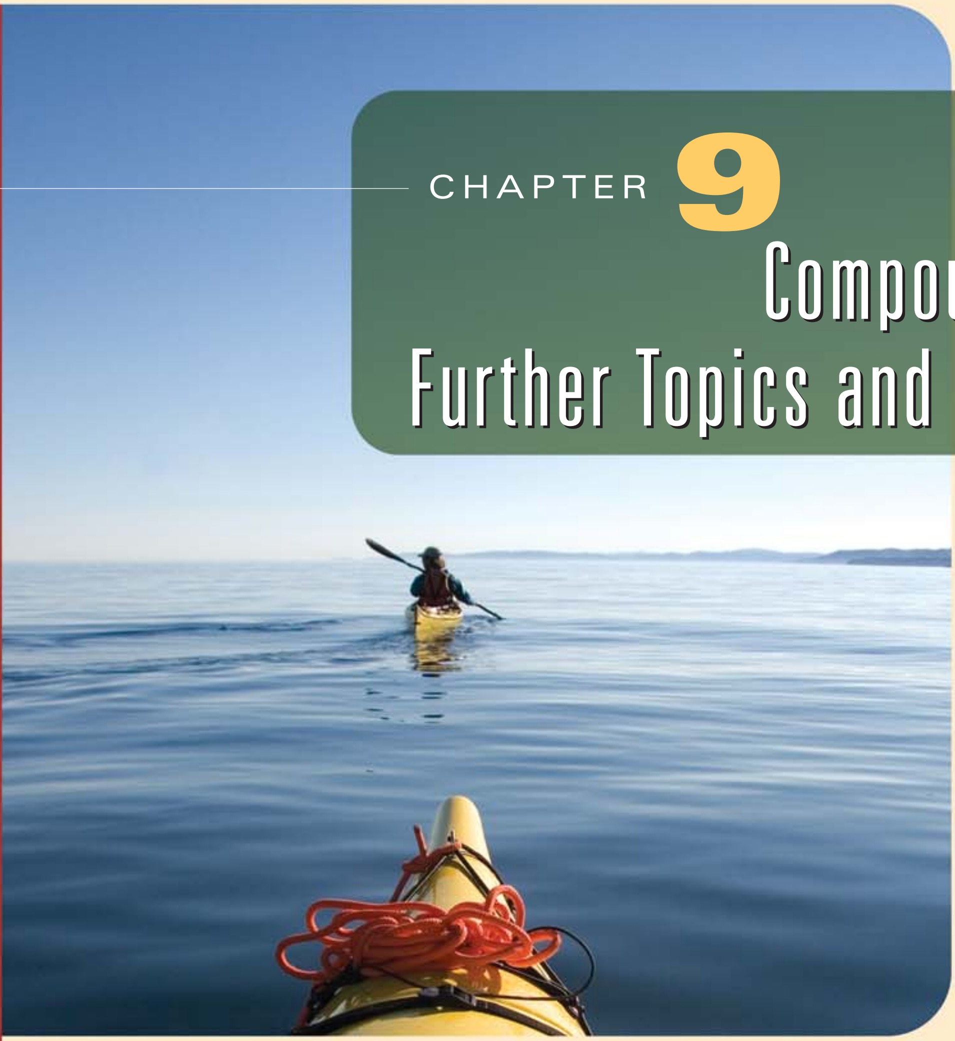
CHAPTER 9

Compound Interest: Further Topics and Applications



CHAPTER OUTLINE

LEARNING OBJECTIVES

After completing this chapter, you will be able to:

- Calculate the interest rate and term in compound interest applications
- Given a nominal interest rate, calculate its effective interest rate
- Given a nominal interest rate, calculate its equivalent interest rate at another compounding frequency
- Calculate the income yield, capital gain yield, and rate of total return on stocks and mutual funds
- Combine rates of total return for successive holding periods

IN ALL OF THE COMPOUND INTEREST PROBLEMS in Chapter 8, the interest rate and the term of the loan or investment were known. With a little reflection, you can think of many situations requiring the calculation of the interest rate, or rate of return, or rate of growth. For example, if you invest $1000 in a mutual fund, what rate of return must it earn to grow to $5000 over a 15-year period? If a stock's price rose from $15.50 to $27.40 over the past five years, what has been its equivalent annual percent increase in price? What was the average annual rate of inflation for the last 10 years if the Consumer Price Index rose from 87.4 to 118.3?

In other circumstances, we want to know the time required for an amount to grow from a beginning value to a target value. How long, for example, will it take an investment to double if it earns 10% compounded annually? By the end of Section 2, you will be able to answer such questions.

Compound interest rates on loans and investments may be quoted with differing compounding frequencies. This gives rise to questions such as: "How do we compare 7.9% compounded semiannually to 8% compounded annually? What semiannually compounded rate is equivalent to 6% compounded monthly?" The techniques you will learn in Sections 3 and 4 will enable you to answer these questions. Later on, in Chapters 10 through 14, this ability will be used routinely in calculations involving annuities.

In Section 5 you will first learn some of the terminology used to describe investment returns from stocks and mutual funds. Then we explain how you can use familiar mathematics from earlier chapters to calculate one-year and long-term returns from stocks and mutual funds.

9.1 CALCULATING THE PERIODIC INTEREST RATE, *i*

In cases where we know values for PV, FV, and n, the periodic and nominal rates of interest may be calculated.

Algebraic Method Rearranging the basic equation $FV = PV(1 + i)^n$ to isolate i is more difficult than isolating PV. First divide both sides of the equation by PV and then interchange the two sides, giving

$$(1 + i)^n = \frac{FV}{PV}$$

Next take the nth root of both sides of the equation. This makes the left side simply $(1 + i)$, and we have

$$1 + i = \sqrt[n]{\frac{FV}{PV}}$$

Therefore,[1]

PERIODIC RATE OF INTEREST

$$i = \sqrt[n]{\frac{FV}{PV}} - 1 = \left(\frac{FV}{PV}\right)^{1/n} - 1 \qquad \textbf{(9-1)}$$

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[1] It was pointed out in Section 2.2 that the nth root of a quantity is equivalent to raising it to the exponent $1/n$.

Financial Calculator Method Enter values for the known variables—*PV*, *FV*, *n*, and *m*—into the appropriate memories. Then press CPT I/Y in sequence to compute *j*, the nominal annual rate of interest. If the value of *i* is required, calculate $i = \frac{j}{m}$.

TRAP Sign Convention Now Mandatory

When you enter values for both *FV* and *PV*, it is imperative that you employ the cash-flow sign convention. If you fail to use it, an error message will appear in your calculator's display.

EXAMPLE 9.1A | CALCULATING THE PERIODIC AND NOMINAL RATES OF INTEREST

The maturity value of a three-year, $5000 compound-interest GIC is $5788.13. To three-figure accuracy, calculate the nominal rate of interest paid on the GIC if interest is compounded:

a. Annually. **b.** Quarterly.

SOLUTION

Given: $PV = \$5000$ and $FV = \$5788.13$

In Part (a), $m = 1$, $n = m(\text{Term}) = 1(3) = 3$ compounding periods.

In Part (b), $m = 4$, $n = m(\text{Term}) = 4(3) = 12$ compounding periods.

Formula (9-1) enables us to calculate the interest rate for one compounding period.

a.

$$\begin{aligned} i &= \left(\frac{FV}{PV}\right)^{1/n} - 1 \\ &= \left(\frac{\$5788.13}{\$5000.00}\right)^{1/3} - 1 \\ &= (1.157626)^{0.\bar{3}} - 1 \\ &= 0.05000 \\ &= 5.000\% \end{aligned}$$

The nominal rate of interest on the GIC is
$j = mi = 1(5.000\%) = 5.00\%$ compounded annually.

P/Y 1 ENTER
(making $C/Y = P/Y = 1$)
3 N
5000 +/− PV
0 PMT
5788.13 FV
CPT I/Y
Ans: 5.000

b.

$$\begin{aligned} i &= \left(\frac{\$5788.13}{\$5000.00}\right)^{1/12} - 1 \\ &= (1.157626)^{0.08\bar{3}} - 1 \\ &= 0.01227 \\ &= 1.227\% \end{aligned}$$

The nominal rate of interest on the GIC is
$j = mi = 4(1.227\%) = 4.91\%$ compounded quarterly.

Same *PV*, *PMT*, *FV*
P/Y 4 ENTER
(making $C/Y = P/Y = 4$)
12 N
CPT I/Y
Ans: 4.909

TRAP Don't Leave Out the Final Step

The calculation of *i* is usually not the last step in a problem. Typically you are asked to determine either the nominal interest rate or the effective interest rate (to be discussed in Section 9.3). Do not forget to complete the extra step needed to directly answer the question.

EXAMPLE 9.1B | CALCULATING A SEMIANNUALLY COMPOUNDED RATE OF RETURN

Mr. Dunbar paid $10,000 for a $40,000 face value strip bond having $19\frac{1}{2}$ years remaining until maturity. (Recall that a strip bond is an investment that returns just one payment, the face value, at maturity.) What semiannually compounded rate of return will Mr. Dunbar earn on his investment?

SOLUTION

Given: $PV = \$10,000$ $FV = \$40,000$ Term $= 19\frac{1}{2}$ years $m = 2$

Then $n = m(\text{Term}) = 2(19.5) = 39$

$$\begin{aligned} i &= \left(\frac{FV}{PV}\right)^{1/n} - 1 \\ &= \left(\frac{\$40,000}{\$10,000}\right)^{1/39} - 1 \\ &= 4^{0.0256410} - 1 \\ &= 0.036185 \\ &= 3.6185\% \end{aligned}$$

$$j = mi = 2(3.6185\%) = 7.24\% \text{ compounded semiannually}$$

P/Y 2 ENTER

(making $C/Y = P/Y = 2$)

39 N

10000 +/– PV

0 PMT

40000 FV

CPT I/Y

Ans: 7.237

Mr. Dunbar will earn 7.24% compounded semiannually on his strip bond investment.

EXAMPLE 9.1C | CALCULATING AN ANNUALLY COMPOUNDED RATE OF RETURN THAT IS EQUIVALENT TO A SERIES OF INDIVIDUAL ANNUAL RETURNS

In the years 2004, 2005, and 2006, the Excel China Fund earned annual rates of return of −4.4%, −6.0%, and 83.2%, respectively. Calculate the fund's equivalent annually compounded rate of return for the three years. (This is the fixed annual rate of return that would produce the same overall growth.)

SOLUTION

The equivalent annually compounded rate of return for the three-year period cannot be obtained by simply averaging the three individual annual returns. Instead, we must use a two-step procedure:

Step 1: Use $FV = PV(1 + i_1)(1 + i_2)(1 + i_3) \ldots (1 + i_n)$ to calculate how much an investment on December 31, 2003 was worth on December 31, 2006.

Step 2: Calculate the annually compounded rate of return that will produce the *same* growth in three years.

Step 1: For the initial investment, choose a "nice, round" amount such as $100 or $1000.

$$\begin{aligned} FV &= PV(1 + i_{2004})(1 + i_{2005})(1 + i_{2006}) \\ &= \$1000(1 - 0.044)(1 - 0.060)(1 + 0.832) \\ &= \$1646.31 \end{aligned}$$

Step 2:

$$\begin{aligned} i &= \left(\frac{FV}{PV}\right)^{1/n} - 1 \\ &= \left(\frac{\$1646.31}{\$1000}\right)^{1/3} - 1 \\ &= 1.64631^{0.333333} - 1 \\ &= 0.18078 \\ &= 18.08\% \end{aligned}$$

$$j = mi = 1(18.08\%) = 18.08\% \text{ compounded annually}$$

P/Y 1 ENTER

(making $C/Y = P/Y = 1$)

3 N

1000 +/− PV

0 PMT

1646.31 FV

CPT I/Y

Ans: 18.078

The mutual fund's equivalent annually compounded rate of return for the 3-year period ended December 31, 2006 was 18.08% compounded annually.

NET @ssets

The *Globe and Mail* maintains one of the most popular Web sites for mutual fund information and analysis. Go to www.mcgrawhill.ca/olc/jerome/ and work your way to the page for the Student Edition. In the Student Edition's navigation bar, select "Chapter 9" in the drop-down box. In the list of resources for Chapter 9, select "Links in Textbook" and then click on the link named "Mutual Funds."

Move your cursor over "GLOBE FUND" in the menu bar and select "Fund Filter" in the drop-down list. The "Fund Filter" allows you to set criteria for searching a comprehensive Canadian mutual fund database. For example, you can easily obtain a listing of all Canadian Equity funds whose equivalent annually compounded rate of return for the past 10 years has been greater than 10%.

To obtain detailed information about a fund whose name you know, choose "Fund Selector" in the list of Tools.

Postscript: At the end of every month, the type of calculation in Example 9.1C is done for about 2000 mutual funds available in Canada. The equivalent compound annual rates of return are calculated for three-year, five-year, and ten-year periods terminating at the month-end. These returns are then published in monthly mutual fund supplements to major newspapers. They are also available on investment Web sites that specialize in mutual funds. (In fact, these equivalent rates of return are easier to find than the year-by-year returns on which they are based.) You may have noticed that mutual fund advertisements commonly quote mutual fund performance in terms of the three-year, five-year, and ten-year compound annual returns. Now you know how they are obtained and how to interpret them.

EXAMPLE 9.1D | CALCULATING AN INFLATION-ADJUSTED (REAL) RATE OF RETURN

Over a 10-year period, Brooke's investment in Suncor stock grew in value from $9480 to $17,580. During the same period, the Consumer Price Index (CPI) rose from 93.6 to 126.1. What was her *real* compound annual rate of return on the stock during the decade? (The real rate of return is the rate of return net of inflation. It represents the rate of increase in purchasing power.)

SOLUTION

With the CPI up from 93.6 to 126.1, Brooke needed $\frac{126.1}{93.6}$ times as many dollars at the end of the decade to purchase the same goods and services as at the beginning. The $9480 value of the stock at the beginning had to grow to

$$\$9480 \times \frac{126.1}{93.6} = \$12{,}772$$

just to maintain her purchasing power. In fact, it grew to $17,580. In terms of end-of-decade dollars, her purchasing power rose from $12,772 to $17,580. Hence, to obtain the real rate of return, use $PV = \$12{,}772$, $FV = \$17{,}580$, and $n = 10$.

$$
\begin{aligned}
i &= \left(\frac{FV}{PV}\right)^{1/n} - 1 \\
&= \left(\frac{\$17{,}580}{\$12{,}772}\right)^{1/10} - 1 \\
&= 1.37645^{0.1} - 1 \\
&= 0.03247 \\
&= 3.247\%
\end{aligned}
$$

$$j = mi = 1(3.247\%) = 3.25\% \text{ compounded annually}$$

P/Y 1 ENTER
(making $C/Y = P/Y = 1$)
10 N
12772 +/− PV
0 PMT
17580 FV
CPT I/Y
Ans: 3.247

The real rate of return on the Suncor stock was 3.25% compounded annually.

Postscript: Two points should be mentioned.

