IV4 – Tea-Leaf Paradox

Anders Hamburgen

With help from:
Abdullah Alkaldi
Sander Leondaridis
Tristan Martinez

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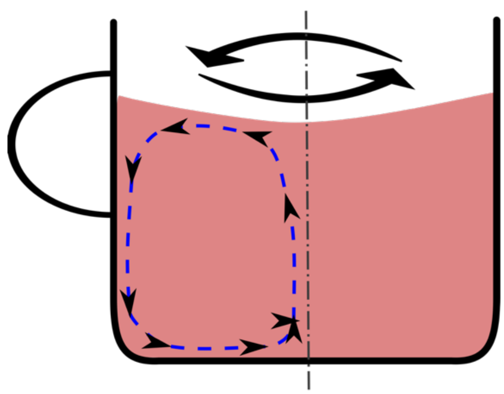
INTRODUCTION

In this report I describe the tea leaf paradox and attempt to explain it. If you stir a cup of loose-leaf tea, you will notice the leaves migrate towards the center of the cup instead of to the outside. This is unexpected, because you might think the leaves would move to the edges of the cup without an obvious centripetal force to keep them in the middle.

[1]

Figure 1: Tea leaves move to the middle of the cup instead of the outside.

James Thomson was the first person to correctly explain why the tea leaves stay in the middle of the cup in 1857 [2]. He identified friction between the water and the cup as supplying the necessary centripetal force on the tea leaves to keep them from moving outward to the edges. The friction at the inner walls and bottom of the cup creates a secondary flow, which has radial and axial velocity components that are not tangential to the direction of the stirring. The motion of this secondary flow is illustrated below.



[3]

Figure 2: The tea-leaf effect is caused by a secondary flow in the container generated by the stirring about the vertical axis.

The flow indicated by the arrow is responsible for keeping the tea leaves in the middle. In my photograph, the dense tea water at the bottom is pushed inward. Albert Einstein was the first solve this problem in a 1926 paper which also discussed the erosion of riverbanks [3].

My team decided to try out the tea leaf paradox ourselves and capture the effect in a way that clearly illustrates the fluid physics at play. We made our first experiment using a combination of water and vegetable oil to get the difference in density needed to produce the effect. Our first attempt was unsatisfactory, but ultimately, we achieved interesting results and discovered a better method for setting up an experiment to show the tea leaf effect.

EXPERIMENTAL SETUP

Our first trial involved a graduated measuring cup, dyed water, and vegetable oil, but I wasn't happy with my results and repeated the experiment at home. There, I used a small glass jar with vertical walls, about 8 cm in diameter 10 cm tall. I filled the bottom with about 20 mL of brewed tea which only filled the jar a small amount. Then I added baby oil to the jar to a level where there was enough liquid to create a strong secondary flow without splashing out. My photography equipment consisted of a Sony Alpha a6300 with 16-50 mm lens, a video light panel with adjustable brightness and color temperature, a Sony HVL-F32M external flash, and a tripod. I set the camera on the tripod about 2 feet away from the jar, and I the video light off to the side of the counter with the jar to avoid glare. I stirred the liquids in the jar with the spoon in one hand while taking photos with the other.

RESULTS AND ANALYSIS

The tea leaf paradox is entirely apparent in my final photo submission. The dense tea water underneath the clear baby oil moves to the middle of the jar, clearly showing the effect. This is evidence for the existence of a secondary flow with components that are orthogonal to the direction of stirring. Gordon Stubley of the Mechanical Engineering department of the University of Waterloo published a chapter titled "*The Fate of Sinking Tea Leaves"* in 2001, where he discusses the fluid physics behind the tea leaf paradox using closed-form analysis and computational methods to explain the effect of the secondary flows. In a simulation with a rectangular packet of water rotating about its edge with uniform angular velocity, he produced the following figure showing the radial and axial velocity vectors of the flow at steady state:

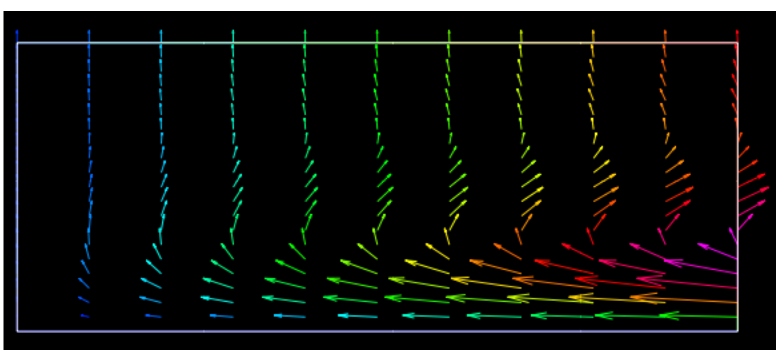

[4]

Figure 3: This CFD simulation of a packet of water rotating about its left edge with a uniform angular velocity shows a component of radial velocity towards the axis of rotation at the bottom and a component away from the axis of rotation in the middle layers of the flow.

