World Applied Sciences Journal 9 (Special Issue of Applied Math): 13-19, 2010
ISSN 1818-4952

Application of Laplace Decomposition Method to Solve Nonlinear Coupled Partial Differential Equations

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Abstract: In this article, we develop a method to obtain approximate solutions of nonlinear coupled partial differential equations with the help of Laplace Decomposition Method (LDM). The technique is based on the application of Laplace transform to nonlinear coupled partial differential equations. The nonlinear term can easily be handled with the help of Adomian polynomials. We illustrate this technique with the help of three examples and results of the present technique have closed agreement with approximate solutions obtained with the help of Adomian Decomposition Method (ADM).

Key words: Approximate solutions . Laplace decomposition method . nonlinear coupled partial differential equations . Adomian decomposition method

INTRODUCTION

The decomposition method has been shown to solve [1-12] efficiently, easily and accurately a large class of linear and nonlinear ordinary, partial, deterministic or stochastic differential equations. The method is very well suited to physical problems since it does not require unnecessary linearization, perturbation and other restrictive methods and assumptions which may change the problem being solved, sometimes seriously.

The Laplace Decomposition Method (LDM) is a numerical algorithm to solve nonlinear ordinary, partial differential equations. Khuri [13, 14] used this method method for the approximate solution of a class of nonlinear ordinary differential equations. Agadjanov [15] applied this method for the solution of Duffing equation. Elgazery [16] exploit this method to solve Falkner-Skan equation. This numerical technique basically illustrates how the Laplace transform may be used to approximate the solutions of the nonlinear differential equations by manipulating the decomposition method which was first introduced by Adomian [17].

The present paper aims at offering an alternative method of solution to the existing ones [18] concerning to the three nonlinear coupled partial differential equations. By using Laplace transform algorithm based on decomposition method for solving coupled nonlinear differential equations the exact solutions of initial value problems are obtained.

LAPLACE DECOMPOSITION METHOD

The aim of this section is to discuss the use of Laplace transform algorithm for the nonlinear partial differential equations. We consider the general form of inhomogeneous nonlinear partial differential equations with initial conditions is given below

$$Lu + Ru + Nu = h(x,t) \tag{2.1}$$

$$u(x,0) = f(x),\ u_t(x,0) = g(x) \tag{2.2}$$

where L is second order differential operator $L = \frac{\partial^2}{\partial t^2}$, R is the is remaining linear operator, Nu represents a general non-linear differential operator and h (x, t) is source term. The methodology consists of applying Laplace transform first on both sides of Eq. (2.1)

$$\mathcal{L}\left[Lu(x,t)\right] + \mathcal{L}\left[Ru(x,t)\right] + \mathcal{L}\left[Nu(x,t)\right] = \mathcal{L}\left[h(x,t)\right] \tag{2.3}$$

Using the differentiation property of Laplace transform we get

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$$s^2 L[u(x,t)] - sf(x) - g(x) + L[Ru(x,t)] + L[Nu(x,t)] = L[h(x,t)] \quad (2.4)$$

$$L[u(x,t)] = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2} L[h(x,t)] - \frac{1}{s^2} L[Ru(x,t)] - \frac{1}{s^2} L[Nu(x,t)] \quad (2.5)$$

The second step in Laplace decomposition method is that we represent solution as an infinite series given below

$$u = \sum_{m=0}^{\infty} u_m(x,t) \quad (2.6)$$

The nonlinear operator is decompose as

$$Nu(x,t) = \sum_{m=0}^{\infty} A_m \quad (2.7)$$

where A_m are Adomian polynomials [12] of u_0, u_1, u_2, …, u_n and it can be calculated by formula given below

$$A_m = \frac{1}{m!} \frac{d^m}{d\lambda^m} \left[N\left(\sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0}, m = 0,1,2,..., \quad (2.8)$$

Putting Eq. (2.6), Eq. (2.7) and Eq. (2.8) in Eq. (2.5) we will get

$$L\left[\sum_{m=0}^{\infty} u_m(x,t) \right] = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2} L[h(x,t)] - \frac{1}{s^2} L[Ru(x,t)] - \frac{1}{s^2} L\left[\sum_{m=0}^{\infty} A_m \right] \quad (2.9)$$

$$\sum_{m=0}^{\infty} L[u_m(x,t)] = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2} L[h(x,t)] - \frac{1}{s^2} L[Ru(x,t)] - \frac{1}{s^2} L\left[\sum_{m=0}^{\infty} A_m \right] \quad (2.10)$$