1. The same answer will be obtained if you choose to adjust for inflation by expressing \$17,580 in terms of beginning-of-decade dollars.
2. An entirely different approach may have occurred to you. Suppose you separately calculate the rate of return on the stock, and the rate of inflation from the CPI data. (You would obtain 6.37% and 3.03% compounded annually, respectively.) You might think that:

$$
\begin{aligned}
\text{Real rate of return} &= \text{Actual rate of return} - \text{Rate of inflation} \\
&= 6.37\% - 3.03\% \\
&= 3.34\%
\end{aligned}
$$

This is a slightly larger value (by 0.09%) than the strictly correct answer we obtained in the "official solution." The reason for the small difference is quite subtle and technical—we will spare you the details. However, real rates of return are, more often than not, calculated this way.[2] Since nominal rates of return and inflation rates are easily obtained from published data, this approximation is an easier approach and is good enough for most purposes.

SPREADSHEET STRATEGIES | Periodic Rate of Interest

The Online Learning Centre (OLC) provides a partially completed template for calculating *i*, the periodic rate of interest. Go to the Student Edition of the OLC, select the "Excel Templates" link in the navigation bar, and then click on the "RATE Function" link. The "Introduction" page of the workbook describes Excel's built-in RATE function.

The main features of a completed Periodic Rate of Interest template are presented on page 340. The formula programmed into cell C9 is displayed in cell C10. Here data are entered to solve Example 9.1B. In this example, we are asked to determine the semiannually compounded rate of return on the \$10,000 purchase price for a \$40,000 face value strip bond with $19\frac{1}{2}$ years remaining until maturity. As an initial "side calculation," we obtain $n = m \times \text{Term} = 2 \times 19.5 = 39$ compounding periods. After entering the known values into the input cells, the computed periodic rate of interest appears immediately in the output cell.

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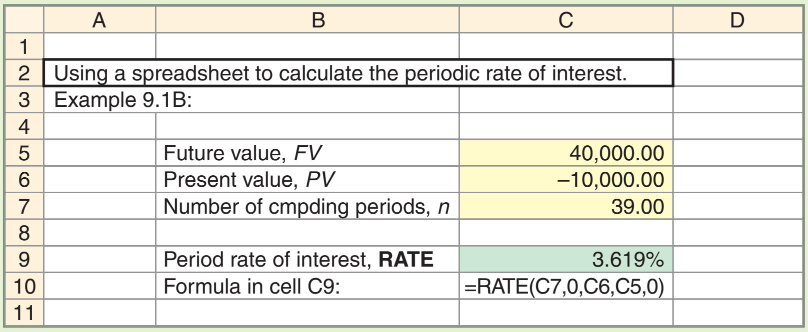
[2] You could say that the real rates of return usually calculated in the real world are not really real.



	A	B	C	D
1				
2	Using a spreadsheet to calculate the periodic rate of interest.			
3	Example 9.1B:			
4				
5		Future value, *FV*	40,000.00	
6		Present value, *PV*	–10,000.00	
7		Number of cmpding periods, *n*	39.00	
8				
9		Period rate of interest, **RATE**	3.619%	
10		Formula in cell C9:	=RATE(C7,0,C6,C5,0)	
11				

The rate of return on the strip bond is $j = mi = 2(3.619\%) = 7.24\%$ compounded semiannually.

CONCEPT QUESTIONS

1. If *FV* is less than *PV*, what can you predict about the value for *i*?
2. Is *FV* negative if you lose money on an investment?
3. Which scenario had the higher periodic rate of return: “\$1 grew to \$2” or “\$3 grew to \$5?” Both investments were for the same length of time at the same compounding frequency. Justify your choice.

EXERCISE 9.1

Spreadsheet template: *A partially completed Excel template for calculating the periodic rate of return is provided in the Online Learning Centre (OLC). Go to the Student Edition of the OLC, click on the “Excel Templates” link in the navigation bar, and then click on the “RATE Function” link. The completed template may be used wherever you need to calculate i in Exercise 9.1*

Answers to the odd-numbered problems are at the end of the book.

Calculate interest rates accurate to the nearest 0.01%.
Calculate the nominal rate of interest in Problems 1 through 6.

Problem	Principal (\$)	Maturity amount (\$)	Compounding frequency	Nominal rate (%)	Term
1.	3400	4297.91	Annually	?	3 years
2.	1000	4016.94	Annually	?	20 years
3.	1800	2299.16	Quarterly	?	2 years, 9 months
4.	6100	13,048.66	Semiannually	?	7 years, 6 months
5.	950	1165.79	Monthly	?	2 years, 5 months
6.	4300	10,440.32	Annually	?	8 years, 6 months

7. When he died in 1790, Benjamin Franklin left \$4600 to the city of Boston, with the stipulation that the money and its earnings could not be used for 100 years. The bequest grew to \$332,000 by 1890. What (equivalent) compound annual rate of return did the bequest earn during the 100-year period?

8. In early 2007, the Templeton Growth Fund ran advertisements containing the message:

$10,000 INVESTED IN TEMPLETON GROWTH FUND IN 1954
WOULD BE WORTH $6.86 MILLION TODAY.

What compound annual rate of return did the fund realize over this period (December 31, 1954 to December 31, 2006)?

9. Anders discovered an old pay statement from 11 years ago. His monthly salary at the time was $2550 versus his current salary of $4475 per month. At what (equivalent) compound annual rate has his salary grown during the period?

10. Mr. and Mrs. Markovich note that the home they purchased 20 years ago for $70,000 is now appraised at $340,000. What was the (equivalent) annual rate of appreciation in the value of their home during the 20-year period?

11. A $1000 five-year compound-interest GIC matured at $1234.01. What semiannually compounded rate of interest did it earn?

12. The maturity value of a $5000 four-year compound-interest GIC was $6147.82. What quarterly compounded rate of interest did it earn?

13. Three years ago Mikhail invested $7000 in a three-year compound interest GIC. He has just received its maturity value of $7867.34. What was the monthly compounded rate of interest on the GIC?

14. **a.** The population of Canada grew from 24,343,000 in 1981 to 32,524,000 in 2006. What was the overall compound annual rate of growth in our population during the period?

b. According to the Canadian Real Estate Association, the average selling price of Canadian homes rose from $67,000 in 1980 to $282,000 in 2006. What has been the overall compound annual appreciation of home prices?

15. The following table contains 1981 and 2006 population figures for five provinces. Calculate each province's equivalent compound annual rate of population increase during the period.

Province	1981 Population	2006 Population
Alberta	2,237,700	3,375,800
British Columbia	2,744,500	4,310,500
Newfoundland	567,700	509,700
Nova Scotia	847,400	934,400
Ontario	8,625,100	12,687,000

16. For an investment to double in value during a 10-year period,

a. What annually compounded rate of return must it earn?

b. What semiannually compounded rate of return must it earn?

c. What monthly compounded rate of return must it earn?

17. For an investment to triple in value during a 15-year period,

a. What annually compounded rate of return must it earn?

b. What quarterly compounded rate of return must it earn?

c. What monthly compounded rate of return must it earn?

18. What compound annual rate of return is required for an investment to double in:

a. 12 years? **b.** 10 years? **c.** 8 years? **d.** 6 years?

For each case, multiply the annual rate of return (in %) by the time period (in years). Compare the four products. Does the comparison suggest a general rule-of-thumb?

19. Monty purchased a strip bond for his RRSP. He paid $3800 for a $5000 face value bond with three years remaining until maturity. What semiannually compounded rate of return will he realize over the three years? (Taken from CIFP course materials.)

20. If the number of workers in the forest industry in Canada declined by 35% from the end of 1993 to the beginning of 2006, what was the compound annual rate of attrition in the industry during the period?

21. The Canadian Consumer Price Index (based on a value of 100 in 1971) rose from 97.2 in 1970 to 210.6 in 1980. What was the (equivalent) annual rate of inflation in the decade of the 1970s?

22. The Consumer Price Index (based on a value of 100 in 1986) rose from 67.2 in 1980 to 119.5 in 1990. What was the (equivalent) annual rate of inflation in the decade of the 1980s?

23. The Consumer Price Index (based on a value of 100 in 1992) rose from 93.3 in 1990 to 113.5 in 2000. What was the (equivalent) annual rate of inflation in the decade of the 1990s?

•24. Using the data given in Problems 21 and 22, calculate the annual rate of inflation for the 1970–1990 period. (Note: Simply averaging the two answers to Problems 21 and 22 will give only an approximation of the correct result.)

25. According to Statistics Canada, undergraduate students paid an average of $4347 in tuition fees for the 2006/2007 academic year compared to fees of $1464 for the 1990/1991 year. During the same period, the Consumer Price Index rose from 94.6 to 129.7.

   a. What would have been the average tuition fees for the 2006/2007 year if tuition fees had grown just at the rate of inflation since the 1990/1991 year?

   b. What was the (equivalent) compound annual rate of increase of tuition fees during the period?

   c. What was the (equivalent) compound annual rate of inflation during the period?

•26. A four-year promissory note for $3800 plus interest at 9.5% compounded semiannually was sold 18 months before maturity for $4481. What quarterly compounded nominal rate of return will the buyer realize on her investment?

•27. A $6000, three-year promissory note bearing interest at 11% compounded semiannually was purchased 15 months into its term for $6854.12. What monthly compounded discount rate was used in pricing the note?

•28. An investor's portfolio increased in value by 93% over a seven-year period in which the Consumer Price Index rose from 95.6 to 115.3. What was the compound annual real rate of return on the portfolio during the period?

•29. An investment grew in value from $5630 to $8485 during a five-year period. The annual rate of inflation for the five years was 2.3%. What was the compound annual real rate of return during the five years?

•30. An investment earned 12% compounded semiannually for two years and 8% compounded annually for the next three years. What was the equivalent annually compounded rate of return for the entire five-year period?

•31. A portfolio earned annual rates of 20%, −20%, 0%, 20%, and −20% in five successive years. What was the portfolio's five-year equivalent annually compounded rate of return?

•32. A portfolio earned annual rates of 20%, 15%, −10%, 25%, and −5% in five successive years. What was the portfolio's five-year equivalent annually compounded rate of return?

•**33.** At the end of 2006, the RBC Canadian Dividend Fund was the largest equity mutual fund in Canada. The aggregate market value of its holdings at the end of 2006 was $8.295 billion. The fund's annual returns in successive years from 1997 to 2006 inclusive were 35.4%, 2.4%, 2.5%, 28.3%, 4.4%, –0.5%, 23.5%, 12.9%, 21.1%, and 15.1%, respectively. For the 3-year, 5-year, and 10-year periods ending December 31, 2006, what were the fund's equivalent annually compounded returns?

•**34.** At the end of 2006, the Mawer New Canada Fund had the best 10-year compound annual return of any Canadian equity mutual fund. During the 10-year period, this fund invested primarily in the shares of smaller Canadian companies. The fund's annual returns in successive years from 1997 to 2006 inclusive were 16.4%, –17.8%, 9.2%, 8.1%, 29.0%, 23.5%, 26.1%, 29.5%, 18.0%, and 15.3%, respectively. For the 3-year, 5-year, and 10-year periods ending December 31, 2006, what were the fund's equivalent annually compounded returns?

•**35.** At the end of 2006, the Mavrix Growth Fund had one of the worst 10-year compound annual returns of any Canadian equity mutual fund. The fund's annual returns in successive years from 1997 to 2006 inclusive were 26.4%, –6.3%, 59.3%, –7.2%, –66.2%, –52.5%, 22.8%, –11.2%, 8.4%, and 35.6%, respectively. For 3-year, 5-year, and 10-year periods ending December 31, 2006, what were the fund's equivalent annually compounded returns?

•**36.** In June of 2006, AIC Limited published full-page advertisements focused on the fact that its AIC Advantage Mutual Fund was Canada's "Best Performing Canadian Equity Fund over the 20 years" ending May 31, 2006. The equivalent annual rate of return during the 20 years was 11.9% compared to 9.9% for the benchmark S&P/TSX Composite Total Return Index. But the advertisement failed to point out that during the second half of that 20-year period, the fund's 9.4% compounded annual return was actually less than the 10.2% growth rate for the S&P/TSX Composite Total Return Index. Furthermore, in the final 5 years of the 20-year period, the fund's 2.4% annual rate of return was far below the index's 9.5% annual growth. The Advantage Fund's five-year performance was even less than the *median* performance of all Canadian equity mutual funds. In short, AIC was still trying to capitalize on the initial 10 years of truly outstanding performance, even though the Advantage Fund's subsequent 10 years' performance was at best mediocre.

a. What would $10,000 invested in the AIC Advantage Fund on May 31, 1986 have grown to after 20 years?

b. What was this investment worth after the first 10 years?

c. What compound annual rate of return did the AIC Advantage Fund earn during the first 10 years of the 20-year period?

d. What was the overall percent increase in the value of an investment in the AIC Advantage Fund during:
(i) The first 10 years? **(ii)** The second 10 years?

37. **Searching a Mutual Fund Data Base** Follow the instructions in the NET @ssets box earlier in this section to locate the "Mutual Funds" link in the textbook's OLC. When the globefund.com page loads, move your cursor over "GLOBE FUND" in the menu bar and select "Fund Selector" from the drop-down list. In the "Option C" area, you can enter the name of a particular fund. Enter "RBC Canadian Dividend" and click on "Go." The table that loads has several tabs along its top. Select "Long-term." This brings up another table with columns giving the fund's compound annual return for 3-year, 5-year, and 10-year periods ending on the last business day of the previous month. How much would $10,000 invested in this fund 10 years earlier be worth at the end of the previous month? Repeat for the "Mawer New Canada" and the "Mavrix Growth" funds.

9.2 CALCULATING THE NUMBER OF COMPOUNDING PERIODS, *n*

If we know values for PV, FV, and i, we can calculate the number of compounding periods and the term of the loan or investment.

Algebraic Method You can take either of two approaches.

1. If you are familiar with the rules of logarithms, you can substitute the values for PV, FV, and i into $FV = PV(1 + i)^n$ and then solve for n.
2. If you are not comfortable manipulating logarithms, you can use formula (9-2). It is, in fact, just a "dressed-up" version of $FV = PV(1 + i)^n$ in which n is already isolated for you.

NUMBER OF COMPOUNDING PERIODS

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1 + i)} \tag{9-2}$$

In Example 9.2A, we will demonstrate both algebraic methods. Thereafter, only the second approach will be used in the text.

Financial Calculator Method Enter values for the four known variables—PV, FV, i, and m—into the appropriate memories. Then press CPT N in sequence to execute the calculation.

TIP **Don't Leave Out the Final Step**

The calculation of n is usually not the last step in a problem. Typically you are asked to determine the total time in years and months (rather than the number of compounding periods). Do not forget to complete the extra step necessary to directly answer the problem.

EXAMPLE 9.2A | CALCULATING THE NUMBER OF COMPOUNDING PERIODS

What is the term of a compound-interest GIC if \$4000 invested at 5.5% compounded annually earns interest totalling \$1227.84?

