## Speed, Distance \& Time Difficulty: Hard

## Model Answers 1

| Level | IGCSE |
| :--- | :--- |
| Subject | Maths (0580/0980) |
| Exam Board | CIE |
| Topic | Algebra \& Graphs |
| Sub-Topic | Speed, Distance \& Time |
| Paper | Paper 2 |
| Difficulty | Hard |
| Booklet | Model Answers 1 |
|  |  |
| Time allowed: | $\mathbf{5 4}$ minutes |
| Score: | $\mathbf{1 4 2}$ |
| Percentage: | $\mathbf{1 0 0}$ |

## Grade Boundaries:

CIE IGCSE Maths (0580)

| A* | A | B | C | D | E |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $>88 \%$ | $76 \%$ | $63 \%$ | $51 \%$ | $40 \%$ | $30 \%$ |

CIE IGCSE Maths (0980)

| 9 | 8 | 7 | 6 | 5 | 4 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $>94 \%$ | $85 \%$ | $77 \%$ | $67 \%$ | $57 \%$ | $47 \%$ | $35 \%$ |

Petra begins a journey in her car.
She accelerates from rest at a constant rate of $0.4 \mathrm{~m} / \mathrm{s}^{2}$ for 30 seconds. She then travels at a constant speed for 40 seconds.

On the grid, draw the speed-time graph for the first 70 seconds of Petra's journey.



Amar cycles at a speed of $18 \mathrm{~km} / \mathrm{h}$.
It takes him 55 minutes to cycle between two villages.

Calculate the distance between the two villages.

Speed distance time relation is

$$
\begin{aligned}
\text { speed } & =\frac{\text { distance }}{\text { time }} \\
\rightarrow \text { distance } & =\text { time } \times \text { speed }
\end{aligned}
$$

Hence, converting time into hours

$$
\text { distance }=\frac{55}{60} \times 18
$$

$=16.5$

The diagram shows information about the first 100 seconds of a car journey.

(a) Calculate the acceleration during the first 20 seconds of the journey.

The acceleration is the gradient

$$
\begin{aligned}
a= & m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{16-0}{20-0} \\
& =\frac{\mathbf{4}}{\mathbf{5}}=\mathbf{0 . 8}
\end{aligned}
$$

(b) Work out the total distance travelled by the car in the 100 seconds.

The distance travelled is the area under the graph


We add the areas of the three shapes (drawn above)

$$
A=\frac{1}{2} \times 20 \times 16+\frac{1}{2} \times 4 \times 30+80 \times 12
$$

$=1180$

A train travels for $m$ minutes at a speed of $x$ metres per second.
(a) Find the distance travelled, in kilometres, in terms of $m$ and $x$. Give your answer in its simplest form.


$$
\begin{aligned}
& \text { Distance }=\text { Speed } \times \text { Time } \\
& \text { Distance }=x \times 60 \mathrm{~m} \text { metres }
\end{aligned}
$$

Multiply minutes by 60 to get seconds

$$
\text { Distance }=\frac{x \times 60 m}{1000} \text { kilometres }
$$

And cancel: $\quad$ Distance $=\frac{\mathbf{3 x m}}{50} \mathrm{~km}$
(b) When $m=5$, the train travels 10.5 km .

Find the value of $x$.

Substitute the values into the equation in (a):

$$
\begin{aligned}
& 10.5=\frac{3 \times x \times 5}{50} \\
& 10.5 \times 50=15 x \\
& x=\frac{10.5 \times 50}{15} \\
& x=\mathbf{3 5} \text { metres per second }
\end{aligned}
$$

A car of length 4.3 m is travelling at $105 \mathrm{~km} / \mathrm{h}$.
It passes over a bridge of length 36 m .
Calculate the time, in seconds, it takes to pass over the bridge completely.

The car is on the bridge when its front is on the bridge and exits when its rear leaves.

The total distance to travel is therefore

$$
36+4.3=40.3 m
$$

Converting the speed into $\mathrm{m} / \mathrm{s}$, first we multiply by 1000 to get the units in $\mathrm{m} / \mathrm{hr}$

$$
\begin{gathered}
\text { Speed }=105 \mathrm{kmh}^{-1} \times 1000 \\
\text { Speed }=105000 \mathrm{mh}^{-1}
\end{gathered}
$$

Next we have to turn it into $\mathrm{m} / \mathrm{s}$

$$
\begin{gathered}
\text { Speed }=105000 m h^{-1} \div\left(60^{2}\right) \\
\text { Speed }=\frac{105000}{3600} \mathrm{~ms}^{-1} \\
\text { Speed }=\frac{175}{6} m s^{-1}
\end{gathered}
$$

Using the speed distance time relation

$$
\begin{aligned}
& \text { speed }=\frac{\text { distance }}{\text { time }} \\
& \frac{175}{6} m s^{-1}=\frac{40.3 m}{\text { time }}
\end{aligned}
$$

Rearrange for time

$$
\begin{gathered}
\text { time }=40.3 \times \frac{6}{175} \\
=\underline{\mathbf{1 . 3 8}} \mathrm{s}(2 \mathrm{dp})
\end{gathered}
$$

A car travels at $56 \mathrm{~km} / \mathrm{h}$.