On comparing both sides of the Eq. (2.10) we have

$$L[u_0(x,t)] = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2} L[h(x,t)] = K(x,s) \quad (2.11)$$

$$L[u_1(x,t)] = -\frac{1}{s^2} L[Ru_0(x,t)] - \frac{1}{s^2} [A_0] \quad (2.12)$$

$$L[u_2(x,t)] = -\frac{1}{s^2} L[Ru_1(x,t)] - \frac{1}{s^2} L[A_1] \quad (2.13)$$

In general, the recursive relation is given by

$$L[u_{n+1}(x,t)] = -\frac{1}{s^2} L[Ru_n(x,t)] - \frac{1}{s^2} L[A_n], \ n \geq 0 \quad (2.14)$$

Applying inverse Laplace transform to Eq. (2.11)-(2.14), So our required recursive relation is given below

$$u_0(x,t) = K(x,t) \quad (2.15)$$

$$u_{n+1}(x,t) = -L^{-1}\left[\frac{1}{s^2} L[Ru_n(x,t)] + \frac{1}{s^2} L[A_n] \right], \ n \geq 0 \quad (2.16)$$

where K(x,t) represent the term arising from source term and prescribe initial conditions. Now first of all we applying Laplace transform of the terms on the right hand side of Eq. (2.16) then applying inverse Laplace transform we get the values of u_1, u_2,…,u_n respectively.

APPLICATIONS

To illustrate this method for coupled nonlinear partial differential equations we take three examples in this section.

Example 1: Consider nonlinear partial differential equation [18]

$$\frac{\partial^2 u(x,t)}{\partial t^2} + u^2 - u_x^2 = 0, \qquad t > 0 \quad (3.1)$$

with initial condition

$$u(x,0) = 0 \quad (3.2)$$

$$u_t(x,0) = e^x \quad (3.3)$$

Applying Laplace transform algorithm we get

$$s^2 u(x,s) - su(x,0) - u_t(x,0) = L\left[u_x^2 - u^2\right] \quad (3.4)$$

$$u(x,s) = \frac{u(x,0)}{s} + \frac{u_t(x,0)}{s^2} + \frac{1}{s^2} L\left[u_x^2 - u^2\right] \quad (3.5)$$

Using given initial condition Eqs. (3.5) becomes

$$u(x,s) = \frac{e^x}{s^2} + \frac{1}{s^2} L\left[u_x^2 - u^2\right] \quad (3.6)$$

Applying inverse Laplace transform to Eqs. (3.6) we get

$$u(x,t) = e^x t + L^{-1}\left[\frac{1}{s^2} L\left[u_x^2 - u^2\right] \right] \quad (3.7)$$

The Laplace Decomposition Method (LDM) [13-16] assumes a series solution of the function u (x, t) is given by

$$u = \sum_{n=0}^{\infty} u_n(x,t) \tag{3.8}$$

Using Eqs. (3.8) into Eqs. (3.7) we get

$$\sum_{n=0}^{\infty} u_n(x,t) = e^x t + L^{-1}\left[\frac{1}{s^2} L\left[\sum_{n=0}^{\infty} A_n(u) - \sum_{n=0}^{\infty} B_n(u)\right]\right] \tag{3.9}$$

In above Eqs. (3.9) A_n (u) and B_n (u) are Adomian polynomials [12] that represents nonlinear terms. So Adomian polynomials are given below

$$\sum_{n=0}^{\infty} A_n(u) = u_x^2 \tag{3.10}$$

$$\sum_{n=0}^{\infty} B_n(u) = u^2 \tag{3.11}$$

The few components of the Adomian polynomials are given as follow

$$A_0(u) = u_{0x}^2 \tag{3.12}$$

$$A_1(u) = 2u_{0x}^2 u_{1x} \tag{3.13}$$

$$\vdots$$

$$A_n(u) = \sum_{i=0}^{n} u_{ix} u_{n-ix}$$

$$B_0(u) = u_0^2 \tag{3.15}$$

$$B_1(u) = 2u_0 u_1 \tag{3.16}$$

$$\vdots$$

$$B_n(u) = \sum_{i=0}^{n} u_i u_{n-i} \tag{3.17}$$

From Eqs. (3.9)-(3.11) our required recursive relation is given below

$$u_0(x,t) = e^x t \tag{3.18}$$

$$u_{n+1}(x,t) = L^{-1}\left[\frac{1}{s^2} L\left[\sum_{n=0}^{\infty} A_n(u) - \sum_{n=0}^{\infty} B_n(u)\right]\right],\ n \geq 0 \tag{3.19}$$