SOLUTION

Given: $PV = \$4000$ $i = \frac{j}{m} = \frac{5.5\%}{1} = 5.5\%$ Total interest = \$1227.84

The maturity value of the GIC is

$$FV = PV + \text{Total interest} = \$4000 + \$1227.84 = \$5227.84$$

Method 1: Use the basic formula $FV = PV(1 + i)^n$ to calculate the number of compounding periods required for \$4000 to grow to \$5227.84. Substitute the known values for PV, FV, and i giving

$$\$5227.84 = \$4000(1.055)^n$$

Therefore, $$1.055^n = \frac{\$5227.84}{\$4000} = 1.30696$$

Now take logarithms of both sides. On the left side, use the rule that: $\ln(a^n) = n(\ln a)$

Therefore, $$n(\ln 1.055) = \ln 1.30696$$

and $$n = \frac{\ln 1.30696}{\ln 1.055} = \frac{0.267704}{0.0535408} = 5.000$$

Since each compounding period equals one year, the term of the GIC is five years.

Method 2: Substitute the known values into the derived formula (9-2). The number of compounding periods required for $4000 to grow to $5227.84 is

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)} = \frac{\ln\left(\frac{\$5227.84}{\$4000.00}\right)}{\ln(1.055)} = \frac{\ln(1.30696)}{\ln(1.055)} = \frac{0.267704}{0.0535408} = 5.000$$

5.5 I/Y
P/Y 1 ENTER
(making $C/Y = P/Y = 1$)
4000 +/− PV
0 PMT
5227.84 FV
CPT N
Ans: 5.000

Since each compounding period equals one year, the term of the GIC is five years.

TIP Efficient Use of Your Calculator

The most efficient keystroke sequence for evaluating formula (9-2) in Method 2 of Example 9.2A is:

5227.84 ÷ 4000 = LN ÷ 1.055 LN =

On the Sharp EL-738 calculator, the (natural) logarithm function ln is the second function of the 2 key.

Noninteger Values for *n* If formula (9-2) or the financial calculator procedure gives a value for n that is not an integer, it means (as you would expect) that the term of the loan or investment includes a partial compounding period. In the final answer, we normally convert the fractional part of n to months, or to months and days (depending on the requested precision).

EXAMPLE 9.2B | CALCULATING AND INTERPRETING A NONINTEGER *N*

Rounded to the nearest month, how long will it take a city's population to grow from 75,000 to 100,000 if the annual growth rate is 2%?

SOLUTION

In effect, we are given:

$$PV = 75,000, FV = 100,000, \text{ and } i = \frac{j}{m} = \frac{2\%}{1} = 2\% \text{ per year}$$

Using formula (9-2) to calculate the required number of compounding periods, we obtain

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)} = \frac{\ln\left(\frac{100,000}{75,000}\right)}{\ln(1.02)} = \frac{0.28768}{0.019803} = 14.527$$

2 I/Y
P/Y 1 ENTER
(making $C/Y = P/Y = 1$
75000 +/− PV
0 PMT
100000 FV
CPT N
Ans: 14.527

It requires 14.527 compounding periods for the population to grow from 75,000 to 100,000. Since a compounding period equals one year,

$$\begin{aligned} 14.527 \text{ compounding periods} &= 14 \text{ years} + 0.527 \times 12 \text{ months} \\ &= 14 \text{ years} + 6.32 \text{ months} \end{aligned}$$

Rounded to the nearest month, it will take 14 years and 6 months for the city's population to reach 100,000.

EXAMPLE 9.2C | CALCULATING AN INVESTMENT'S DOUBLING TIME

How long will it take an investment to double in value if it earns:

a. 6% compounded annually? **b.** 10% compounded annually?

Include accrued interest and round the answer to the nearest month.

SOLUTION

We require the maturity value of an investment to be twice the initial investment. Therefore, we can simply set *PV* = \$1 and *FV* = \$2.

In part (a), $i = \frac{j}{m} = \frac{6\%}{1} = 6\%$ per year. In part (b), $i = \frac{j}{m} = \frac{10\%}{1} = 10\%$ per year.

a. Substituting in formula (9-2),

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)} = \frac{\ln(2)}{\ln(1.06)} = 11.896$$

6 I/Y
P/Y 1 ENTER
(making *C/Y* = *P/Y* = 1)
1 +/− PV
0 PMT
2 FV
CPT N
Ans: 11.896

The doubling time is

$$11.896 \text{ years} = 11 \text{ years} + 0.896 \times 12 \text{ months} = 11 \text{ years} + 10.75 \text{ months}$$

An investment earning 6% compounded annually will double in 11 years and 11 months (rounded to the nearest month).

b. Substituting in formula (9-2),

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)} = \frac{\ln(2)}{\ln(1.10)} = 7.2725$$

Same *P/Y, C/Y*
Same *PV, PMT, FV*
10 I/Y
CPT N
Ans: 7.2725

The doubling time is

$$7.2725 \text{ years} = 7 \text{ years} + 0.2725 \times 12 \text{ months} = 7 \text{ years} + 3.27 \text{ months}$$

An investment earning 10% compounded annually will double in 7 years and 3 months (rounded to the nearest month).

Rule of 72 Investors have a rule-of-thumb to quickly *estimate* the number of years it will take an investment to double.[3] Known as the **Rule of 72,** it says:

$$\text{Doubling time (in years)} \approx \frac{72}{\text{Percent annual rate of return}}$$

For example, an investment earning 9% compounded annually will double in approximately $\frac{72}{9} = 8$ years. If the investment earns 12% compounded annually, it will double in about $\frac{72}{12} = 6$ years.

The answers in the preceding Example 9.2C provide an indication of the accuracy of the Rule of 72. At an annually compounded rate of 6%, we calculated a doubling time of 11.90 years (vs. 12.0 years using the Rule of 72). At 10% compounded annually, we calculated a doubling time of 7.27 years (vs. 7.2 years using the Rule of 72). In both cases, the estimate is within 1% of the correct value.

Valuing Strip Bonds and Other Single-Payment Investments Most loans and investments are structured so that the full term equals an integer multiple of the compounding period. However, an existing loan or investment contract may be sold and transferred to a new investor on a date that does not coincide with an interest-conversion date. The time remaining until maturity includes a partial compounding interval which must be taken into account in the price calculation. Consequently, n is not an integer in such cases.

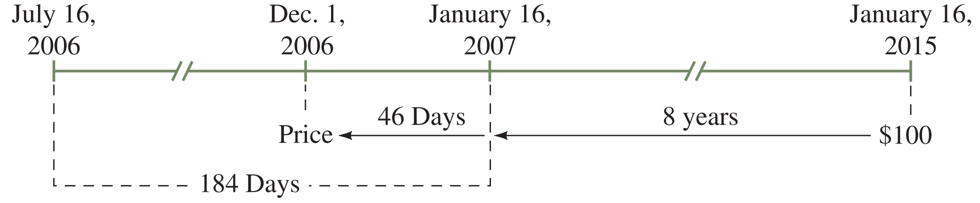
In Table 9.1, consider the Government of Quebec strip bond maturing on January 16, 2015. Let us see how the market's required rate of return (4.161% compounded semiannually) on December 1, 2006 dictates the quoted market price of $71.56. (Recall that bond prices are quoted in the media as the price per $100 of face value.)

TABLE 9.1 Strip Bond Price and Yield Quotations (December 1, 2006)

Issuer	Maturity Date	Price ($)	Yield (%)
Government of British Columbia	June 9, 2014	74.34	3.981
Nova Scotia Power Corp.	February 26, 2019	58.67	4.406
Government of Quebec	January 16, 2015	71.56	4.161
Government of Newfoundland	April 17, 2014	74.77	3.981
Government of Ontario	September 8, 2027	39.83	4.482
TD Bank	August 4, 2014	71.96	4.333

SOURCE: *Globe and Mail Report on Business*, December 1, 2006

As indicated on the following timeline, the price should be the present value of the $100 face value discounted at 4.161% compounded semiannually all the way back from the maturity date (January 16, 2015) to December 1, 2006. We need to determine the number of compounding periods in this interval. Working back from the maturity date, there are 16 compounding periods in the 8 years between January 16, 2007 and January 16, 2015. Then there are an additional 46 days from December 1, 2006 to January 16, 2007. This 46-day



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[3] The approximation is very good for annual interest rates between 5% and 11%; the value estimated for the doubling time is within 2% of its true value.

interval is the fraction $\frac{46}{184}$ of the full 184-day compounding period from July 16, 2006 to January 16, 2007. Therefore, $n = 16\frac{46}{184} = 16.2500$ compounding periods, $i = \frac{4.161\%}{2} = 2.0805\%$, and

$$\text{Price, } PV = FV(1 + \mathrm{i})^{-n} = \$100(1.020805)^{-16.2500} = \$71.56$$

This equals the quoted price accurate to the cent. (In general, a yield rounded to four figures guarantees only three-figure accuracy in the price.)

EXAMPLE 9.2D | CALCULATING THE TIME UNTIL MATURITY OF A STRIP BOND

A $10,000 face value strip bond was purchased for $4188.77. At this price, the bond provided the investor with a return of 5.938% compounded semiannually until the maturity date. To the nearest day, how long before the maturity date was the bond purchased? Assume that each half year is exactly 182 days long.

SOLUTION

The purchase price of a strip bond equals the present value, on the date of purchase, of the bond's face value. The prevailing market rate of return should be used as the discount rate. In this example, $4188.77 is the present value of $10,000 discounted at 5.938% compounded semiannually. To determine the time interval used in the present value calculation, we must first calculate the number of compounding periods. We are given:

$$PV = \$4188.77, FV = \$10,000, \text{ and } i = \frac{j}{m} = \frac{5.938\%}{2} = 2.969\%$$

Substituting in formula (9-2),

$$\begin{aligned} n &= \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)} \\ &= \frac{\ln\left(\frac{\$10,000}{\$4188.77}\right)}{\ln(1.02969)} \\ &= 29.74176 \end{aligned}$$

5.938 I/Y
P/Y 2 ENTER
(making *C/Y* = *P/Y* = 2)
4188.77 +/− PV
0 PMT
10000 FV
CPT N
Ans: 29.74176

Since each compounding period is 0.5 year, the time remaining to maturity is

$$(0.50 \times 29) \text{ years} + (0.74176 \times 182) \text{ days} = 14.5 \text{ years} + 135.00 \text{ days}$$

Hence, the bond was purchased with 14 years, 6 months, and 135 days remaining until its maturity date.

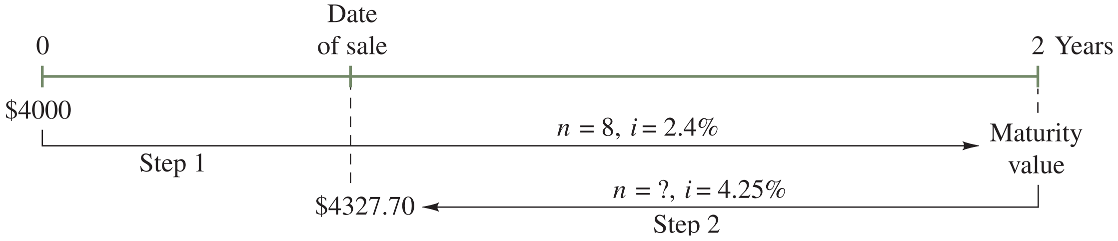
EXAMPLE 9.2E | SOLVING FOR A NONINTEGER "*n*" IN A DISCOUNTING PROBLEM

A loan contract requires the payment of $4000 plus interest two years after the contract's date of issue. The interest rate on the $4000 face value is 9.6% compounded quarterly. Before the maturity date, the original lender sold the contract to an investor for $4327.70. The sale price was based on a discount rate of 8.5% compounded semiannually from the date of sale. How many months before the maturity date did the sale take place?

SOLUTION

The selling price represents the present value (on the date of sale) of the loan's maturity value. In other words, \$4327.70 was the present value of the maturity value, discounted at 8.5% compounded semiannually. Therefore, the solution requires two steps as indicated in the following time diagram.

1. Calculate the maturity value of the debt.
2. Determine the length of time over which the maturity value was discounted to give a present value of \$4327.70.



Step 1: For the maturity value calculation,

$$n = m(\text{Term}) = 4(2) = 8 \text{ and } i = \frac{j}{m} = \frac{9.6\%}{4} = 2.4\%.$$

The maturity value of the contract is

$$\begin{aligned} FV &= PV(1+i)^n \\ &= \$4000(1.024)^8 \\ &= \$4835.70 \end{aligned}$$

9.6 I/Y
P/Y 4 ENTER
(making *C/Y* = *P/Y* = 4)
8 N
4000 +/− PV
0 PMT
CPT FV
Ans: 4835.70

Step 2: For discounting the maturity value,

$$i = \frac{j}{m} = \frac{8.5\%}{2} = 4.25\%.$$

The number of compounding periods between the date of sale and the maturity date is

$$\begin{aligned} n &= \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)} \\ &= \frac{\ln\left(\frac{\$4835.70}{\$4327.70}\right)}{\ln(1.0425)} \\ &= 2.6666 \end{aligned}$$

Same *PMT*, *FV*
8.5 I/Y
P/Y 2 ENTER
(making *C/Y* = *P/Y* = 2)
4327.70 +/− PV
CPT N
Ans: 2.6666

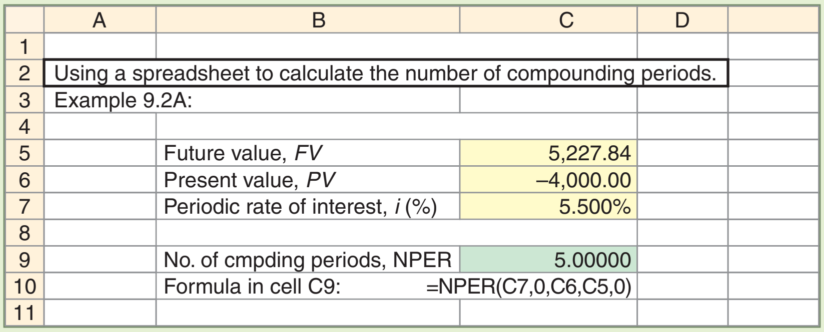
Each compounding period is six months long.
Therefore, the date of sale was

$$2.6666 \times 6 \text{ months} = 16.00 \text{ months before the maturity date.}$$

SPREADSHEET STRATEGIES | Number of Compounding Periods

The Online Learning Centre (OLC) provides a partially completed template for calculating *n*, the number of compounding periods. Go to the Student Edition of the OLC, select the "Excel Templates" link in the navigation bar, and then click on the "NPER Function" link. The "Introduction" page of the workbook describes Excel's built-in NPER function.

The main features of a completed "NPER Function" template are presented below. The formula programmed into cell C9 is displayed in cell C10. Here data are entered to solve Example 9.2A. In this example, $4000 invested in a GIC earning 5.5% compounded annually grew to a maturity value of $5227.84. We are asked to determine the term of the GIC. After entering the known values into the input cells, the computed number of compounding periods appears immediately in the output cell.



	A	B	C	D	
1					
2	Using a spreadsheet to calculate the number of compounding periods.				
3	Example 9.2A:				
4					
5		Future value, *FV*	5,227.84		
6		Present value, *PV*	–4,000.00		
7		Periodic rate of interest, *i* (%)	5.500%		
8					
9		No. of cmpding periods, NPER	5.00000		
10		Formula in cell C9:	=NPER(C7,0,C6,C5,0)		
11					

Since each compounding period equals one year, the term of the GIC was five years.

