How to Solve 2 Dimension Relative Velocity Problems

Any two dimension relative velocity problem is an application of vector addition. There are two scenarios you must learn to identify, when reading a question. Also, it is important to note that what happens in one direction is independent of what happens in the other and that only time of travel is common to both.

1. Read the problem carefully! At this point you need to determine if we have a case 1 or case 2 problem. In case 1, the object of interest is compared to a moving surface or medium, for example a boat in a river or a balloon in the wind. In case 2, the object of interest is compared to another moving object. The both objects must be on a common surface or in a common medium and this surface or medium has a constant velocity, usually it is at rest, for example a shark chasing a fish.

   Read the following practice exercise and determine what case of relative velocity is described. Then make a statement about the why you think this is so.

   i. Practice 1: A large panel van has a velocity of 30 m/s at 20° S of E in still air. A gust of wind of 20 m/s due East relative to the Earth strikes the van. What is the resultant velocity of the van relative to the Earth, during the gust?

ANSWER: Case 1. The van is the object of interest and is affected by a moving medium, the wind. The speed of the wind (air) is relative to the Earth. The speed of the van is relative to the air.

   ii. Practice 2: U-531 fires a torpedo at a freighter in the North Atlantic. The torpedo travels at 27 knots NW. The freighter travels at 7 knots 30° S of E. What is the velocity of the torpedo relative to the freighter?

ANSWER: Case 2. The object of interest is the torpedo and is compared to a freighter, another moving object. Both the torpedo's and freighter's velocity are relative to the ocean (water). We assume the water is still.

2. Now you need to determine what you are looking for. This means you must read the problem and determine the velocity you need to find and its reference frame.

   i. Practice 1: A large panel van has a velocity of 30 m/s at 20° S of E in still air. A gust of wind of 20 m/s due East relative to the Earth strikes the van. What is the resultant velocity of the van relative to the Earth, during the gust?

ANSWER: The velocity of the van to the earth. We write this using subscripts V_{vE}. The first letter of the subscript identifies the object, 'v' means van. The second letter of the subscript identifies the frame of reference, 'E' means Earth. Literally translated V_{vE} means "the velocity of the van relative to the Earth."

ii. Practice 2: U-531 fires a torpedo at a freighter in the North Atlantic. The torpedo travels at 27 knots NW. The freighter travels at 7 knots 30° S of E. What is the velocity of the torpedo relative to the freighter?

Answer: The velocity of the torpedo to the freighter. We write this using subscripts V_{tf}. The first letter of the subscript identifies the object, 't' means torpedo. The second letter of the subscript identifies the frame of reference, 'f' means freighter. Literally translated V_{tf} means "the velocity of the torpedo relative to the freighter."

3. Now you need determine the velocities of all important objects / surfaces. This means you must read the problem and identify all the listed velocities, again paying attention to the subscripts.

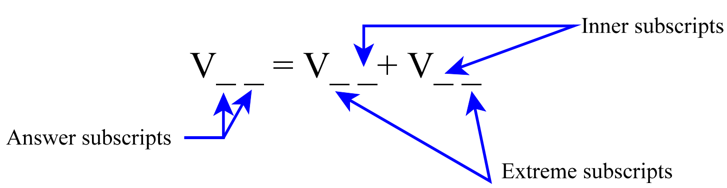
i. Practice 1: A large panel van has a velocity of 30 m/s at 20° S of E in still air. A gust of wind of 20 m/s due East relative to the Earth strikes the van. What is the resultant velocity of the van relative to the Earth, during the gust?

ANSWER: We are given the velocity of the van to the air. V_{va} = 30 m/s at 20° S of E. Also, we are given the velocity of the air to the Earth. V_{aE} = 20 m/s E. Note the subscript 'a' here means air.

ii. Practice 2: U-531 fires a torpedo at a freighter in the North Atlantic. The torpedo travels at 27 knots NW. The freighter travels at 7 knots 30° S of E. What is the velocity of the torpedo relative to the freighter?

Answer: We are given the velocity of the torpedo to the water. V_{tw} = 27 kts at NW. Also, we are given the velocity of the freighter to the water. V_{tw} = 7 kts 30° S of E. Note the subscript 'w' here means water.

4. Now you are ready to write the vector addition equation to solve your problem. Below is a template equation. The subscripts in the answer match the extreme subscripts in the added velocities. The inner subscripts, of the added velocities, ought to match each other.