The first few components of u_n (x,t) follows immediately upon setting

$$u_1(x,t) = L^{-1}\left[\frac{1}{s^2} L\left[A_0(u) - B_0(u)\right]\right]$$

$$= L^{-1}\left[\frac{1}{s^2} L\left[u_{0x}^2 - u_0^2\right]\right]$$

$$= L^{-1}\left[\frac{1}{s^2} L\left[t^2 e^{2x} - t^2 e^{2x}\right]\right]$$

$$= L^{-1}\left[\frac{1}{s^2} L[0]\right]$$

$$u_1(x,t) = 0 \tag{3.20}$$

Therefore the solution obtained by LDM is given below

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = e^x t \tag{3.21}$$

Which is same as solution obtained by ADM [18].

Example 2: Consider system of nonlinear coupled partial differential equations [18]

$$\frac{\partial u(x,y,t)}{\partial t} - v_x w_y = 1 \tag{3.22}$$

$$\frac{\partial v(x,y,t)}{\partial t} - w_x u_y = 5 \tag{3.23}$$

$$\frac{\partial w(x,y,t)}{\partial t} - u_x v_y = 5 \tag{3.24}$$

with initial conditions (3.14)

$$u(x,y,0) = x + 2y \tag{3.25}$$

$$v(x,y,0) = x - 2y \tag{3.26}$$

$$w(x,y,0) = -x + 2y \tag{3.27}$$

Applying the Laplace decomposition method

$$su(x,y,s) - u(x,y,0) = L\left[1 + v_x w_y\right] \tag{3.28}$$

$$sv(x,y,s) - v(x,y,0) = L\left[5 + w_x u_y\right] \tag{3.29}$$

$$sw(x,y,s) - w(x,y,0) = L\left[5 + u_x v_y\right] \tag{3.30}$$

Using initial conditions Eqs. (3.28)-(3.30) becomes

$$u(x,y,s) = \frac{1}{s^2} + \frac{1}{s}(x + 2y) + \frac{1}{s} L\left[v_x w_y\right] \tag{3.31}$$

$$v(x,y,s) = \frac{5}{s^2} + \frac{1}{s}(x-2y) + \frac{1}{s}L\left[w_x u_y\right] \tag{3.32}$$

$$w(x,y,s) = \frac{5}{s^2} + \frac{1}{s}(-x+2y) + \frac{1}{s}L\left[u_x v_y\right] \tag{3.33}$$

Applying inverse Laplace transform we get

$$u(x,y,t) = t + (x+2y) + L^{-1}\left[\frac{1}{s}L\left[v_x w_y\right]\right] \tag{3.34}$$

$$v(x,y,t) = 5t + (x-2y) + L^{-1}\left[\frac{1}{s}L\left[w_x u_y\right]\right] \tag{3.35}$$

$$w(x,y,t) = 5t + (-x+2y) + L^{-1}\left[\frac{1}{s}L\left[u_x v_y\right]\right] \tag{3.36}$$

The recursive relations are

$$u_0(x,y,t) = t + (x+2y) \tag{3.37}$$

$$u_{n+1}(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty} C_n(v,w)\right]\right],\ n \ge 0 \tag{3.38}$$

$$v_0(x,y,t) = 5t + (x-2y) \tag{3.39}$$

$$v_{n+1}(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty} D_n(u,w)\right]\right],\ n \ge 0 \tag{3.40}$$

$$w_0(x,y,t) = 5t + (-x+2y) \tag{3.41}$$

$$w_{n+1}(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[\sum_{n=0}^{\infty} E_n(u,v)\right]\right],\ n \ge 0 \tag{3.42}$$