Find the time it takes to travel 300 metres.
Give your answer in seconds correct to the nearest second.

First, we convert km/h to $\mathrm{m} / \mathrm{s}$.

$$
56 \mathrm{~km} / \mathrm{h}=56000 \mathrm{~m} / \mathrm{h}=\frac{56000 \mathrm{~m} / \mathrm{s}}{3600 \text { seconds per hour }}=15.55 \mathrm{~m} / \mathrm{s}
$$

Then we calculate the time taken by dividing the distance travelled by the speed.

$$
\text { Time } \text { taken }=\frac{\text { distance }}{\text { speed }}=\frac{300 \mathrm{~m}}{15.55 \mathrm{~m} / \mathrm{s}}
$$

Time taken $=19.3 \mathrm{~s}$

Round the answer to the nearest second.

$$
\text { Time } \text { taken }=\mathbf{1 9 s}
$$



The diagram shows the speed-time graph of a car.
The car travels at $45 \mathrm{~km} / \mathrm{h}$ for 20 seconds.
The car then decelerates for 10 seconds until it stops.
(a) Change $45 \mathrm{~km} / \mathrm{h}$ into $\mathrm{m} / \mathrm{s}$.

We convert km/h to m/s.

$$
\begin{aligned}
45 \frac{\mathrm{~km}}{\mathrm{~h}}=45000 \frac{\mathrm{~m}}{\mathrm{~h}} & =\frac{45000 \frac{\mathrm{~m}}{\mathrm{~s}}}{3600 \text { seconds per hour }} \\
& =\mathbf{1 2 . 5} \mathbf{~ m} / \mathrm{s}
\end{aligned}
$$

(b) Find the deceleration of the car, giving your answer in $\mathrm{m} / \mathrm{s}^{2}$.

The deceleration is defined as the change of velocity over the change of time.
The speed went from $12.5 \mathrm{~m} / \mathrm{s}$ to $0 \mathrm{~m} / \mathrm{s}$, so the change of velocity was $12.5 \mathrm{~m} / \mathrm{s}$.
This happened between 20 second and 30 second since the measurement begun, so the change of time was 10 seconds.

$$
\begin{aligned}
\text { deceleration }= & \frac{\text { change of velocity }}{\text { changle of time }}=\frac{12.5 \mathrm{~m} / \mathrm{s}}{10 \mathrm{~s}} \\
& =\mathbf{1 . 2 5} \mathbf{m s}^{-2}
\end{aligned}
$$

(c) Find the distance travelled by the car during the 30 seconds, giving your answer in metres.

The total distance travelled by the car is the area under the graph.

To get the distance, we sum the rectangle and the triangle
(the area of a triangle is $1 / 2$ times the product of two perpendicular sides).
Use the speed in $\mathrm{m} / \mathrm{s}$ we have calculated in part a).

$$
\begin{gathered}
\text { distance }=\text { rectange }+ \text { triangle } \\
\text { distance }=20 \mathrm{~s} \times 12.5 \mathrm{~m} / \mathrm{s}+\frac{1}{2} \times(30 \mathrm{~s}-20 \mathrm{~s}) \times 12.5 \mathrm{~m} / \mathrm{s} \\
\text { distance } \\
=\mathbf{3 1 2 . 5} \mathbf{m}
\end{gathered}
$$



A tram leaves a station and accelerates for 2 minutes until it reaches a speed of 12 metres per second. It continues at this speed for 1 minute.
It then decelerates for 3 minutes until it stops at the next station.
The diagram shows the speed-time graph for this journey.
Calculate the distance, in metres, between the two stations.

Distance travelled by the train is the area under the graph. To get the distance, we sum the two triangles and the rectangle.
(The area of a triangle is $1 / 2$ times the product of two perpendicular sides)

Convert minutes into seconds by multiplying by 60 ( 60 seconds per minute).

$$
\begin{gathered}
\text { distance }=\text { first triangle }+ \text { rectangle }+ \text { second triangle } \\
\text { distance }=\frac{1}{2} \times 120 s \times 12 \mathrm{~m} / \mathrm{s}+(180-120) s \times 12 \mathrm{~m} / \mathrm{s}+\frac{1}{2} \times(360-180) \mathrm{s} \times 12 \mathrm{~m} / \mathrm{s} \\
\text { distance }=720 \mathrm{~m}+720 \mathrm{~m}+1080 \mathrm{~m} \\
\text { distance travelled }=\mathbf{2 5 2 0 m}
\end{gathered}
$$



A car starts from rest and accelerates for $u$ seconds until it reaches a speed of $10 \mathrm{~m} / \mathrm{s}$. The car then travels at $10 \mathrm{~m} / \mathrm{s}$ for $2 u$ seconds.
The diagram shows the speed-time graph for this journey.
The distance travelled by the car in the first $3 u$ seconds is 125 m .
(a) Find the value of $u$.