CONCEPT QUESTIONS

1. Under what circumstance does the value calculated for *n* equal the number of years in the term of the loan or investment?
2. Which investment scenario requires more time: "$1 growing to $2" or "$3 growing to $5?" Both investments earn the same rate of return. Justify your choice.

EXERCISE 9.2

Spreadsheet template: *A partially completed Excel template for calculating the number of compounding periods is provided in the Online Learning Centre (OLC). Go to the Student Edition of the OLC, click on the "Excel Templates" link in the navigation bar, and then click on the "NPER Function" link. The completed template may be used wherever you need to calculate n in Exercise 9.2.*

Answers to the odd-numbered problems are at the end of the book.

Calculate the term of each loan or investment in Problems 1 through 6.

Problem	Principal ($)	Maturity amount ($)	Compounding frequency	Nominal rate (%)	Term
1.	1100	4483.92	Annually	6.3	? years, ? months
2.	4625	8481.61	Annually	7.875	? years, ? months
3.	5670	10,365.39	Semiannually	9.5	? years, ? months
4.	2000	3172.42	Annually	8.75	? years, ? months
5.	2870	3837.30	Monthly	10	? years, ? months
6.	3250	4456.90	Quarterly	7.5	? years, ? months

7. A number of years ago, your client invested $6000 at a rate of return of 9% compounded annually. If the investment is currently worth $10,968.25, for how long has she held the investment? (Taken from CIFP course materials.)

8. A few years ago Avtar invested $6000 in a compound-interest GIC that earned 4.5% compounded semiannually. He recently received the maturity value of $7168.99. What was the term of the GIC?

9. Your client wants to invest a $250,000 inheritance and grow it to $325,000. Rounded to the nearest month, how long will this take if the investment earns 7% compounded annually? (Taken from CIFP course materials.)

10. Your client invests $10,000 today at a rate of return of 7.7% compounded quarterly. Rounded to the nearest month, how long will it take the investment to grow to $22,000? (Taken from CIFP course materials.)

11. Marilyn was supposed to pay $1450 to Bernice on March 1. Some time later Marilyn paid Bernice an equivalent payment of $1528.01, allowing for a time value of money of 4.5% compounded monthly. When did Marilyn make the payment?

12. What is the remaining time until the maturity date of a $10,000 strip bond if it is purchased for $4011.33 to yield 6.4% compounded semiannually until maturity?

13. Rounded to the nearest month, how long will it take a town's population to:

a. Grow from 32,500 to 40,000 if the annual growth rate is 3%?

b. Shrink from 40,000 to 32,500 if the annual rate of decline is 3%?

14. Rounded to the nearest month, how long will it take an investment to double if it earns:

a. 8.4% compounded annually? **b.** 10.5% compounded semiannually?

15. Rounded to the nearest month, how long will it take an investment to triple if it earns:

a. 9% compounded annually? **b.** 8% compounded quarterly?

16. Rounded to the nearest quarter year, how long will it take an investment to quadruple if it earns:

a. 8% compounded annually? **b.** 9% compounded semiannually?

•17. Rounded to the nearest month, how long will it take money to lose half of its purchasing power if the annual inflation rate is:

a. 2.5%? **b.** 3.5%?

•18. Rounded to the nearest month, how long will it take money to lose 25% of its purchasing power if the annual rate of inflation is:

a. 2%? **b.** 4%?

•19. When discounted to yield 10.5% compounded monthly, a $2600 three-year promissory note bearing interest at 12.25% compounded annually was priced at $3283.57. How many months after the issue date did the discounting take place?

•20. The proceeds from the sale of a $4500 five-year promissory note bearing interest at 9% compounded quarterly were $6055.62. How many months before its maturity date was the note sold if it was discounted to yield 10.5% compounded monthly?

•21. A $4000 loan at 7.5% compounded monthly was settled by a single payment of $5000 including accrued interest. Rounded to the nearest day, how long after the initial loan was the $5000 payment made? For the purpose of determining the number of days in a partial month, assume that a full month has 30 days.

•22. If money is worth 8% compounded quarterly, how long (to the nearest day) before a scheduled payment of $6000 will $5000 be an equivalent payment? For the purpose of determining the number of days in a partial calendar quarter, assume that a full quarter has 91 days.

•23. Wilf paid $557.05 for a $1000 face value strip bond. At this price the investment will yield a return of 5.22% compounded semiannually. How long (to the nearest day) before its maturity date did Wilf purchase the bond? Assume that each half-year has exactly 182 days.

•24. A $5000 face value strip bond may be purchased today for $1073.36 yielding the purchaser 7.27% compounded semiannually. How much time (to the nearest day) remains until the maturity date? Assume that each half-year has exactly 182 days.

•25. $7500 was borrowed for a four-year term at 9% compounded quarterly. The terms of the loan allow prepayment of the loan based on discounting the loan's maturity value at 7% compounded quarterly. How long (to the nearest day) before the maturity date was the loan prepaid if the payout amount was $9380.24 including accrued interest? For the purpose of determining the number of days in a partial calendar quarter, assume that a full quarter has 91 days.

•26. Consider the Government of British Columbia strip bond in Table 9.1.

  a. Calculate the bond's market price on December 1, 2006 based on the quoted yield of 3.981% compounded semiannually.

  b. What would the price be one year later if the bond's yield remains the same?

•27. Consider the Government of Ontario strip bond in Table 9.1.

  a. Calculate the bond's market price on December 1, 2006 based on the quoted yield of 4.482% compounded semiannually.

  b. What would the price be on December 1, 2020 if the bond's yield remains the same?

•28. Consider the Government of Newfoundland strip bond in Table 9.1.

  a. Calculate the bond's yield to four-figure accuracy on December 1, 2006 based on the quoted price of $74.77.

  b. What would the yield be one year later if the bond's price remains the same?

•29. Consider the Nova Scotia Power Corp. strip bond in Table 9.1.

  a. Calculate the bond's yield to four-figure accuracy on December 1, 2006 based on the quoted price of $58.67.

  b. What would the yield be one year later if the bond's price remains the same?

9.3 EFFECTIVE INTEREST RATE

The future value of $100 invested for one year at 10% compounded semiannually is $110.25. The future value of $100 invested for one year at 10.25% compounded annually is also $110.25. Therefore, an interest rate of 10% compounded semiannually has the *same effect* as a rate of 10.25% compounded annually. The **effective interest rate,** *f*, is defined as the *annually* compounded rate[4] that produces the *same* future value after one year as the given nominal rate. In the present example, 10% compounded semiannually has an effective rate[5] of 10.25%.

---

[4] There is a natural preference in business for discussing interest rates on the basis of annual compounding. This is because an *annually* compounded rate of return represents the *actual* percentage increase in a year. For example, at a return of 9% compounded annually, you can immediately say that $100 will grow by 9% ($9) in the next year. But at a return of 9% compounded monthly, you cannot say how much $100 will grow in a year without a short calculation. In the second case, the *actual* percentage increase will be more than 9%.

[5] When an effective interest rate is quoted or calculated, the compounding frequency does not need to be specified. Everyone understands from the definition of effective interest rate that "effective" implies "annual compounding."

TIP Intuitive Approach for Calculating *f*

Note in the preceding example that the effective interest rate (10.25%) is numerically equal to the actual amount of interest ($10.25) that $100 will earn in one year at the given nominal rate. This is a general result for all nominal interest rates. We can use this idea for our financial calculator method for determining *f*. That is, we can calculate the future value of $100 after one year at the given nominal rate. Then we can just inspect the future value to see the amount of interest earned (and thereby identify the value of *f*.)

We can readily derive a formula for f. Suppose you invest $100 for one year at the effective rate f (compounded annually) and another $100 for one year at the nominal rate $j = mi$. Their future values are calculated in parallel columns below.

The first $100 will undergo just one compounding of the effective rate f.	The second $100 will undergo m compoundings of the periodic rate i.
$FV = PV(1+i)^n$ $= \$100(1+f)^1$	$FV = PV(1+i)^n$ $= \$100(1+i)^m$

For f to be equivalent to the nominal rate j, these future values must be equal. That is,

$$\$100(1+f) = \$100(1+i)^m$$

$$1 + f = (1+i)^m$$

EFFECTIVE INTEREST RATE

$$f = (1+i)^m - 1 \tag{9-3}$$

TIP Comparing Nominal Rates of Interest

To compare two nominal rates of interest, convert each to its effective interest rate. Then you can directly compare the effective rates, and thereby rank the given nominal rates.

EXAMPLE 9.3A | CONVERTING A NOMINAL INTEREST RATE TO AN EFFECTIVE INTEREST RATE

What is the effective rate of interest corresponding to 10.5% compounded monthly?

SOLUTION

Given: $j = 10.5\%$ and $m = 12$

For the financial calculator solution, we will use the intuitive approach described in the first TIP above.

Then $i = \frac{j}{m} = \frac{10.5\%}{12} = 0.875\%$ per month and

$$\begin{aligned} f &= (1+i)^m - 1 \\ &= (1.00875)^{12} - 1 \\ &= 1.11020 - 1 \\ &= 0.11020 \\ &= 11.02\% \end{aligned}$$

The effective interest rate is 11.02% (compounded annually).

10.5 I/Y

P/Y 12 ENTER

(making $C/Y = P/Y = 12$)

12 N

100 +/− PV

0 PMT

CPT FV

Ans: $1\underbrace{11.020}_{f}$

EXAMPLE 9.3B | COMPARING ALTERNATIVE NOMINAL INTEREST RATES

Which is the most attractive of the following interest rates offered on five-year GICs?

a. 5.70% compounded annually
b. 5.68% compounded semiannually
c. 5.66% compounded quarterly
d. 5.64% compounded monthly

SOLUTION

The preferred rate is the one having the highest effective rate. The algebraic calculations of the effective rates are presented in the table below.

j	m	i	$f = (1 + i)^m - 1$
a. 5.70%	1	0.057	$f = j$ when $m = 1$; $f = 5.700\%$
b. 5.68%	2	0.0284	$f = (1.0284)^2 - 1 = 0.05761 = 5.761\%$
c. 5.66%	4	0.01415	$f = (1.01415)^4 - 1 = 0.05781 = 5.781\%$
d. 5.64%	12	0.0047	$f = (1.0047)^{12} - 1 = 0.05788 = 5.788\%$

In Appendix 9A, we describe and demonstrate the use of the Texas Instruments BA II PLUS's Interest Conversion Worksheet (ICONV). Let us now use this worksheet to calculate the effective interest rates in Parts (b), (c), and (d).

Part (b)

2nd ICONV
5.68 ENTER
↓ ↓ 2 ENTER
↑ CPT
Ans: 5.761

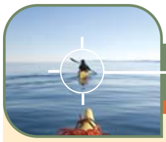
Part (c)

Press ↑ (to scroll back to the *NOM* line.)
5.66 ENTER
↓ ↓ 4 ENTER
↑ CPT
Ans: 5.781

Part (d)

Press ↑ (to scroll back to the *NOM* line.)
5.64 ENTER
↓ ↓ 12 ENTER
↑ CPT
Ans: 5.788

The most attractive rate is 5.64% compounded monthly since it has the highest effective rate. For the alternative rates in this example, the ranking in terms of effective rates is in the reverse order of the nominal rates.



POINT OF INTEREST

Not in Your Best Interest

The Canada Revenue Agency (CRA) charges interest on overdue income tax, Canada Pension Plan contributions, and Employment Insurance Premiums. The prescribed rate is adjusted by the CRA every 3 months based on changes in the Bank of Canada rate. For the second quarter of 2007, the prescribed nominal rate was 9%.

You now know that the compounding frequency matters. The more frequently a given nominal rate is compounded, the higher the effective rate of interest. Considering who is setting the rate in this case, you would probably guess that the prescribed rate is compounded monthly (the highest compounding frequency normally encountered in commerce). Not so! But before you think the CRA has some compassion after all, think again—the prescribed rate is compounded daily!

Questions

1. Calculate the effective rate of interest corresponding to 9% compounded daily.
2. How much more interest would accrue on a \$1000 debt in a year at 9% compounded daily than at 7% compounded monthly?

EXAMPLE 9.3C | FINDING THE EFFECTIVE RATE GIVEN THE PRINCIPAL AND MATURITY VALUE

Calculate the effective rate of interest if \$100 grew to \$150 in $3\frac{1}{2}$ years with quarterly compounding.

SOLUTION

The problem could be solved by first finding the quarterly compounded nominal rate that produces the given maturity value. Then the corresponding effective rate could be calculated. But this two-step solution is unnecessarily long.

The essential question (which may be answered in one step) is: At what annually compounded rate will \$100 grow to \$150 after $3\frac{1}{2}$ years?

With $PV = \$100$, $FV = \$150$, $m = 1$, and $n = 3.5$, formula (9-1) gives

$$\left.\begin{aligned} i &= \left(\frac{FV}{PV}\right)^{1/n} - 1 \\ &= \left(\frac{\$150}{\$100}\right)^{1/3.5} - 1 \\ &= 1.5^{0.28571} - 1 \\ &= 0.1228 \end{aligned}\right\}$$

P/Y 1 ENTER
(making $C/Y = P/Y = 1$)
3.5 N
100 +/− PV
0 PMT
150 FV
CPT I/Y
Ans: 12.28

Since \$100 will grow to \$150 in $3\frac{1}{2}$ years at 12.28% compounded annually, the effective interest rate is 12.28%.

TIP Clarification of Terminology

Be clear on the distinction between the descriptions "compounded semiannually" and "per half year." The former refers to the compounding *frequency*. The latter refers to the compounding *period*. For example, if you hear or read "6% compounded semiannually," you are being given the value for "*j*." Then $i = j/m = 6\%/2 = 3\%$ (per half year). On the other hand, if an interest rate is described as "6% per half year," you are being given the value for "*i*" and the period to which it applies. Then $j = mi = 2i = 12\%$ compounded semiannually.

EXAMPLE 9.3D | CALCULATING THE EFFECTIVE INTEREST RATE ON A CHARGE CARD

A department store credit card quotes a rate of 1.75% per month on the unpaid balance. Calculate the effective rate of interest being charged.

SOLUTION

Since accrued interest is paid or converted to principal each month, we have monthly compounding with $i = 1.75\%$ per month, $m = 12$, and $j = mi = 12(1.75\%) = 21\%$ compounded monthly.

Therefore,

$$\begin{aligned} f &= (1 + i)^m - 1 \\ &= (1.0175)^{12} - 1 \\ &= 0.23144 \\ &= 23.14\% \end{aligned}$$

2nd ICONV
21 ENTER
↓ ↓ 12 ENTER
↑ CPT
Ans: 23.14

The effective rate on the credit card is 23.14%.

EXAMPLE 9.3E | CONVERTING AN EFFECTIVE INTEREST RATE TO A NOMINAL INTEREST RATE

What monthly compounded (nominal) rate of interest has an effective rate of 10%?

SOLUTION

Given: $f = 10\%$ $m = 12$

Substitute these values into formula (9-3) and solve for i.

$$\begin{aligned} f &= (1 + i)^m - 1 \\ 0.10 &= (1 + i)^{12} - 1 \\ 1.10 &= (1 + i)^{12} \end{aligned}$$

Now use the rule that

If $x^m = a$, then $x = a^{1/m}$

Therefore, $1.1^{1/12} = 1 + i$

$$\begin{aligned} 1 + i &= 1.1^{0.08\overline{3}} \\ i &= 1.007974 - 1 \\ &= 0.007974 \\ &= 0.7974\% \end{aligned}$$

2nd ICONV
↓ 10 ENTER
↓ 12 ENTER
↓ CPT
Ans: 9.57

Then $j = mi = 12(0.7974\%) = 9.57\%$ compounded monthly.

