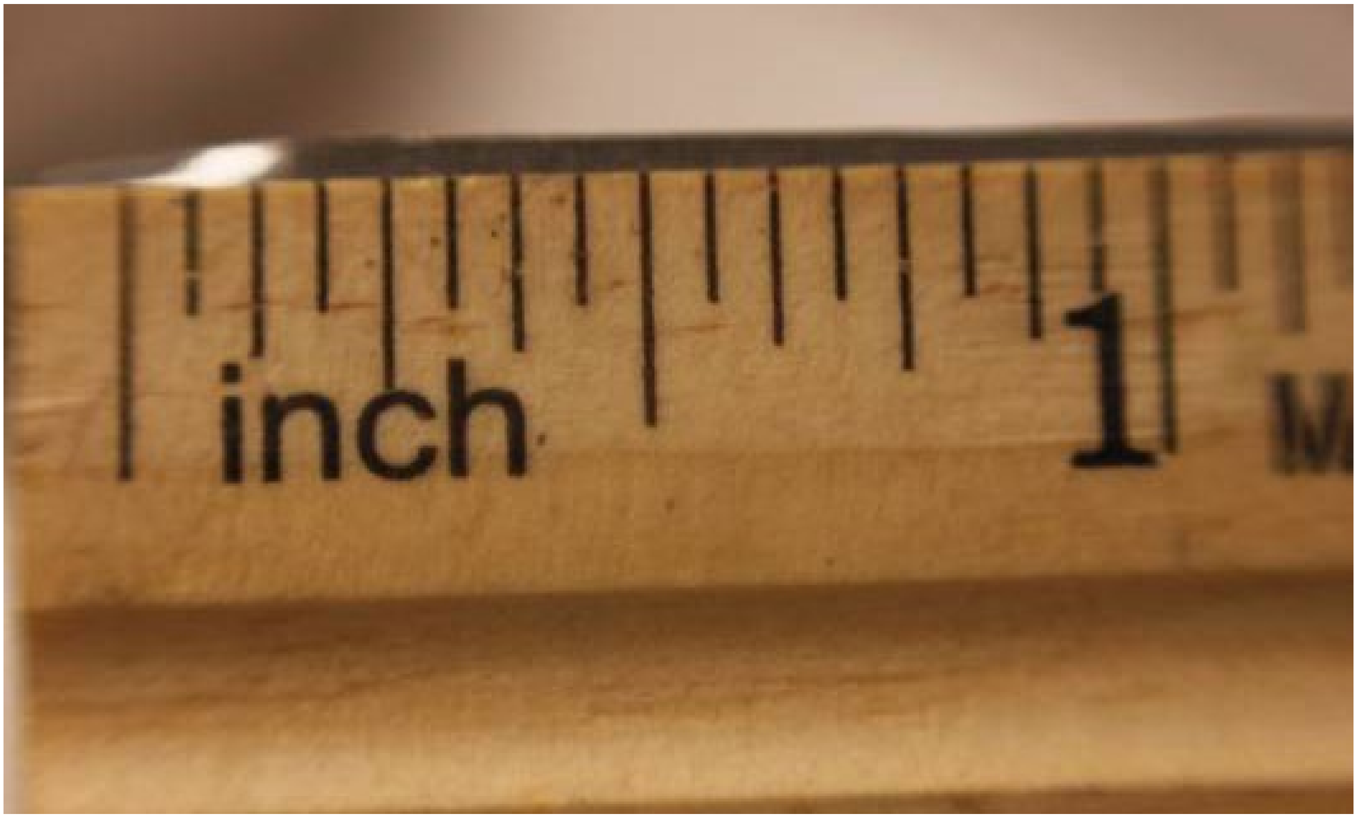
The *Measurement Chart* asks students to make estimates and then calculate precise measurements. As an extension with the *Measurement Chart*, you can ask students to figure out how far off each actual measurement is from their estimate.

Making a Round Table looks at precision and tricks of the trade in a professional context. Read through the *Finding the Center* handout with students to address any questions before giving out the *Making a Table* handout.

5 If you are using the paper rulers, some students may start by labeling all the increments. You may have other students who only write the increments as needed. And still yet other students who have a different strategy. Let them figure out what works best for them. If they are struggling, you might help them break down the task by suggesting they focus on labeling the ruler first.

6 For additional practice reading the fractions of a ruler, students can go to **https://www.rulergame.net/**.

Stand and Talk: Ruler



Measurement Chart*

Working with a partner, complete the following chart. For each object, estimate how long you think it will be. Write your estimate in the table. Then measure with your paper ruler. Add that information to your chart and calculate how far off you were. Once you have done this with all of the objects, choose at least four more objects around you and add that data to the chart.