where C_n (v, w), D_n (u, w) and E_n (u, v) are Adomian polynomials representing the nonlinear terms [12] in above Eqs. (3.37)-(3.42). The few components of Adomian polynomials are given as follow

$$C_0(v,w) = v_{0x} w_{0y} \tag{3.43}$$

$$C_1(v,w) = v_{1x} w_{0y} + v_{0x} w_{1y} \tag{3.44}$$

$$\vdots$$

$$C_n(v,w) = \sum_{i=0}^{n} v_{ix} w_{n-iy} \tag{3.45}$$

$$D_0(u,w) = u_{0y} w_{0x} \tag{3.46}$$

$$D_1(u,w) = u_{1y} w_{0x} + u_{0y} w_{1x} \tag{3.47}$$

$$\vdots$$

$$D_n(v,w) = \sum_{i=0}^{n} w_{ix} u_{n-iy} \tag{3.48}$$

$$E_0(u,v) = u_{0x} v_{0y} \tag{3.49}$$

$$E_1(u,v) = u_{1x} v_{0y} + u_{0x} v_{1y} \tag{3.50}$$

$$\vdots$$

$$E_n(u,v) = \sum_{i=0}^{n} u_{ix} v_{n-iy} \tag{3.51}$$

In view of the recursive relations (3.37)-(3.42) we obtained other components as follows

$$\begin{aligned} u_1(x,y,t) &= L^{-1}\left[\frac{1}{s}L\left[C_0(v,w)\right]\right] \\ &= L^{-1}\left[\frac{1}{s}L\left[v_{0x} w_{0y}\right]\right] \\ &= L^{-1}\left[\frac{1}{s}L\left[(1)(2)\right]\right] \\ &= 2t \end{aligned} \tag{3.52}$$

$$\begin{aligned} v_1(x,y,t) &= L^{-1}\left[\frac{1}{s}L\left[D_0(u,w)\right]\right] \\ &= L^{-1}\left[\frac{1}{s}L\left[w_{0x} u_{0y}\right]\right] \\ &= L^{-1}\left[\frac{1}{s}L\left[(-1)(2)\right]\right] \\ &= -2t \end{aligned} \tag{3.53}$$

$$\begin{aligned} w_1(x,y,t) &= L^{-1}\left[\frac{1}{s}L\left[E_0(u,v)\right]\right] \\ &= L^{-1}\left[\frac{1}{s}L\left[u_{0x} v_{0y}\right]\right] = -2t \end{aligned} \tag{3.54}$$

$$\begin{aligned} u_2(x,y,t) &= L^{-1}\left[\frac{1}{s}L\left[C_1(v,w)\right]\right] \\ &= L^{-1}\left[\frac{1}{s}L\left[v_{1x} w_{0y} + v_{0x} w_{1y}\right]\right] = 0 \end{aligned} \tag{3.55}$$

$$\begin{aligned} v_2(x,y,t) &= L^{-1}\left[\frac{1}{s}L\left[D_1(u,w)\right]\right] \\ &= L^{-1}\left[\frac{1}{s}L\left[w_{1x} u_{0y} + w_{0x} u_{1y}\right]\right] = 0 \end{aligned} \tag{3.56}$$

$$\begin{aligned} w_2(x,y,t) &= L^{-1}\left[\frac{1}{s}L\left[E_1(u,v)\right]\right] \\ &= L^{-1}\left[\frac{1}{s}L\left[u_{1x} v_{0y} + u_{0x} v_{1y}\right]\right] = 0 \end{aligned} \tag{3.57}$$

So the solution of above system of nonlinear partial differential equations are given below

$$u(x,y,t) = \sum_{m=0}^{\infty} u_m(x,t) = x + 2y + 3t \tag{3.58}$$

$$v(x,y,t) = \sum_{m=0}^{\infty} v_m(x,t) = x - 2y + 3t \tag{3.59}$$

$$w(x,y,t) = \sum_{m=0}^{\infty} w_m(x,t) = -x + 2y + 3t \tag{3.60}$$

Example 3: Consider system of nonlinear coupled partial differential equations [18]