EXAMPLE 9.3F | CONVERTING AN INTEREST RATE FROM NOMINAL TO EFFECTIVE AND BACK TO NOMINAL

The department store mentioned in Example 9.3D has been charging 1.75% per month on its credit card. In response to lower prevailing interest rates, the Board of Directors has agreed to reduce the card's effective interest rate by 4%. To the nearest 0.01%, what will be the new periodic rate (per month)?

SOLUTION

To solve this problem, we must first calculate the effective rate corresponding to $i = 1.75\%$ per month. Then we must calculate the new monthly compounded rate whose effective rate is 4% lower.

Step 1: See the solution to Example 9.3D for this calculation. We obtained $f = 23.14\%$.

Step 2: The new effective rate must be $23.14\% - 4\% = 19.14\%$.

Substitute $f = 19.14\%$ and $m = 12$ into formula (9-3).

$$\begin{aligned} f &= (1+i)^m - 1 \\ 0.1914 &= (1+i)^{12} - 1 \\ 1.1914 &= (1+i)^{12} \\ 1 + i &= 1.1914^{1/12} \\ i &= 1.1914^{0.08\overline{3}} - 1 \\ &= 1.01470 - 1 \\ &= 0.01470 \\ &= 1.47\% \end{aligned}$$

2nd ICONV

↓ 19.14 ENTER

↓ 12 ENTER

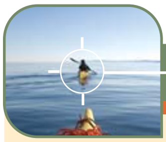
↓ CPT

Ans: 17.641

The new periodic rate will be

$$i = \frac{j}{m} = \frac{17.641\%}{12} = 1.47\%$$

The new periodic interest rate on the credit card will be 1.47% per month.



POINT OF INTEREST

Payday-Loan Operations Receive Unwanted Interest

Since 1990, storefront cheque-cashing and payday-loan businesses have proliferated in Canada. There are many franchise operations with names like Money Mart, Stop 'N' Cash, Premier Cash Advance, Money Sense, The Cash Store, and A OK Payday Loans. In a 2005 survey, the Financial Consumer Agency of Canada found that 7% of Canadians have used a cheque-cashing or payday-loan company.

In a typical payday-loan transaction, the amount of the loan can be up to 30% of the borrower's net pay, which must be verified by a recent pay stub. Payday lenders do not require credit checks or collateral. The loan may be advanced on the spot if the borrower provides proof of:

- Identification (by a suitable photo ID)
- Residence (by a recent utility bill)
- An active chequing account (by a bank statement for the preceding month)

The interest rate charged on payday loans is high but less than the 60% effective rate established as a "criminal rate of interest" by Section 347 of the Criminal Code. Payday lenders also apply other charges or "item fees" in addition to interest on the loan. For example, one operation's Loan Agreement specified the following charges:

- Interest "at the effective rate of 59% per annum."
- A "first-party cheque-cashing fee of 4.99% of the amount of the loan plus interest."
- A "$12.99 per item fee."

At the time of receiving a loan, the borrower provides a cheque (post-dated to the next payday) for the amount needed to repay the loan.

Questions

1. What periodic rate for one week is equivalent to an effective rate of 59%? (Assume there are exactly 52 weeks in a year.)
2. Under the terms of the Loan Agreement, what would the total interest and other charges be for a one-week loan of $300? (Note that the cheque-cashing fee is 4.99% of the *combined* principal and interest.)
3. Treating all charges as interest, what is the overall effective rate of interest?

Since the beginning of 2003, class action lawsuits have been filed against most payday-loan operations. The Statements of Claim typically make the case that the effective rates of interest charged on payday loans exceed the 60% limit set by the Criminal Code. "Interest" has a broad definition in the Criminal Code—it includes other charges and fees associated with a loan.

4. Instead of charging the sort of "item fees" previously listed, Stop 'N' Cash charged a 48% effective interest rate plus an insurance premium to insure the loan against death or disability of the customer. The insurance premium was 24.75% of the money advanced! (The lawyers filing a class action against Stop 'N' Cash in Ontario claim that "the alleged insurance company is not licensed to conduct business in Ontario … and is not in good standing in the country of its incorporation, Barbados.") Treating the insurance premium as interest, what is the overall effective rate of interest on a $400 loan for two weeks? (Assume there are 52 weeks in a year.)
5. August of 2006 brought the first class-action civil ruling in Canada regarding payday loan companies. Justice Brenda Brown of the BC Supreme Court ruled that the processing fees and late fees charged by A OK Payday Loans were effectively interest and, consequently, A OK was charging a criminal rate of interest. A OK Payday Loans was ordered to pay back everything it had charged customers in excess of an effective rate of 60%.

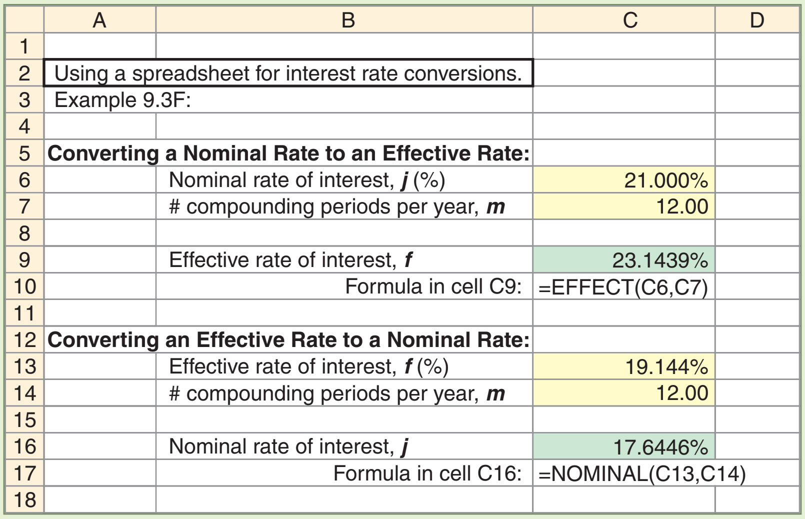
   A OK's practice had been to charge a 21% effective rate of interest plus a processing fee of $9.50 for every $50 borrowed. Treating the processing fee as additional interest, what effective rate of interest did A OK charge on a $400 loan for two weeks? (Assume there are 52 weeks in a year.) Under the judge's ruling, how much must A OK refund to that customer?

SPREADSHEET STRATEGIES | Interest Rate Conversion

The Online Learning Centre (OLC) provides a partially completed template for (1) converting a nominal rate of interest to an effective rate and (2) converting an effective rate to a nominal rate. Go to the Student Edition of the OLC, select the "Excel Templates" link in the navigation bar, and then click on the "Interest Rate Conversion" link. The "Introduction" page of the workbook describes Excel's built-in EFFECT and NOMINAL functions.

The main features of a completed Interest Rate Conversion template are presented below. The formulas programmed into cells C9 and C16 are displayed in cells C10 and C17, respectively. Data are entered to solve Example 9.3F. In this example, a department store has been charging 1.75% per month on its credit card. The Board of Directors have decided to reduce the effective annual rate on the card by 4%. The question asks us to determine the new periodic interest rate (per month).

Enter the values for $j = 12(1.75\%) = 21.0\%$ and $m = 12$ into cells C6 and C7. The effective rate immediately appears in C9. Next we reduce the effective rate to 23.144% − 4% = 19.144% and enter this new rate into C13. Also enter $m = 12$ in C14. The new nominal rate appears in C16.



	A	B	C	D
1				
2	Using a spreadsheet for interest rate conversions.			
3	Example 9.3F:			
4				
5	**Converting a Nominal Rate to an Effective Rate:**			
6		Nominal rate of interest, ***j*** (%)	21.000%	
7		# compounding periods per year, ***m***	12.00	
8				
9		Effective rate of interest, ***f***	23.1439%	
10		Formula in cell C9:	=EFFECT(C6,C7)	
11				
12	**Converting an Effective Rate to a Nominal Rate:**			
13		Effective rate of interest, ***f*** (%)	19.144%	
14		# compounding periods per year, ***m***	12.00	
15				
16		Nominal rate of interest, ***j***	17.6446%	
17		Formula in cell C16:	=NOMINAL(C13,C14)	
18				

It remains only to calculate the new periodic rate: $i = \frac{j}{m} = \frac{17.6446\%}{12} = 1.47\%$ per month.

CONCEPT QUESTIONS

1. What is meant by the effective rate of interest?
2. Is the effective rate of interest ever smaller than the nominal interest rate? Explain.
3. Is the effective rate of interest ever equal to the nominal interest rate? Explain.
4. A semiannually compounded nominal rate and a monthly compounded nominal rate have the same effective rate. Which has the larger nominal rate? Explain.
5. From a lender's point of view, would you rather disclose to borrowers the nominal interest rate or the effective interest rate?

EXERCISE 9.3

Spreadsheet template: *A partially completed Excel template for converting interest rates is provided in the Online Learning Centre (OLC). Go to the Student Edition of the OLC, click on the "Excel Templates" link in the navigation bar, and then click on the "Interest Rate Conversion" link. The completed template may be used throughout Exercise 9.3.*

Answers to the odd-numbered problems are at the end of the book.

Calculate interest rates and growth rates accurate to the nearest 0.01%.

Calculate the missing interest rates in Problems 1 through 20.

Problem	Nominal rate and compounding frequency	Effective rate (%)
1.	10.5% compounded semiannually	?
2.	10.5% compounded quarterly	?
3.	10.5% compounded monthly	?
4.	7.5% compounded semiannually	?
5.	7.5% compounded quarterly	?
6.	7.5% compounded monthly	?
7.	?% compounded semiannually	10.5
8.	?% compounded quarterly	10.5
9.	?% compounded monthly	10.5
10.	?% compounded semiannually	7.5
11.	?% compounded quarterly	7.5
12.	?% compounded monthly	7.5
13.	12% compounded monthly	?
14.	18% compounded monthly	?
15.	11.5% compounded quarterly	?
16.	9.9% compounded semiannually	?
17.	?% compounded semiannually	10.25
18.	?% compounded quarterly	7
19.	?% compounded monthly	10
20.	?% compounded monthly	8

21. What is the effective interest rate corresponding to a nominal annual rate of:

- **a.** 9% compounded semiannually?
- **b.** 9% compounded quarterly?
- **c.** 9% compounded monthly?

22. What is the effective rate of interest on a credit card that calculates interest at the rate of 1.8% per month?

23. What is the effective rate of interest corresponding to a nominal rate of:

- **a.** 8% compounded semiannually?
- **b.** 12% compounded quarterly?

24. If an invoice indicates that interest at the rate of 2% per month will be charged on overdue amounts, what effective rate of interest will be charged?

25. If the nominal rate of interest paid on a savings account is 2% compounded monthly, what is the effective rate of interest?

26. A company reports that its sales have grown 3% per quarter for the last eight fiscal quarters. What annual growth rate has the company been experiencing for the last two years?

27. If a $5000 investment grew to $6450 in 30 months of monthly compounding, what effective rate of return was the investment earning?

28. After 27 months of quarterly compounding, a $3000 debt had grown to $3810. What effective rate of interest was being charged on the debt?

29. Lisa is offered a loan from a bank at 7.2% compounded monthly. A credit union offers similar terms, but at a rate of 7.4% compounded semiannually. Which loan should she accept? Present calculations that support your answer.

30. Craig can buy a three-year compound-interest GIC paying 4.6% compounded semiannually or 4.5% compounded monthly. Which option should he choose? Present calculations that support your answer.

31. Camille can obtain a residential mortgage loan from a bank at 6.5% compounded semiannually, or from an independent mortgage broker at 6.4% compounded monthly. Which source should she pick if other terms and conditions of the loan are the same? Present calculations that support your answer.

32. ABC Ltd. reports that its sales are growing at the rate of 1.3% per month. DEF Inc. reports sales increasing by 4% each quarter. What is each company's effective annual rate of sales growth?

33. Columbia Trust wants its annually, semiannually, and monthly compounded five-year GICs all to have an effective interest rate of 5.75%. What nominal annual rates should it quote for the three compounding options?

34. Belleville Credit Union has established interest rates on its three-year GICs so that the effective rate of interest is 7% on all three compounding options. What are the monthly, semiannually, and annually compounded rates?

•35. A department store chain currently charges 18% compounded monthly on its credit card. To what amount should it set the monthly compounded annual rate if it wants to add 2% to the effective interest rate?

•36. An oil company wants to drop the effective rate of interest on its credit card by 3%. If it currently charges a periodic rate of 1.7% per month, at what amount should it set the periodic rate?

9.4 EQUIVALENT INTEREST RATES

Preamble The main purpose of this section is to prepare you for a routine calculation you will carry out for a broad category of annuities in Chapters 10, 11, 12, and 14. The concept behind the calculation is developed here because it is an extension of ideas from Section 9.3.

Equivalent interest rates are interest rates that produce the *same* future value after one year. For example, 8% compounded quarterly and 8.08% compounded semiannually are equivalent *nominal* interest rates. If you calculate the future value of $100 invested at either rate for one year, you will obtain $108.24. You can see that equivalent interest rates have *different numerical values* but produce the *same effect.*

If *nominal* rates are equivalent, so also are their respective *periodic* rates. From the preceding example, we can conclude that:

$$i = \frac{8\%}{4} = 2\% \text{ per quarter} \qquad \text{is } \textit{equivalent} \text{ to} \qquad i = \frac{8.08\%}{2} = 4.04\% \text{ per half year}$$

They will both produce the same future value when compounded over a one-year term.

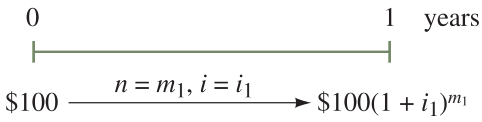
We want to be able to answer questions such as:

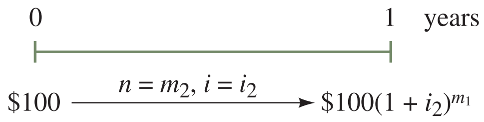
"What periodic rate per half year is equivalent to 2.5% per quarter?"

To answer this and similar questions, we will derive a formula that answers the general question:

"What i_2 with a given m_2 is equivalent to a given i_1 with a given m_1?"

For equivalence, $100 invested at each rate for one year must have the same future value. The two investments are shown in the following diagrams. Both future values are obtained using $FV = PV(1 + i)^n$.