	Estimate	Actual Measurement
Length of hand		
Length of arm		
Length of foot		
Length between your eyes		
Length around your wrist		
Length of your wingspan		
Length around your head		

Bonus question:
Approximately how many of your hands would it take to measure your height?

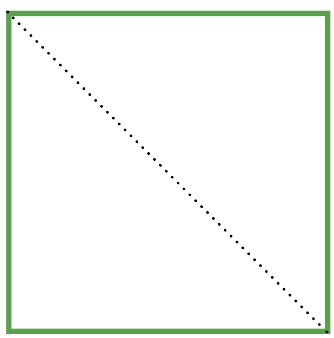
*Adapted from *About Teaching Mathematics* by Marilyn Burns

Finding the Center

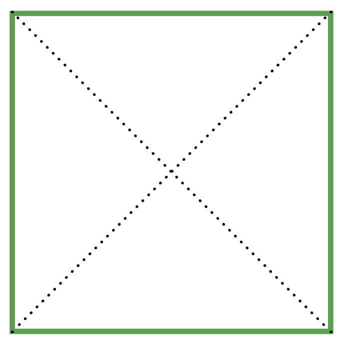
In carpentry, finding the centers of things is pretty important. For example, if you want to make a table with a support in the middle, you need to be able find the exact center. To find the center of a square or a rectangle, you can draw two diagonal lines from the opposite corners. The point where they cross is the center of your material.

Here's how you can find the center:

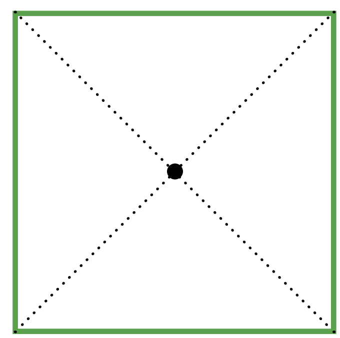
1. Draw a diagonal line from one corner to the other.



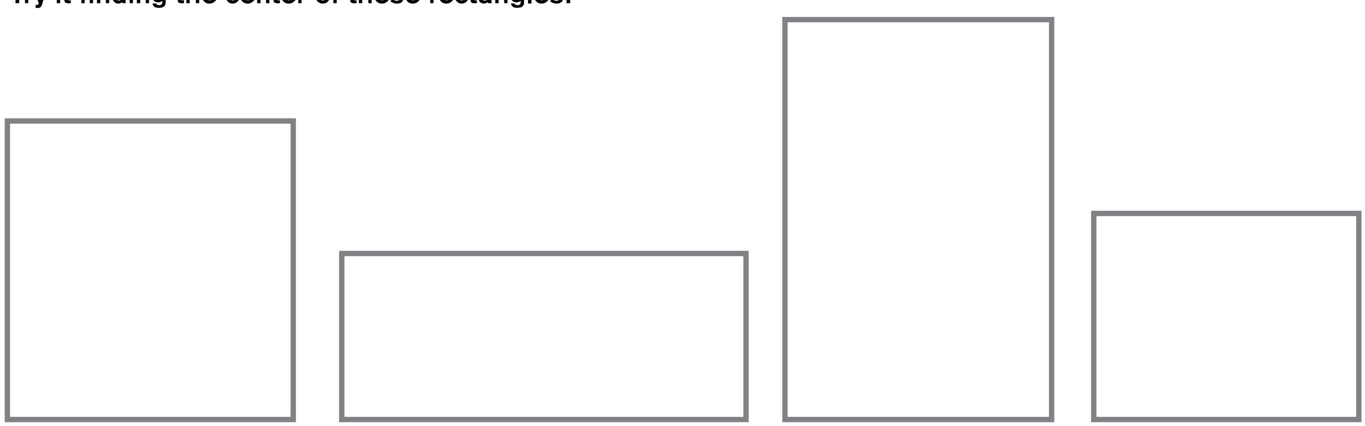
2. Draw a diagonal line to connect the other two corners.



3. Mark the spot where the two lines intersect. That's the center of the square!



Try it finding the center of these rectangles:

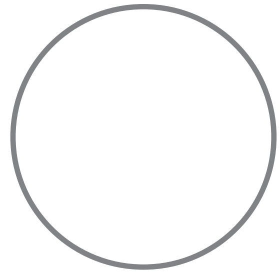


But what if you wanted to find the center of a circular piece of wood?

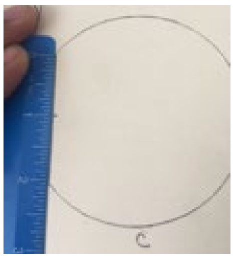
Circles don't have corners that you can connect.