$$\frac{\partial u(x,y,t)}{\partial t} + v_x w_y - v_y w_x = -u \tag{3.61}$$

$$\frac{\partial v(x,y,t)}{\partial t} + w_x u_y + u_x w_y = v \tag{3.62}$$

$$\frac{\partial w(x,y,t)}{\partial t} + u_x v_y + u_y v_x = w \tag{3.63}$$

with initial conditions

$$u(x,y,0) = e^{x+y} \tag{3.64}$$

$$v(x,y,0) = e^{x-y} \tag{3.65}$$

$$w(x,y,0) = e^{-x+y} \tag{3.66}$$

Applying the same procedure as applied in previous examples we arrive at recursive relations as follows

$$u_0(x,y,t) = e^{x+y} \tag{3.67}$$

$$u_{n+1}(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[\begin{matrix}\sum_{n=0}^{\infty} F_n(v,w) \\ -\sum_{n=0}^{\infty} G_n(v,w) - u_n\end{matrix}\right]\right], \quad n \geq 0 \tag{3.68}$$

$$v_0(x,y,t) = e^{x-y} \tag{3.69}$$

$$v_{n+1}(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[\begin{matrix}v_n - \sum_{n=0}^{\infty} H_n(u,w) \\ -\sum_{n=0}^{\infty} I_n(u,w)\end{matrix}\right]\right], \quad n \geq 0 \tag{3.70}$$

$$w_0(x,y,t) = e^{-x+y} \tag{3.71}$$

$$w_{n+1}(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[\begin{matrix}w_n - \sum_{n=0}^{\infty} J_n(u,v) \\ -\sum_{n=0}^{\infty} K_n(u,v)\end{matrix}\right]\right], \quad n \geq 0 \tag{3.72}$$

where F_n (v,w), G_n (v,w), H_n (u,w), I_n (u,w), J_n (u,v), and K_n (u,v) are Adomian polynomials [12] representing nonlinearities arising in above system of nonlinear coupled partial differential equations. The few components of above Adomian polynomials are given below

$$F_0(v,w) = v_{0y} w_{0x} \tag{3.73}$$

$$F_1(v,w) = v_{1y} w_{0x} + v_{0y} w_{1x} \tag{3.74}$$

$$\vdots$$

$$F_n(v,w) = \sum_{i=0}^{n} v_{iy} w_{n-ix} \tag{3.75}$$

$$G_0(v,w) = v_{0x} w_{0y} \tag{3.76}$$

$$G_1(v,w) = v_{1x} w_{0y} + v_{0x} w_{1y} \tag{3.77}$$

$$\vdots$$

$$G_n(v,w) = \sum_{i=0}^{n} v_{ix} w_{n-iy} \tag{3.78}$$

$$H_0(u,w) = w_{0x} u_{0y} \tag{3.79}$$

$$H_1(u,w) = w_{1x} u_{0y} + w_{0x} u_{1y} \tag{3.80}$$

$$\vdots$$

$$H_n(u,w) = \sum_{i=0}^{n} w_{ix} u_{n-iy} \tag{3.81}$$

$$I_0(u,w) = w_{0y} u_{0x} \tag{3.82}$$

$$I_1(u,w) = w_{1y} u_{0x} + w_{0y} u_{1x} \tag{3.83}$$

$$\vdots$$

$$I_n(u,w) = \sum_{i=0}^{n} w_{iy} u_{n-ix} \tag{3.84}$$

$$J_0(u,v) = u_{0x} v_{0y} \tag{3.85}$$

$$J_1(u,v) = u_{1x} v_{0y} + u_{0x} v_{1y} \tag{3.86}$$

$$\vdots$$

$$J_n(u,v) = \sum_{i=0}^{n} u_{ix} v_{n-iy} \tag{3.87}$$

$$K_0(u,v) = u_{0y}v_{0x} \tag{3.88}$$

$$K_1(u,v) = u_{1y}v_{0x} + u_{0y}v_{1x} \tag{3.89}$$

$$\vdots$$

$$K_n(u,v) = \sum_{i=0}^{n} u_{iy}v_{n-ix} \tag{3.90}$$

Therefore other components of the solutions are given below

$$u_1(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[F_0(v,w) - G_0(v,w) - u_0\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[v_{0y}w_{0x} - v_{0x}w_{0y} - u_0\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[e^{x-y}e^{-x+y} - e^{x-y}e^{-x+y} - e^{x+y}\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[-e^{x+y}\right]\right]$$