We want to solve for the value of i_2 that makes the two future values equal. That is, solve for i_2 in

$$\$100(1 + i_2)^{m_2} = \$100(1 + i_1)^{m_1}$$
$$(1 + i_2)^{m_2} = (1 + i_1)^{m_1}$$

Divide both exponents by m_2, giving

$$1 + i_2 = (1 + i_1)^{m_1/m_2}$$

Hence,

EQUIVALENT PERIODIC RATE

$$i_2 = (1 + i_1)^{m_1/m_2} - 1 \qquad \textbf{(9-4)}$$

To answer the question: "What periodic rate per half year is equivalent to 2.5% per quarter?" substitute $m_2 = 2$, $i_1 = 2.5\% = 0.025$, and $m_1 = 4$ into formula (9-4).

$$i_2 = (1 + i_1)^{m_1/m_2} - 1 = (1.025)^{4/2} - 1 = 1.025^2 - 1 = 0.050625 = 5.0625\% \text{ per half year}$$

EXAMPLE 9.4A | CALCULATION OF THREE EQUIVALENT INTEREST RATES

For a given interest rate of 10% compounded quarterly, what is the equivalent nominal rate of interest with:

a. Annual compounding? **b.** Semiannual compounding? **c.** Monthly compounding?

SOLUTION

The given rate is $j_1 = 10\%$ with $m_1 = 4$. Therefore, $i_1 = 2.5\%$ per quarter.

In the following columns, we substitute the given values for m_1, m_2, and i_1 into formula (9-4).

a. $m_2 = 1$

$$i_2 = (1.025)^{4/1} - 1$$
$$= 0.10381$$
$$= 10.381\% \text{ per year}$$

$$j_2 = m_2 \times i_2$$
$$= 1 \times 10.381\%$$
$$= 10.381\% \text{ compounded annually}$$

b. $m_2 = 2$

$$i_2 = (1.025)^{4/2} - 1$$
$$= 0.050625$$
$$= 5.0625\% \text{ per half-year}$$

$$j_2 = m_2 \times i_2$$
$$= 2 \times 5.0625\%$$
$$= 10.125\% \text{ compounded semiannually}$$

c. $m_2 = 12$

$$i_2 = (1.025)^{4/12} - 1$$
$$= 0.0082648$$
$$= 0.82648\% \text{ per month}$$

$$j_2 = m_2 \times i_2$$
$$= 12 \times 0.82648\%$$
$$= 9.918\% \text{ compounded monthly}$$

To use the ICONV worksheet, first compute the effective rate corresponding to the given nominal interest rate. Then compute the requested nominal rates that are equivalent to this effective rate.

Part (a)

2nd ICONV
10 ENTER
↓ ↓ 4 ENTER
↑ CPT
Ans: 10.381

Part (b)

Press ↓ (to scroll
down to the *C/Y* line.)
2 ENTER
↑ ↑ CPT
Ans: 10.125

Part (c)

Press ↓ ↓ (to scroll
down to the *C/Y* line.)
12 ENTER
↑ ↑ CPT
Ans: 9.918

a. $j = f = 10.381\%$ compounded annually

b. $j = 10.125\%$ compounded semiannually

c. $j = 9.918\%$ compounded monthly

CONCEPT QUESTIONS

1. What is the significance of two nominal interest rates being equivalent?
2. Suppose the periodic rate for six months is 4%. Is the equivalent periodic rate for three months (pick one):
   (i) equal to $4\% \times \frac{3}{6} = 2\%$?
   (ii) less than 2%?
   (iii) greater than 2%?
   Answer the question without doing any calculations. Explain your choice.
3. Suppose the periodic rate for one month is 0.5%. Is the equivalent periodic rate for six months (pick one):
   (i) equal to $6(0.5\%) = 3\%$?
   (ii) less than 3%?
   (iii) greater than 3%?
   Answer the question without doing calculations. Explain your choice.

EXERCISE 9.4

Answers to the odd-numbered problems are at the end of the book.

Throughout this Exercise, calculate interest rates accurate to the nearest 0.01%.
Calculate the equivalent interest rates in Problems 1 through 12.

Problem	Given interest rate	Equivalent interest rate
1.	10% compounded annually	?% compounded semiannually
2.	10% compounded annually	?% compounded quarterly
3.	10% compounded annually	?% compounded monthly
4.	10% compounded semiannually	?% compounded annually
5.	10% compounded semiannually	?% compounded quarterly
6.	10% compounded semiannually	?% compounded monthly
7.	10% compounded quarterly	?% compounded annually
8.	10% compounded quarterly	?% compounded semiannually
9.	10% compounded quarterly	?% compounded monthly
10.	10% compounded monthly	?% compounded annually
11.	10% compounded monthly	?% compounded semiannually
12.	10% compounded monthly	?% compounded quarterly

Calculate the equivalent interest rates in Problems 13 through 20.

Problem	Given interest rate	Equivalent interest rate
13.	9% compounded semiannually	?% compounded annually
14.	10% compounded quarterly	?% compounded annually
15.	8.25% compounded annually	?% compounded monthly
16.	4% compounded monthly	?% compounded semiannually
17.	7.5% compounded semiannually	?% compounded quarterly
18.	6% compounded quarterly	?% compounded monthly
19.	8.5% compounded quarterly	?% compounded semiannually
20.	10.5% compounded monthly	?% compounded quarterly

21. What annually compounded interest rate is equivalent to 6% compounded:
a. semiannually? **b.** quarterly? **c.** monthly?

22. What monthly compounded interest rate is equivalent to 6% compounded:
a. annually? **b.** semiannually? **c.** quarterly?

23. For a three-year GIC investment, what nominal rate compounded monthly would put you in the same financial position as 5.5% compounded semiannually?

24. A trust company pays 5.5% compounded semiannually on its three-year GIC. For you to prefer an annually compounded GIC of the same maturity, what value must its nominal interest rate exceed?

25. You are offered a loan at a rate of 9% compounded monthly. Below what nominal rate of interest would you choose semiannual compounding instead?

26. Banks usually quote residential mortgage interest rates on the basis of semiannual compounding. An independent mortgage broker is quoting rates with monthly compounding. What rate would the broker have to give to match 6.5% compounded semiannually available from a bank?

27. A credit union pays 5.25% compounded annually on five-year compound-interest GICs. It wants to set the rates on its semiannually and monthly compounded GICs of the same maturity so that investors will earn the same total interest. What should the rates be on the GICs with the higher compounding frequencies?

28. A bank offers a rate of 5.0% compounded semiannually on its four-year GIC. What monthly compounded rate should the bank offer on four-year GICs to make investors indifferent between the alternatives?

9.5 INVESTMENT RETURNS FROM STOCKS AND MUTUAL FUNDS

There are two ways you can benefit from an investment. The first is by receiving **income**—money you receive without selling any part of the investment. Aside from any income it generates, an investment may grow in value. The increase in value of the investment is called the **capital gain.** The forms taken by the income and capital gain from various types of investments are presented in Table 9.2.

TABLE 9.2 Forms of Income and Capital Gain From Various Types of Investments

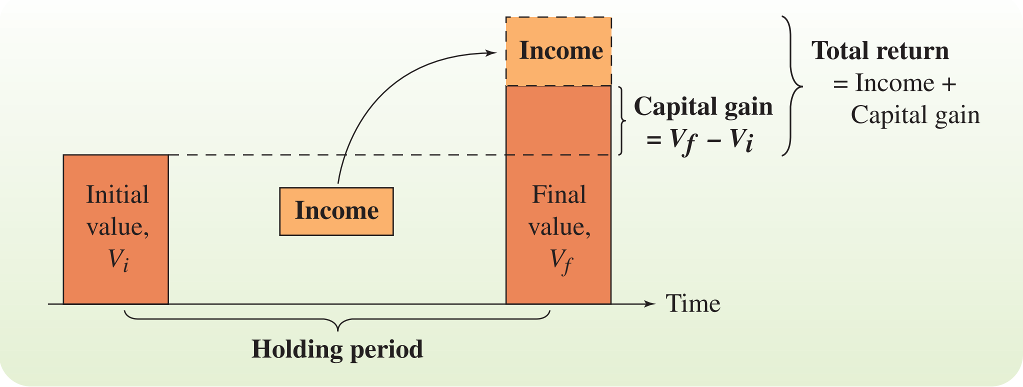
Investment	Income	Capital gain
Stocks (or shares)	Dividends	Change in share price
Canada Savings Bonds (Sec. 8.5)	Interest	None (Redeemable at purchase price)
Marketable bonds (Chapter 15)	Interest	Change in bond price
Mutual fund units	Distributions	Change in unit value
Gold bullion	None	Change in market price of gold
GICs (Sec. 8.5)	Interest	None (Redeemable at purchase price)

A typical investment scenario is represented in Figure 9.1. The initial value (or beginning value) of the investment is V_i. In finance, the term **holding period** is used for the time interval over which we are calculating the income and capital gain. Income may be received from the investment at one or more points during the holding period. The final value of the investment at the end of the holding period is V_f. The capital gain is the increase in value, $V_f - V_i$. (If the value of an investment declines during the holding period, the capital gain will be negative, indicating a **capital loss.**) The sum of the income and capital gain from the investment is called the **total return.** In summary,

$$\text{Capital gain} = \text{Final value}\,(V_f) - \text{Initial value}\,(V_i)$$

$$\text{Total return} = \text{Income} + \text{Capital gain}$$

FIGURE 9.1 The Components of Investment Returns



The distinction between the "income" and "capital gain" components of the "total return" is important for two reasons. One is that "income" represents actual cash inflow that can be used by the investor to cover living expenses. To convert the "capital gain" component into a usable cash inflow, the investor must first sell the investment. When the investment is sold, we say that "the capital gain is realized." (Up to that point it was an "unrealized capital gain," sometimes described in everyday language as a "paper gain.") The second reason for separating the "income" and "capital gain" components of "total return" is that capital gains are subject to a lower rate of income tax than investment income.

The income, capital gain, and total return amounts are all in terms of dollars. When discussing the performance of investments, investors prefer to express these amounts as a percentage of the initial value, V_i. Accordingly, we define:

INVESTMENT YIELDS AND RATE OF TOTAL RETURN

$$\left.\begin{aligned} \textit{Income yield} &= \frac{\textit{Income}}{V_i} \times 100\% \\ \textit{Capital gain yield} &= \frac{\textit{Capital gain}}{V_i} \times 100\% = \frac{V_f - V_i}{V_i} \times 100\% \\ \textit{Rate of total return} &= \textit{Income yield} + \textit{Capital gain yield} \\ &= \frac{\textit{Income} + \textit{Capital gain}}{V_i} \times 100\% \end{aligned}\right\} \quad \textbf{(9-5)}$$

In their advertising, mutual funds emphasize the rates of total return, but give little or no information about the income and capital gain components. They do so because (1) the rate of total return is the figure of greatest importance to investors, and (2) most investors in mutual funds do not understand how the income (distribution) and capital gain components of total return are determined for a mutual fund. When we used the term "rate of return" in connection with mutual funds in Section 9.1, we were really referring to "rate of *total* return." To get information about the income and capital gain components of a mutual fund's total return, you need to go to the mutual fund's annual financial report or to a mutual fund database such as globefund.com (mentioned in the NET @ssets box in Section 9.1).

TIP Watch for Key Words

The words "yield" and "rate" indicate amounts calculated as a percentage of the initial investment. Particular names may be used for specific types of income yield. For example, shareholders refer to a stock's "dividend yield," but bond investors speak of a bond's "current yield."

In everyday life, terms are sometimes used with less precision than in an academic environment. For example, "yield" may be dropped from "capital gain yield" when investors discuss capital gains.

Tables 9.3 and 9.4 provide price and income data for selected stocks and mutual funds. These data are used in Example 9.5A and Exercise 9.5.

TABLE 9.3 Income and Price Data for Selected Stocks

Company Name	Price ($) at the end of: 2004	2005	2006	Dividends ($) paid in: 2005	2006
TD Bank	46.72	58.95	68.71	1.64	1.84
Loblaw Companies Ltd.	69.68	55.23	48.58	0.84	0.84
Cameco Corporation	20.79	36.71	47.15	0.12	0.16
Petro-Canada	30.02	46.14	47.61	0.30	0.40
Research in Motion Ltd.	98.78	76.75	149.00	0.00	0.00

TABLE 9.4 Income and Price Data for Selected Mutual Funds

Name of mutual fund	Price ($) at the end of: 2004	2005	2006	Distribution ($) in: 2005	2006
PH&N Canadian Equity Fund (Series A)	66.73	79.26	91.26	0.11	0.35
Mawer New Canada Fund	32.71	38.46	42.11	0.15	2.22
Sprott Canadian Equity Fund	24.46	27.69	38.50	0.00	0.15
RBC Canadian Dividend Fund	35.61	42.76	48.17	0.33	1.02
PH&N Bond Fund (Series A)	9.99	10.00	9.95	0.63	0.42

EXAMPLE 9.5A | CALCULATING INVESTMENT RETURNS FROM SECURITIES

To the nearest 0.01%, calculate the income yield, capital gain yield, and rate of total return in each of 2005 and 2006 for TD Bank shares and for PH&N Canadian Equity Fund units. Use the data in Tables 9.3 and 9.4.

SOLUTION

Security	Income yield $\left(\frac{\text{Income}}{V_i} \times 100\%\right)$	Capital gain yield $\left(\frac{V_f - V_i}{V_i} \times 100\%\right)$	Rate of total return (Income yield + Capital gain yield)
TD Bank shares (2005)	$\frac{\$1.64}{\$46.72} \times 100\%$ $= 3.51\%$	$\frac{\$58.95 - \$46.72}{\$46.72} \times 100\%$ $= 26.18\%$	$3.51\% + 26.18\%$ $= 29.69\%$
TD Bank shares (2006)	$\frac{\$1.84}{\$58.95} \times 100\%$ $= 3.12\%$	$\frac{\$68.71 - \$58.95}{\$58.95} \times 100\%$ $= 16.56\%$	$3.12\% + 16.56\%$ $= 19.68\%$
PH&N Cdn. Equity (2005)	$\frac{\$0.11}{\$66.73} \times 100\%$ $= 0.16\%$	$\frac{\$79.26 - \$66.73}{\$66.73} \times 100\%$ $= 18.78\%$	$0.16\% + 18.78\%$ $= 18.94\%$
PH&N Cdn. Equity (2006)	$\frac{\$0.35}{\$79.26} \times 100\%$ $= 0.44\%$	$\frac{\$91.26 - \$79.26}{\$79.26} \times 100\%$ $= 15.14\%$	$0.44\% + 15.14\%$ $= 15.58\%$

EXAMPLE 9.5B | CALCULATING V_f GIVEN V_i, INCOME, AND RATE OF TOTAL RETURN

An investor is prepared to buy common shares of Eagle Brewing Ltd. at the current share price of $11.50 if he can expect at least a 15% rate of total return over the next year. Assuming that the company repeats last year's $0.40 per share dividend, what will the minimum share price have to be one year from now for the investment objective to be achieved?

SOLUTION

Given: $V_i = \$11.50$ Rate of total return = 15% Income = $0.40

We want to determine what V_f must be one year from now for the rate of total return to be 15%. If the annual dividend is $0.40 per share,

$$\text{Income yield} = \frac{\$0.40}{\$11.50} \times 100\% = 3.478\%$$

The rest of the 15% rate of total return must come from capital gain. That is,

$$\text{Capital gain yield} = 15\% - 3.478\% = 11.522\%$$

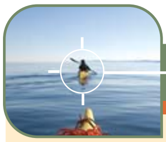
Hence,

$$\text{Capital gain} = 0.11522 \times V_i = 0.11522 \times \$11.50 = \$1.33 \text{ per share}$$

and

$$V_f = V_i + \text{Capital gain} = \$11.50 + \$1.33 = \$12.83$$

The share price must be at least $12.83 one year from now for the investor to achieve the minimum desired rate of total return (15%).