Finding the Center of a Circle

There are many tricks of the trade when it comes to finding the center of a circle. Here are two methods that require only a ruler and a square edge. A square edge is something that forms a right angle. The corner of a piece of paper or the corner of a ruler will work.

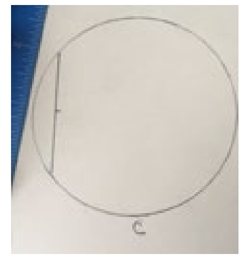


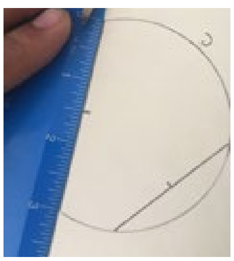
THE CHORD METHOD



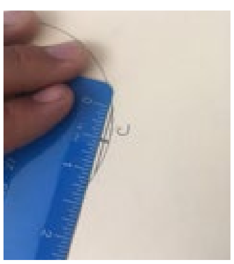
Using a ruler, draw a line from one point of the circle to another point on the circle. Any line connecting two points on a circle is called a **chord**.

Once you draw a chord, make a tick mark halfway.



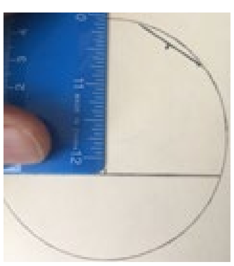


Next draw another chord, again marking the halfway point.



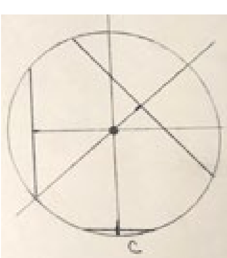
Draw a third chord and again mark the halfway point.

You will end up with something like the picture on the right.



Line up a straight edge with the halfway point on one of the chords and draw a perpendicular line. **Perpendicular lines** are lines that form 90° angles.

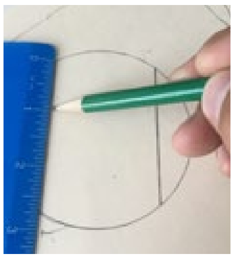
Repeat this step for all three chords.



The place where all three lines intersect is the center of the circle.

THE PARALLEL LINES METHOD

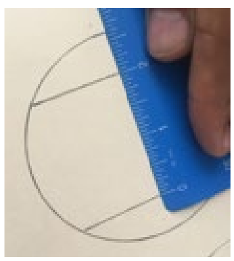
Draw a chord.

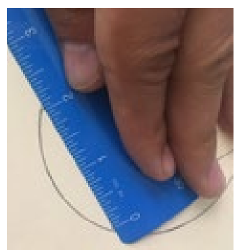


Draw a second chord. Two things need to be true about the second chord:

- It must be the same length as the first chord.
- The second chord must be parallel to the first chord.

Parallel lines are lines that never meet. They are always the same distance apart.



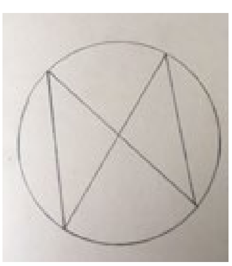


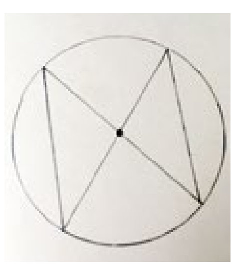
It's a good idea to measure the distance between your two chords and make sure they are the same in a few different places.

If the distance is the same, the lines are parallel.

Once you are confident your lines are parallel, you are ready for the next step.

Connect the end of one of the chords with the end of the other chord, crossing over the middle of the circle.





Connect the other two ends of the parallel chords.

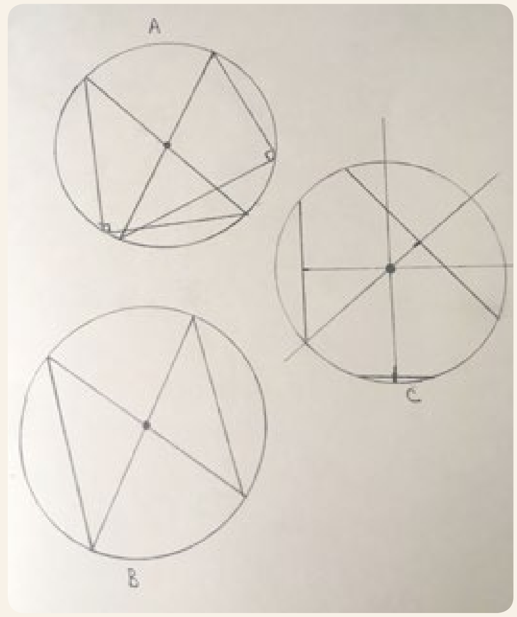
The point where the two lines intersect is the center of the circle!