This model would explain why the tea water in my experiment is pushed towards the center of the container. But what force is sending it there? The friction from the walls of the container creates a pressure gradient that overcomes the centrifugal tendency of the dense tea water [4]. At steady state, assuming no angular dependence for pressure or velocity, the Navier-Stokes equation for the radial direction in cylindrical form [5] can be simplified to

$$-\rho \frac{v_t^2}{r} = -\frac{\partial P}{\partial r}, \tag{1}$$

where ρ is the density of the fluid $[kg/m^3]$, v_t is the velocity in $[m/s^2]$ of the tangential direction of flow (the direction of the stirring), r is the radius $[m]$, and P is the pressure $[Pa]$. Note, this simplification only applies at a distance well above the bottom of the container, where friction at the base affects the flow [4]. Now, if we substitute $v_t = \omega r$, where ω is the angular velocity of the fluid $[rad/s]$, separate variables, and integrate each side, we find the following expression for pressure as a function of radius:

$$P(r, z \to \infty) = \rho \frac{\omega^2 r^2}{2}. \tag{2}$$

It is immediately apparent that pressure is proportional to the square of the radius. This pressure gradient is what provides the centripetal force to push the heavier water to the inside. The geometry of the problem is lucky in that we can use the Navier-Stokes equations to obtain such a tidy, closed-form solution.

VISUALIZATION TECHNIQUES

To do the tea leaf effect justice, we used two liquids of different densities and colors to show the effect, rather than suspending a solid particle like an actual tea leaf. Using two different liquids provided a clear, well-defined boundary layer to give structure to the flow effect in question. The brewed tea as the denser liquid was the obvious choice: It's darkly colored, and nothing would suit a "tea-leaf paradox" experiment better. Otherwise, it was fortunate the baby oil was clear, as it provided excellent contrast to the dark tea water. Lighting was also important. To illuminate the setup, I used a Pixel G1s video light panel position on the counter surface to the side of the glass jar. This removed some of the glare from having it aimed in the same direction of the camera lens. I set it to 3700 K and 100% brightness yielding the full power of the light. Yet again, this 12 W light has been wonderful this semester for close-range flow visualization experiments in my otherwise dim and drab kitchen. To add even more light, I used a Sony HVL-F32M external flash, which has a color temperature of 5500K [6]. I learned how quickly you can fully discharge two AA batteries using a powerful flash.

PHOTOGRAPHIC TECHNIQUES

I strove for a fine time resolution with this picture, which meant I needed a fast shutter speed and lot of light. I used a Sony a6300 Mirrorless digital camera with a 16-50 mm lens. The camera was placed approximately 24 inches away from the glass jar in the same horizontal plane and was zoomed to 50 mm. I used an aperture of f/5.6 to allow a lot of light onto the sensor and an ISO of 1200 to balance light sensitivity with graininess. The original image size was 6000 x 4000 px, later cropped to 3840 x 3840 px. The unedited and edited photos are shown below in Figure 4 for comparison.

Figure 4: The unedited image (left) versus the final, edited image (right).

As far as photo editing, I started by cropping the image to a square form to channel focus towards the fluid in the jar. I also did an angular crop to make the sides of the jar parallel to the frame. I adjusted the RGB curve a slightly to increase contrast in the midtones, and I changed the white balance to make the photo appear brighter and truer to the naked-eye appearance.

OUTCOMES

I would not say this is my favorite work of the semester, but I think I did manage to make lemonade from lemons. After all, I did get a good shot of the tea-leaf paradox and am satisfied with the outcome. I was limited by the materials at hand, and I have since been gifted a larger glass beaker I suspect would do a better job of displaying the effect. Also, others have done this experiment using mineral spirits as the less dense fluid instead of baby oil, and I would be curious to try the experiment with that as the less-dense fluid instead. I am grateful to Gordon Stubley's publication which enabled me to analyze the physics of this problem clearly with tools I already understand.

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