POINT OF INTEREST

False Profits and Bull Marketing

The following tale reveals how little scrutiny is given to claims of investment performance. It also exposes our tendency to suspend critical analysis in the face of a good story.

In 1994 Hyperion published *The Beardstown Ladies' Common-Sense Investment Guide*. The book describes the homespun stock-picking methods of an investment club of sixteen women living in Beardstown, Illinois (population 6000). We are told they range in age from 41 to 87. According to the book's introduction, "their hand-picked portfolio of fewer than 20 stocks has earned an average annual return of 23.4%" over the preceding 10 years (1984–1993). This return was almost twice the rate of return earned by the benchmark Standard & Poor's 500 (S&P 500) portfolio, and more than twice the average annual return achieved by professional mutual fund managers! Naturally the story found great appeal among the general public — here was a group of savvy septuagenarians emerging from the Illinois cornfields to trounce Wall Street's overpaid MBAs. (The book also contained the ladies' favourite recipes, such as Ann's Kentucky Cream Cake and Helen's Springtime Pie.)

The Beardstown Ladies became celebrities almost overnight. The book was at the top of the *New York Times* bestseller list for three months and eventually sold over 800,000 copies in seven languages. The Beardstown Ladies were the subject of articles in scores of publications and appeared as guests on several television talk shows. Four more books followed in rapid succession. Along the way, the geriatric gurus produced their own home video titled *Cookin' up Profits on Wall Street*. After reviewing the video, the late great movie critic Gene Siskel wrote what proved to be a prescient one-liner: "Wait till you see what these ladies are up to!"

In early 1998, Shane Tristch, a managing editor at *Chicago Magazine*, started to write yet another warm fuzzy story about the lovable ladies from Beardstown. However, he was troubled by the following disclaimer which appeared in the front material of the Beardstown Ladies' first book.

> *"Investment clubs commonly compute their annual 'return' by calculating the increase in their total club balance over a period of time. Since this increase includes the dues that the members pay regularly, this 'return' may be different from the return that might be calculated for a mutual fund or a bank. Since the regular contributions are an important part of the club philosophy, the Ladies' returns described in this book are based on this common calculation."*

The "dues" refer to regular contributions of new money that most investment clubs require from each member. In the case of the Beardstown Ladies, each member contributed $25 per month right from the start in 1984. Anyone with a basic knowledge of investing should have an "Excuse me?" moment upon reading the disclaimer. It is preposterous to treat new injections of investment capital as part of the total return from a portfolio! Frankly, it is highly doubtful that this method of calculating returns is "commonly" used by investment clubs.

Tristch wrote an article for the March 1998 issue of *Chicago Magazine* exposing the flaw and challenging the claim of a 23.4% average annual return. The Beardstown Ladies allowed the international accounting firm Price Waterhouse to audit their records. Instead of the 23.4% average annual return, Price Waterhouse determined that the average annual rate of return was a modest 9.1%. This was far short of the publicized 23.4% return and well short of the 14.9% annual return on the unmanaged S&P 500 portfolio. The Ladies beat the S&P 500 in only two of the 10 years. The Beardstown Ladies and their publisher had built an empire based on a profoundly flawed calculation that went unchallenged by over 800,000 readers for four years!

Compounding Rates of Return Suppose you read that a stock's price rose by 10% in Year 1 and 20% in Year 2. Does this mean that the overall price increase for the two-year period was 10% + 20% = 30%? No—the understanding is that, in Year 2, the price rose 20% from its value at the *end* of Year 1. In other words, the understanding is that you should *compound* the returns given for successive periods. This corresponds to what you did in Section 8.4 with the interest rates earned in successive periods by GICs and Canada Savings Bonds. (For these investments, the interest rate equals the rate of *total* return each year because GICs and CSBs do not experience capital gains or losses.) We can use

$$FV = PV(1 + i_1)(1 + i_2)(1 + i_3)\ldots(1 + i_n) \tag{8-4}$$

for compounding rates of total return. Simply view i_1, i_2, i_3, … , i_n as the rates of total return in each of the n successive periods.

EXAMPLE 9.5C | COMPOUNDING RATES OF TOTAL RETURN ON AN INVESTMENT

The China Fund is a mutual fund that can be purchased on the New York Stock Exchange (trading symbol CHN). As its name suggests, the fund invests in the stocks of companies in China. The investment research firm Morningstar Inc. reports that the rates of total return provided by the fund for individual years 2002 to 2006 were 12.4%, 215.6%, −9.3%, −21.6%, and 66.5%, respectively. (As these rates of return indicate, Chinese stocks are notoriously volatile.)

a. If you had invested $1000 in the fund at the beginning of 2002, what was your investment worth at the end of 2006?

b. What would have been the dollar amount of your total return in 2004?

SOLUTION

a. We can directly substitute the given values into formula (8-4).

$$\begin{aligned} FV &= PV(1+i_1)(1+i_2)(1+i_3)(1+i_4)(1+i_5) \\ &= \$1000(1+0.124)(1+2.156)[1+(-0.093)][1+(-0.216)](1+0.665) \\ &= \$1000(1.124)(3.156)(0.907)(0.784)(1.665) \\ &= \$4199.92 \end{aligned}$$

b. In 2004 the rate of total return was −9.3%. The dollar amount of the total return was

$$\begin{aligned} &-0.093 \times (\text{Value of the investment at the end of 2003}) \\ &\quad = -0.093 \times \$1000(1.124)(3.156) \\ &\quad = -\$329.90 \end{aligned}$$

The value of your investment would have fallen by $329.90 in 2004.

EXAMPLE 9.5D | COMPARING THE PERFORMANCE OF A MUTUAL FUND TO A BENCHMARK

The following table presents the rates of total return on the Phillips Hager & North (PH&N) Dividend Income mutual fund for each year from 2002 to 2006 inclusive. Corresponding figures are also given for the Toronto Stock Exchange's S&P/TSX Composite Total Return Index. (This Index measures the performance of a portfolio of the common shares of over 250 of the largest Canadian companies trading on the Toronto Stock Exchange. The index is often used as the benchmark for evaluating the performance of other portfolios of Canadian stocks.)

What is the difference between the overall rate of total return on the mutual fund and on the benchmark portfolio represented by the S&P/TSX Index?

	Rate of total return (%)				
Fund name	**2002**	**2003**	**2004**	**2005**	**2006**
PH&N Dividend Income Fund (Series A)	−7.2	24.9	14.5	13.6	16.5
S&P/TSX Composite Total Return Index	−12.4	26.7	14.5	24.1	17.3

SOLUTION

Suppose you invested an amount PV in the PH&N Dividend Income Fund at the beginning of 2002. By the end of 2006, the investment would have grown to

$$\begin{aligned} FV &= PV(1+i_1)(1+i_2)\,(1+i_3)(1+i_4)(1+i_5) \\ &= PV(1-0.072)(1+0.249)(1+0.145)(1+0.136)(1+0.165) \\ &= PV(1.756) \end{aligned}$$

Since the final value is 1.756 times the initial value, the investment in the mutual fund grew by a total of 75.6%. For the same initial investment in the portfolio represented by the S&P/TSX Composite Index,

$$\begin{aligned} FV &= PV(1+i_1)(1+i_2)(1+i_3)(1+i_4)(1+i_5) \\ &= PV(1-0.124)(1+0.267)(1+0.145)(1+0.241)(1+0.173) \\ &= PV(1.850) \end{aligned}$$

This portfolio grew by 85.0%. Therefore, the PH&N Dividend Income Fund grew by

$$85.0\% - 75.6\% = 9.4\%$$

less than the unmanaged S&P/TSX Index portfolio.

EXAMPLE 9.5E | CALCULATING ONE OF A SERIES OF PERCENT CHANGES

Hanako invested in a stock that doubled in the first year and rose another 50% in the second year. Now at the end of the third year, Hanako is surprised to discover that the stock's price is up only 80% from her purchase price. By what percentage did the stock's price change in the third year?

SOLUTION

The stock's price is up 80% over the entire three-year period. Therefore,

$$FV = 1.80PV$$

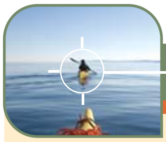
In terms of the capital gain yield in individual years,

$$\begin{aligned} FV &= PV(1+i_1)(1+i_2)(1+i_3) \\ &= PV(1+1.00)(1+0.50)(1+i_3) \\ &= PV(3.0)(1+i_3) \end{aligned}$$

For the same final value in both cases, we require

$$\begin{aligned} PV(1.80) &= PV(3.0)(1+i_3) \\ 1.80 &= 3.0(1+i_3) \\ \frac{1.80}{3.0} &= 1+i_3 \\ i_3 &= 0.60 - 1 = -0.40 = -40\% \end{aligned}$$

The stock's price declined 40% in Year 3.



POINT OF INTEREST

A 20% Gain Doesn't Offset a 20% Loss

Equal positive and negative rates of return in successive years have a curious lack of symmetry in their effects on the overall return from an investment. A 20% gain in Year 2 will not recoup a 20% loss in Year 1—you need a 25% gain to break even after a 20% loss!

Conversely, a 20% gain followed by a 20% loss also leaves you in a net loss position. A 20% loss will, in fact, erase an earlier 25% gain!

The *Globe and Mail's* Personal Finance writer Rob Carrick pointed out effects of this sort in an October 4, 2001 article titled "*In denial about your stocks? It's time to get real.*" Quoting from the article:

> *"Lose half your money and you need a 100% rise (to recover your losses.) Lose 60% and you need a 150% gain. To get back to even, a stock down 80% would have to surge 400%. A stock down 90%—hello Nortel—would have to rise 900% to get back to where you bought it."*

Question 1:

Verify Rob Carrick's numbers. That is, show the calculation of the percent gains required to break even after losses of 50%, 60%, 80%, and 90%.

When a series of annual returns are given for an investment, you may be tempted to calculate the average of the annual returns to obtain a measure of the overall long-term performance of the investment. At best, this average is only an approximation of the investment's annual rate of growth. At worst, it can give a very misleading result. We can dramatically illustrate this point using the rather extreme example of a 100% gain followed by a 50% loss in successive years. The simple average of a 100% gain and a 50% loss is $[100\% + (-50\%)] / 2 = 25\%$ per year. But if a \$1000 investment doubles to \$2000 (a 100% gain) and then loses half its value (50% loss), the investment will be right back at its beginning value of \$1000. The actual two-year gain was 0%! The average (25%) of the individual rates of return for the two years is meaningless!

You must, therefore, be very cautious about drawing conclusions based on a simple average of a series of annual rates of return. It will always *overstate* the actual performance. The more volatile the year-to-year rates of return, the greater will be the degree of overstatement.

Question 2:

For the ten years 1997 to 2006 inclusive, the AIM Global Technology Fund had rates of return of 37.9%, 34.5%, 200.3%, −27.9%, −43.8%, −48.9%, 16.2%, −5.2%, −4.2%, and 7.4%, respectively.

a. What was the average annual rate of return for the ten years?

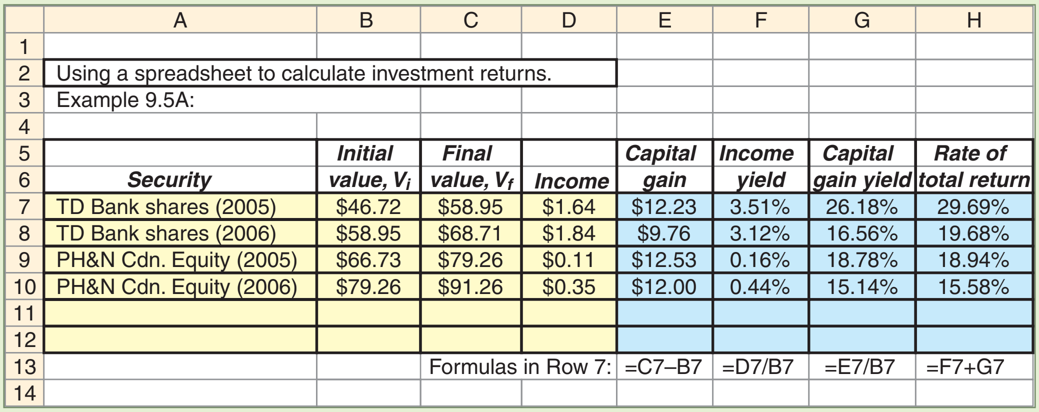
b. If a \$1000 investment earned this average rate of return each and every year for ten years, what was its final value?

c. If \$1000 was invested in the AIM Global Technology Fund at the beginning of 1997, what was the actual value of the investment at the end of 2006?

SPREADSHEET STRATEGIES | Calculating Investment Returns

The Online Learning Centre (OLC) presents a partially completed template for calculating the income yield, capital gain yield, and rate of total return for each of several investments, given their initial value, final value, and income for the holding period. Go to the Student Edition of the OLC, select the "Excel Templates" link in the navigation bar, and then click on the "Investment Returns" link.

The main features of the completed template are presented below. The formulas programmed into cells E7, F7, G7, and H7 are displayed in Row 13. Data are entered for answering Example 9.5A. In this example, we are to determine the income yield, capital gain yield, and rate of total return in both 2005 and 2006 for TD Bank shares and the PH&N Canadian Equity Fund based on data provided in Tables 9.3 and 9.4. After entering the given information for each investment in the yellow input cells, the answers appear in the blue output cells.



	A	B	C	D	E	F	G	H
1								
2	Using a spreadsheet to calculate investment returns.							
3	Example 9.5A:							
4								
5		***Initial***	***Final***		***Capital***	***Income***	***Capital***	***Rate of***
6	***Security***	***value, V_i***	***value, V_f***	***Income***	***gain***	***yield***	***gain yield***	***total return***
7	TD Bank shares (2005)	$46.72	$58.95	$1.64	$12.23	3.51%	26.18%	29.69%
8	TD Bank shares (2006)	$58.95	$68.71	$1.84	$9.76	3.12%	16.56%	19.68%
9	PH&N Cdn. Equity (2005)	$66.73	$79.26	$0.11	$12.53	0.16%	18.78%	18.94%
10	PH&N Cdn. Equity (2006)	$79.26	$91.26	$0.35	$12.00	0.44%	15.14%	15.58%
11								
12								
13			Formulas in Row 7:		=C7–B7	=D7/B7	=E7/B7	=F7+G7
14								

CONCEPT QUESTIONS

1. What is meant by a "capital loss?"
2. What is meant by the "total return" from an investment?
3. Can the income yield from an investment be negative? Explain or give an example.
4. Is it possible for the capital gain yield to exceed 100%? Explain or give an example.
5. Is it possible for a capital loss to be worse than −100%? Explain or give an example.
6. Does the combined effect of a 20% increase followed by a 20% decrease differ from the combined effect of a 20% decrease followed by a 20% increase? Justify your answer.
7. How much will an investment of $100 be worth after 20 years if it increases in value by 25% in each of 10 years, but declines by 20% in each of the other 10 years?
8. If a series of compound percent changes are all positive, is the overall percent increase larger or smaller than the sum of the individual percent changes? Justify your answer.
9. If a series of compound percent changes are all negative, is the overall percent decrease larger or smaller (in magnitude) than the sum of the individual percent changes? Justify your answer.