How Precise Were Your Measurements?

The definition of a **circle** is the set of all points that are an equal distance from a point.

You can check to see if the point you found is actually in the center of your circle. Measure the distance from your center point to a few different points on the circle. If those distances are equal that you have a perfect center point. If the distances are different, your point isn't in the exact center of the circle.

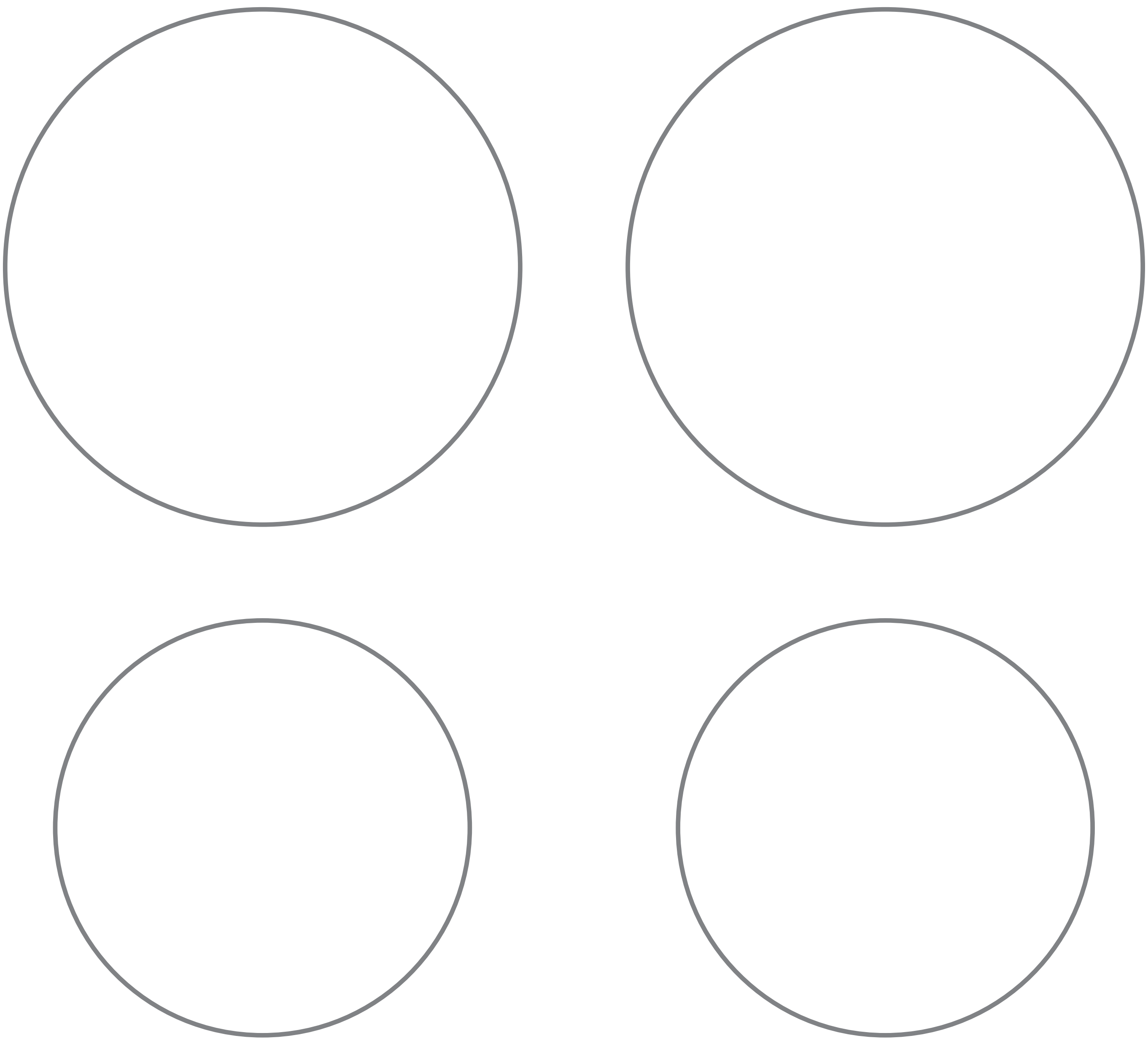
There is a special name for any line that connects the center of a circle to a point on the circle. It's called the **radius**.



Making a Round Table

Imagine you are making a round table with a pedestal support in the middle. You would need to find the center so that the tabletop is supported evenly and won't tip over.

As practice for finding the center of a round tabletop, use the CHORD METHOD or the PARALLEL LINES METHOD to find the center of the circles below.



Paper Rulers (note: print at 100%)