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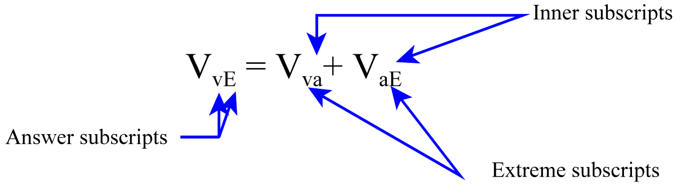
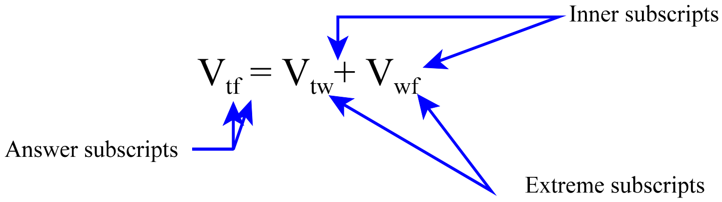


It is worth pointing out now that we know V_{va} = 30 m/s at 20° S of E, and V_{aE} = 20 m/s E.

ii. Practice 2: U-531 fires a torpedo at a freighter in the North Atlantic. The torpedo travels at 27 knots NW. The freighter travels at 7 knots 30° S of E. What is the velocity of the torpedo relative to the freighter?



It is worth pointing out now that we know V_{tw} = 27 kts NW, and V_{fw} = 7 kts at 30° S of E. However, we really need to know “the velocity of the water relative to the freighter”, V_{wf} and not “the velocity of freighter to the water,” V_{fw}!!!!

Before proceeding to the next step, lets determine “the velocity of the water relative to the freighter”, $V_{wf.}$ The following is a unique feature of case 2 relative velocity problems. That is $V_{wf} = - V_{fw}$. This means the size of both velocities are the same, but the directions of each are perfectly opposing. Therefore, V_{wf} = 7 kts at 30° N of W!

5. Last step before the math, draw a labelled tip to tail diagram based on the vector equation you wrote.

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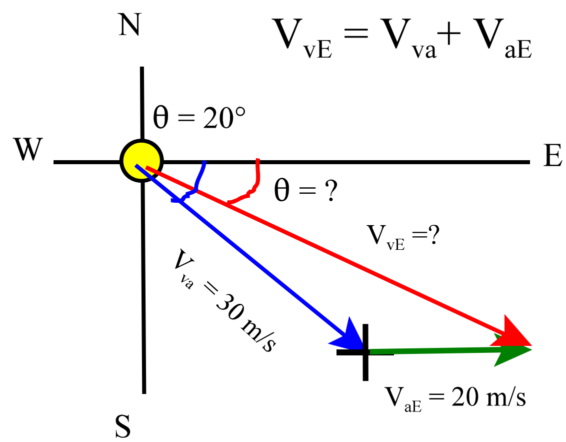

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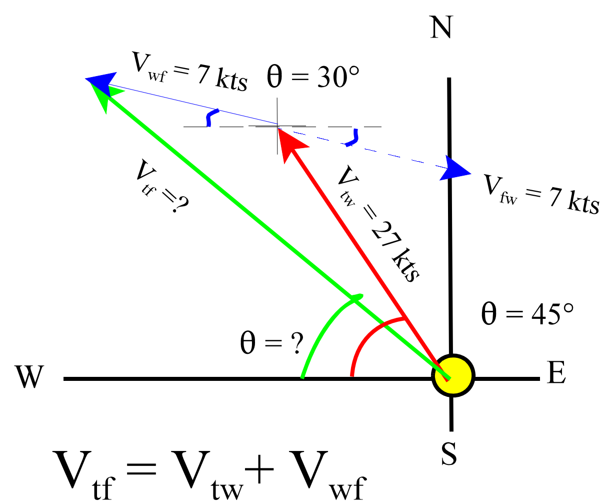
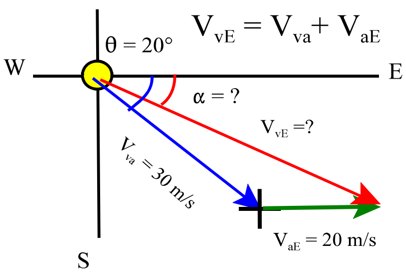


6. The math, with vector addition there are many options and these are classified by graphical and analytical means. In this tutorial we will concentrate on the most efficient analytical means. When adding two vectors the Law of Sines and Cosines is probably best. However, the component method is easier to visualize. I will show both solutions for each problem.

Component diagram



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(1) Component method.