$$= -e^{x+y}L^{-1}\left[\frac{1}{s}\frac{1}{s}\right] = -e^{x+y}t \tag{3.91}$$

$$v_1(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[v_0 - H_0(u,w) - I_0(u,w)\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[e^{x-y} - w_{0x}u_{0y} - w_{0y}u_{0x}\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[e^{x-y} + e^{-x+y}e^{x+y} - e^{-x+y}e^{x+y}\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[e^{x-y}\right]\right]$$

$$= e^{x-y}L^{-1}\left[\frac{1}{s}\frac{1}{s}\right] = e^{x-y}t \tag{3.92}$$

$$w_1(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[w_0 - J_0(u,v) - K_0(u,v)\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[w_0 - u_{0x}v_{0y} - u_{0y}v_{0x}\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[e^{-x+y} + e^{x+y}e^{x-y} - e^{x+y}e^{x-y}\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[e^{-x+y}\right]\right]$$

$$= e^{-x+y}L^{-1}\left[\frac{1}{s}\frac{1}{s}\right] = e^{-x+y}t \tag{3.93}$$

$$u_2(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[F_1(v,w) - G_1(v,w) - u_1\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[(v_{1y}w_{0x} + v_{0y}w_{1x}) - (v_{1x}w_{0y} + v_{0x}w_{1y}) - u_1\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[te^{x+y}\right]\right] = e^{x+y}L^{-1}\left[\frac{1}{s^3}\right] = \frac{e^{x+y}}{2!}t^2 \tag{3.94}$$

$$v_2(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[v_1 - H_1(u,w) - I_1(u,w)\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[v_1 - (w_{1x}u_{0y} + w_{0x}u_{1y}) - (w_{1y}u_{0x} + w_{0y}u_{1x})\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[e^{x-y}t\right]\right] = \frac{e^{x-y}}{2!}t^2 \tag{3.95}$$

$$w_2(x,y,t) = L^{-1}\left[\frac{1}{s}L\left[w_1 - J_1(u,v) - K_1(u,v)\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[w_1 - (u_{1x}v_{0y} + u_{0x}v_{1y}) - (u_{1y}v_{0x} + u_{0y}v_{1x})\right]\right]$$

$$= L^{-1}\left[\frac{1}{s}L\left[e^{-x+y}t\right]\right] = \frac{e^{-x+y}}{2!}t^2 \tag{3.96}$$

So our required solutions are given below

$$u(x,y,t) = \sum_{n=0}^{\infty} u_n(x,y,t)$$

$$= e^{x+y} + e^{x+y}t + e^{x+y}\frac{t^2}{2!} + \ldots\ldots\ldots\ldots$$

$$= e^{x+y}\left[1 + t + \frac{t^2}{2!} + \ldots\ldots\ldots\ldots\right] = e^{x+y+t} \tag{3.97}$$

$$v(x,y,t) = \sum_{n=0}^{\infty} v_n(x,y,t)$$

$$= e^{x-y} + e^{x-y}t + e^{x-y}\frac{t^2}{2!} + \ldots\ldots\ldots$$

$$= e^{x-y}\left[1 + t + \frac{t^2}{2!} + \ldots\ldots\ldots\ldots\right] = e^{x-y+t} \tag{3.98}$$

$$w(x,y,t) = \sum_{n=0}^{\infty} w_n(x,y,t)$$

$$= e^{-x+y} + e^{-x+y}t + e^{-x+y}\frac{t^2}{2!} + \ldots\ldots\ldots\ldots$$

$$= e^{-x+y}\left[1 + t + \frac{t^2}{2!} + \ldots\ldots\ldots\ldots\right] = e^{-x+y+t} \tag{3.99}$$

From (3.21), (3.58)-(3.60) and (3.98)-(3.99), approximate solutions obtained by Laplace decomposition method is similar to the solution obtained by Adomian decomposition method [18].

CONCLUSION

In this article, Laplace dcomposition method (LDM) is applied to solve nonlinear coupled partial differential equations with initial conditions. The results of three examples are compared with ADM [18]. The results of these three examples tell us that both methods can be used alternatively for the solution of high-order initial value problems.

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