The total distance travelled by the car is the area under the graph.
To get the distance, we sum the triangle and the rectangle
(the area of a triangle is $1 / 2$ times the product of two perpendicular sides).

$$
\begin{gathered}
\text { distance }=\text { triangle }+ \text { rectangle } \\
\text { distance }=\frac{1}{2} \times u \times 10 \mathrm{~m} / \mathrm{s}+2 u \times 10 \mathrm{~m} / \mathrm{s} \\
\text { distance }=(5 u+20 u) \mathrm{m}
\end{gathered}
$$

We know that the distance travelled is 125 m .

$$
125 m=(25 u) m
$$

Divide both sides of the equation by 25 to get the value of $u$.

$$
\begin{gathered}
u=\frac{125}{25} \\
\boldsymbol{u}=\mathbf{5}
\end{gathered}
$$

(b) Find the acceleration in the first $u$ seconds.

The acceleration is defined as the change of velocity over the change of time

$$
\begin{gathered}
\text { Acceleration }=\frac{\text { change of velocity }}{\text { changle of time }}=\frac{10 \mathrm{~m} / \mathrm{s}}{5 \mathrm{~s}} \\
=\mathbf{2} \mathbf{m s}^{-2}
\end{gathered}
$$

A container ship travelled at $14 \mathrm{~km} / \mathrm{h}$ for 8 hours and then slowed down to $9 \mathrm{~km} / \mathrm{h}$ over a period of 30 minutes.

It travelled at this speed for another 4 hours and then slowed to a stop over 30 minutes.
The speed-time graph shows this voyage.

(a) Calculate the total distance travelled by the ship.

The area under the speed-time graph represents the distance travelled.

We separate the area under the graph in the 2 trapezia represented in the figure below to make the calculation easier.


The area of a trapezium is represented by the formula:

$$
A=\frac{(a+b) h}{2}
$$

In our case, for the big trapezium, the height would be $\mathrm{h}=$ $9 \mathrm{~km} / \mathrm{h}$, and the 2 bases would be $\mathrm{a}=12.5$ hours and $\mathrm{b}=$

13 hours
$A=\frac{(12.5 \mathrm{~h}+13 \mathrm{~h}) \times 9 \mathrm{~km} / \mathrm{h}}{2}$
$A=114.75 \mathrm{~km}$

For the small trapezium, $\mathrm{h}=14 \mathrm{~km} / \mathrm{h}-9 \mathrm{~km} / \mathrm{h}=5 \mathrm{~km} / \mathrm{h}$
$a=8$ hours, $b=8.5$ hours

```
\(A=\frac{(8.5 \mathrm{~h}+8 \mathrm{~h}) \times 5 \mathrm{~km} / \mathrm{h}}{2}\)
\(\mathrm{A}=41.25 \mathrm{~km}\)
Total area \(=\mathrm{A}(\) big trapezium \()+\mathrm{A}(\) small trapezium \()\)
Total area \(=41.25 \mathrm{~km}+114.75 \mathrm{~km}\)
Total area \(=156 \mathbf{k m}\)
```

(b) Calculate the average speed of the ship for the whole voyage.

To work out the average speed we need to divide the total distance travelled by the total time taken to travel that distance.

Average speed $=\frac{156 \mathrm{~km}}{13 \text { hours }}$

Average speed $=12 \mathrm{~km} / \mathrm{h}$


The graph shows the speed of a truck and a car over 60 seconds.
(a) Calculate the acceleration of the car over the first 45 seconds.

Acceleration $=\frac{\text { Speed }}{\text { TIme }}$. This is also the gradient of a speed time graph.

To calculate the acceleration over the first 45 seconds, we calculate the gradient for that part of the graph:

$$
\text { Acceleration } \approx \frac{\text { Change in } y}{\text { Change in } x}=\frac{36}{45}=0.8
$$

(b) Calculate the distance travelled by the car while it was travelling faster than the truck.

The distance travelled can be found from a speed time graph as the area under the curve, and the distance travelled by the car while it was travelling faster than the truck is shown by the shaded area on the diagram below:


This area is calculated in two parts: Firstly the triangular part which can be calculated as follows

$$
\begin{aligned}
& \text { Area of triangle }=\frac{1}{2} \times \text { base } \times \text { height } \\
& \text { Area of triangle }=\frac{1}{2} \times 40 \times 24=480
\end{aligned}
$$

Secondly the area of the rectangle is calculated as follows:

$$
\begin{aligned}
& \text { Area of rectangle }=\text { width } \times \text { height } \\
& \text { Area of rectangle }=40 \times 12=480
\end{aligned}
$$

The total area is then 960, which is the total distance travelled

Therefor the total distance travelled by the car while it is travelling faster than the truck is

960m