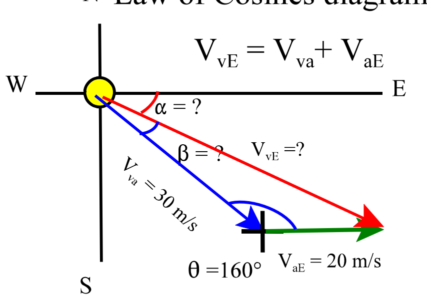
Vector (v) and angle (θ)	X-comp $v_x = v\cos\theta$	Y-comp $v_y = v\sin\theta$
V_{va} = 30 m/s, 20°	V_{vax} = 28.19	V_{vay} = -10.26
V_{vE} = 20 m/s, 0°	V_{vax} = 20	V_{vEy} = 0
Resultant	V_{vEx} = 48.19	V_{vEy} = -10.26

Note using your diagram, YOU determine is a component is positive or negative.

$$v_{vE} = \sqrt{{v_{vEx}}^2 + {v_{vEy}}^2} = \sqrt{48.19^2 + (-10.26)^2} = 49.3\, {}^{m}\!/\!{}_{s}$$

$$\theta = \tan^{-1}\left(\frac{v_{vEy}}{v_{vEx}}\right) = \tan^{-1}\left(\frac{-10.26}{48.19}\right) = 12.0°\text{ S of E}$$

Law of Cosines diagram



(2) Law of sines and cosines

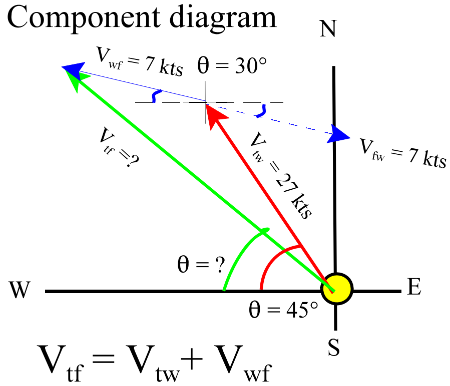
$$v_{vE} = \sqrt{{v_{va}}^2 + {v_{aE}}^2 - 2(v_{va})(v_{aE})\cos\theta}$$

$$v_{vE} = \sqrt{30^2 + 20^2 - 2(30)(20)\cos 160} = 49.2\, {}^{m}\!/\!{}_{s}$$

$$\beta = \sin^{-1}\left(\frac{v_{aE}\sin\theta}{v_{vE}}\right) = \sin^{-1}\left(\frac{20\sin 160}{49.2}\right) = 8.0°$$

$$\alpha = 20° - \beta = 12.0°$$

ii. Practice 2: U-531 fires a torpedo at a freighter in the North Atlantic. The torpedo travels at 27 knots NW. The freighter travels at 7 knots 30° S of E. What is the velocity of the torpedo relative to the freighter?



(1) Component method.

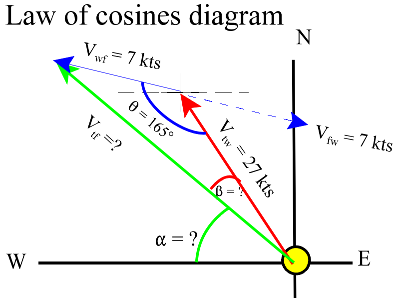
Vector (v) and angle (θ)	X-comp $v_x = v\cos\theta$	Y-comp $v_y = v\sin\theta$
V_{tw} = 27 kts, 45°	$V_{twx} = -19.1$	$V_{twy} = 19.1$
V_{wf} = 7 kts, 30°	$V_{wfx} = -6.1$	$V_{wfy} = 3.5$
Resultant	$V_{tfx} = -25.2$	$V_{tfy} = 22.6$

Note using your diagram, YOU determine is a component is positive or negative.

$$v_{tf} = \sqrt{v_{tfx}^2 + v_{tfy}^2} = \sqrt{(-25.2)^2 + (22.6)^2} = 33.8\ kts$$

$$\theta = \tan^{-1}\left(\frac{v_{tfy}}{v_{tfx}}\right) = \tan^{-1}\left(\frac{22.6}{-25.2}\right) = 41.8° \text{ N of W}$$

(2) Law of sines and cosines



$$v_{tf} = \sqrt{v_{tw}^2 + v_{wf}^2 - 2v_{tw}v_{wf}\cos\theta}$$

$$v_{tf} = \sqrt{27^2 + 7^2 - 2(27)(7)\cos 165°}$$

$$v_{tf} = 33.8 kts$$

$$\beta = \sin^{-1}\left(\frac{v_{wf}\sin\theta}{v_{tf}}\right)$$

$$\beta = \sin^{-1}\left(\frac{7\sin 165°}{33.8}\right) = 3.1°$$

$$\alpha = 45° - \beta = 41.9° \text{ N of W}$$