EXERCISE 9.5

Spreadsheet templates: *The "Investment Returns" template provided in the Online Learning Centre (OLC) may be used in Problems 1-8 and 17-26. The "Formula (8-4) Calculator" template in the OLC is helpful in answering Problems 29 and 42-46.*

Answers to the odd-numbered problems are at the end of the book.

Calculate the missing quantities in Problems 1 through 16. Determine yields and rates of return to the nearest 0.01%. Calculate dollar amounts accurate to the cent.

Problem	Initial value ($)	Income ($)	Final value($)	Income yield	Capital gain yield	Rate of total return
1.	100	10	110	?	?	?
2.	100	10	90	?	?	?
3.	90	10	86	?	?	?
4.	135	0	151	?	?	?
5.	1367	141	1141	?	?	?
6.	879	280	1539	?	?	?
7.	2500	200	0	?	?	?
8.	1380	250	2875	?	?	?
•9.	2000	?	2200	5%	?	?
•10.	4300	?	3950	?	?	−5%
•11.	3730	250	?	?	?	5%
•12.	1800	50	?	?	150%	?
•13.	?	?	1800	?	−40%	−30%
•14.	?	100	?	5%	15%	?
•15.	1600	?	?	8%	?	0%
••16.	?	150	2700	?	?	80%

17. Calculate the income yield, capital gain yield, and rate of total return in each of 2005 and 2006 for Loblaw Companies' shares and Mawer New Canada Fund units. Use the data in Tables 9.3 and 9.4.

18. Calculate the income yield, capital gain yield, and rate of total return in each of 2005 and 2006 for Cameco Corporation shares and Sprott Canadian Equity Fund units. Use the data in Tables 9.3 and 9.4.

19. Calculate the income yield, capital gain yield, and rate of total return in each of 2005 and 2006 for Research in Motion shares and PH&N Bond Fund units. Use the data in Tables 9.3 and 9.4.

•20. Assume that the TD Bank shares in Table 9.3 will pay a $2.04 per share dividend in 2007. What must the share price be at the end of 2007 for a total rate of return in 2007 of 10%?

•21. Assume that the Loblaw shares in Table 9.3 will pay a $0.84 per share dividend in 2007. What must the share price be at the end of 2007 for a total rate of return in 2007 of 7%?

22. One year ago, Art Vandelay bought Norwood Industries shares for $37 per share. Today they are worth $40 per share. During the year, Art received dividends of $0.60 per share. What was his income yield, capital gain yield, and rate of total return for the year?

23. Rose purchased units of the Trimark Fund one year ago at $24.10 per unit. Today they are valued at $25.50. On the intervening December 31, there was a distribution of $0.83 per unit. ("Distribution" is the term used by most mutual funds for income paid to unitholders.) Calculate Rose's income yield, capital gain yield, and rate of total return for the year.

24. The market value of Stephanie's bonds has declined from $1053.25 to $1021.75 per bond during the past year. In the meantime she has received two semiannual interest payments of $35. Calculate Stephanie's income yield, capital gain yield, and rate of total return for the year.

25. Vitaly's shares of Dominion Petroleum have dropped in value from $36.75 to $32.25 during the past year. The shares paid a $0.50 per share dividend six months ago. Calculate Vitaly's income yield, capital gain yield, and rate of total return for the year.

26. Jeff purchased some Mitel preferred shares on the Toronto Stock Exchange for $13.50. The shares pay a quarterly dividend of $0.50. Twelve months later the shares were trading at $15.25. What was Jeff's rate of total return for the year?

•27. One year ago, Morgan invested $5000 to purchase 400 units of a mutual fund. He has just noted in the *Financial Post* that the fund's rate of return on investment for the year was 22% and that the current price of a unit is $13.75. What amount did the fund distribute as income per unit during the year?

••28. The *Globe and Mail* Report on Business noted that shares of Compact Computers produced a 55% rate of total return in the past year. The shares paid a dividend of $0.72 per share during the year, and they currently trade at $37.50. What was the price of the shares one year ago?

29. Adjusted for stock splits, the price of Microsoft shares rose 88.3%, 56.4%, 114.6%, and 68.4% in the years 1996 to 1999 respectively. In 2000, the share prices fell 62.8%.

a. What was the overall five-year percent change in the price of Microsoft shares?

b. If the share price at the end of 2000 was $43.38, what was the price at the beginning of 1996?

30. The federal government cut transfer payments to the provinces by a total of 20% over a five-year period. In the next budget speech, the Minister of Finance announced "the level of transfer payments will be restored to their former level by a 20% increase to be phased in over the next two years." Is this an accurate statement? Explain briefly.

31. The price of Bionex Inc. shares rose by 25% in each of two successive years. If they began the two-year period at $12 per share, what was the percent increase in price over the entire two years?

32. The price of Biomed Corp. shares also began the same two-year period (as in Problem 31) at $12, but fell 25% in each year. What was their overall percent decline in price?

33. What rate of return in the second year of an investment will wipe out a 50% gain in the first year?

34. What rate of return in the second year of an investment will nullify a 25% return on investment in the first year?

35. What rate of return in the second year of an investment is required to break even after a 50% loss in the first year?

36. What rate of return in the second year of an investment is required to break even after a rate of return of -20% in the first year?

37. After two consecutive years of 10% rates of return, what rate of return in the third year will produce a cumulative gain of 30%?

38. After two consecutive years of 10% losses, what rate of return in the third year will produce a cumulative loss of 30%?

•39. Victor cannot find the original record of his purchase four years ago of units of the Imperial Global Fund. The current statement from the fund shows that the total current value of the units is $47,567. From a mutual fund database, Victor found that the fund's rates of return for Years 1 to 4 have been 15.4%, 24.3%, 32.1%, and −3.3%, respectively.

a. What was Victor's original investment in the fund?

b. What was the dollar increase in the value of his investment in Year 3?

•40. The S&P/TSX Composite Index rose 3.4%, dropped 1.4%, and then rose 2.1% in three successive months. The Index ended the three-month period at 9539.

a. What was the Index at the beginning of the three-month period?

b. How many points did the Index drop in the second month?

•41. In three successive years the price of the common shares of Abysmal Resources Ltd. fell 35%, 55%, and 80%, ending the third year at 75 cents.

a. What was the share price at the beginning of the three-year skid?

b. How much (in dollars and cents) did the share price drop in the third year?

The following table lists the rates of return from five well-known Canadian equity mutual funds during each of five years. The last line in the table gives the corresponding percent changes in the S&P/TSX Composite Total Return Index. For each fund, calculate:

- **The percent increase (over the entire five years) in the value of an investment made at the beginning of 2002.**
- **The difference between the fund's overall percent increase and the S&P/TSX Index's overall percent increase. Did the fund outperform the Index?**

Problem	Fund name	2002 return (%)	2003 return (%)	2004 return (%)	2005 return (%)	2006 return (%)
42.	Altamira Equity	−16.1	8.6	13.2	20.4	19.2
43.	AGF Canadian Stock	−14.0	22.2	12.2	20.3	18.6
44.	Ethical Growth	−15.1	17.5	8.2	15.7	19.6
45.	Sceptre Canadian Equity	−11.7	15.5	16.2	21.5	29.8
46.	Sprott Canadian Equity	39.3	30.0	37.9	13.2	39.6
	S&P/TSX Total Return Index	−12.4	26.7	14.5	24.1	17.3

KEY TERMS

Capital gain *p. 365*
Capital gain yield *p. 366*
Capital loss *p. 365*
Effective interest rate *p. 352*
Equivalent interest rate *p. 362*
Holding period *p. 365*
Income *p. 365*
Income yield *p. 366*
Rate of total return *p. 366*
Rule of 72 *p. 347*
Total return *p. 365*

SUMMARY OF NOTATION AND KEY FORMULAS

f = Effective rate of interest

FORMULA (9-1) $$i = \sqrt[n]{\frac{FV}{PV}} - 1 = \left(\frac{FV}{PV}\right)^{1/n} - 1$$ Finding the periodic interest rate (or periodic rate of return)

FORMULA (9-2) $$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)}$$ Finding the number of compounding periods

FORMULA (9-3) $$f = (1+i)^m - 1$$ Finding the effective rate of interest (or effective rate of return)

FORMULA (9-4) $$i_2 = (1+i_1)^{m_1/m_2} - 1$$ Finding an equivalent periodic interest rate

FORMULA (9-5)
$$\left\{\begin{aligned} \text{Income yield} &= \frac{\text{Income}}{V_i} \times 100\% \\ \text{Capital gain yield} &= \frac{\text{Capital gain}}{V_i} \times 100\% = \frac{V_f - V_i}{V_i} \times 100\% \\ \text{Rate of total return} &= \text{Income yield} + \text{Capital gain yield} \\ &= \frac{\text{Income} + \text{Capital gain}}{V_i} \times 100\% \end{aligned}\right\}$$
Finding investment yields and rate of total return

REVIEW PROBLEMS

Answers to the odd-numbered review problems are at the end of the book.

Calculate percentages accurate to the nearest 0.01%.

1. A sum of $10,000 invested in the TD Science and Technology Fund at the end of 1999 would have declined to $2961 by the end of 2006. What compound annual rate of return did the fund realize during this period?

2. Maxine found an old pay statement from nine years ago. Her hourly wage at the time was $13.50 versus her current wage of $20.80 per hour. At what equivalent (compound) annual rate has her wage grown over the period?

3. If a company's annual sales grew from $165,000 to $485,000 in a period of eight years, what has been the compound annual rate of growth of sales during the period?

•4. An investor's portfolio increased in value by 53% over a five-year period while the Consumer Price Index rose from 121.6 to 135.3. What was the annually compounded real rate of return on the portfolio for the five years?

•5. A portfolio earned −13%, 18%, 5%, 24%, and −5% in five successive years. What was the portfolio's five-year compound annual return?

•6. The Front Street Special Opportunities Canadian Fund was the top-performing fund in Canada for the five years ending December 31, 2006. What three-year and five-year compound annual returns to December 31, 2006, did the fund report if its annual returns in successive years from 2002 to 2006 inclusive were 14.5%, 121.2%, 19.2%, 46.8%, and 22.5%, respectively?

•7. Terry was supposed to pay $800 to Becky on March 1. At a later date, Terry paid Becky an equivalent payment in the amount of $895.67. If they provided for a time value of money of 8% compounded monthly, on what date did Terry make the payment?

•8. What is the time remaining until the maturity date of a $50,000 strip bond if it has just been purchased for $20,822.89 to yield 5.38% compounded semiannually until maturity?

•9. When discounted to yield 9.5% compounded quarterly, a $4500 four-year promissory note bearing interest at 11.5% compounded semiannually was priced at $5697.84. How long after the issue date did the discounting take place?

•10. The population of a mining town declined from 17,500 to 14,500 in a five-year period. If the population continues to decrease at the same compound annual rate, how long, to the nearest month, will it take for the population to drop by another 3000?

11. To the nearest day, how long will it take a $20,000 investment to grow to $22,000 (including accrued interest) if it earns 7% compounded quarterly? Assume that a quarter year has 91 days.

12. What monthly compounded nominal rate would put you in the same financial position as 5.5% compounded semiannually?

13. You are offered a loan at a rate of 10.5% compounded monthly. Below what figure would a semiannually compounded nominal rate have to be to make it more attractive?

14. A bank offers a rate of 5.3% compounded semiannually on its four-year GICs. What monthly and annually compounded rates should it quote in order to have the same effective interest rate at all three nominal rates?

15. If an invoice indicates that interest at the rate of 1.2% per month will be charged on overdue amounts, what effective rate of interest will be charged?

16. If the nominal rate of interest paid on a savings account is 3% compounded monthly, what is the effective rate of interest paid?

17. If an interest rate of 6.9% compounded semiannually is charged on a car loan, what effective rate of interest should be disclosed to the borrower?

18. If a $15,000 investment grew to $21,805 in $4\frac{1}{2}$ years of quarterly compounding, what effective rate of return was the investment earning?

19. Camille can obtain a residential mortgage loan from a bank at 8.75% compounded semiannually or from an independent mortgage broker at 8.6% compounded monthly. Which source should she pick if other terms and conditions of the loan are the same? Present calculations that support your answer.

20. One year ago, Christos bought 1000 units of the Dominion Aggressive Growth Fund at $20.35 per unit. Today a unit's value is $19.10. During the year, the fund made a distribution of $0.40 per unit. On this investment, what is Christos's:

a. Income yield?

b. Capital gain yield?

c. Total return in dollars?

d. Rate of total return?

21. A company's annual report states that its common shares had price gains of 23%, 10%, –15%, and 5% during the preceding four fiscal years. The share price stood at $30.50 after last year's 5% gain.

a. What was the price of the shares at the beginning of the four-year period?

b. How much (in dollars and cents) did the share price decline in the third year?

•22. An $8600 investment was worth only $7900 one year later. If the rate of total return for the year was –5%, how much income was received from the investment during the year?

SELF-TEST EXERCISE

Answers to the self-test problems are at the end of the book.

Calculate percentages accurate to the nearest 0.01%.

1. The home the Bensons purchased 13 years ago for $85,000 is now appraised at $215,000. What has been the annual rate of appreciation of the value of their home during the 13-year period?

2. If the Consumer Price Index rose from 109.6 to 133.8 over an $8\frac{1}{2}$-year period, what was the equivalent compound annual inflation rate during the period?

•3. A company's sales dropped 10% per year for five years.

- **a.** What annual rate of sales growth for the subsequent five years would return the sales to the original level?
- **b.** To the nearest month, how long would it take for sales to return to the original level if they increased at 10% per year?

•4. An investor's portfolio increased in value from $35,645 to $54,230 over a six-year period. At the same time, the Consumer Price Index rose by 26.5%. What was the portfolio's annually compounded real rate of return?

•5. One of the more volatile mutual funds in recent years has been the AGF China Focus Fund. The fund's annual returns in successive years from 2001 to 2006 inclusive were 20.8%, −11.0%, 63.4%, −9.3%, 8.6%, and 67.6%, respectively. What was the fund's equivalent compound annual return for the six years ended December 31, 2006?

•6. To the nearest month, how long will it take an investment to increase in value by 200% if it earns 7.5% compounded semiannually?

•7. Rounded to the nearest month, how long will it take money to lose one-third of its purchasing power if the annual inflation rate is 3%?

•8. An investor paid $4271.17 to purchase a $10,000 face value strip bond for her RRSP. At this price the investment will provide a return of 6.47% compounded semiannually. How long (to the nearest day) after the date of purchase will the bond mature? Assume that each half-year is exactly 182 days long.

•9. A trust company pays 5.375% compounded annually on its five-year GICs. What semiannually compounded interest rate would produce the same maturity value?

10. Which of the following rates would you prefer for a loan: 7.6% compounded quarterly, 7.5% compounded monthly, or 7.7% compounded semiannually?

11. A $10,000 investment grew to $12,000 after 39 months of semiannual compounding. What effective rate of return did the investment earn?

12. One thousand shares of Frontier Mining were purchased at $6.50 per share. The share price rose 110% in the first year after purchase, declined 55% in the second year, and then dropped another 55% in the third year.

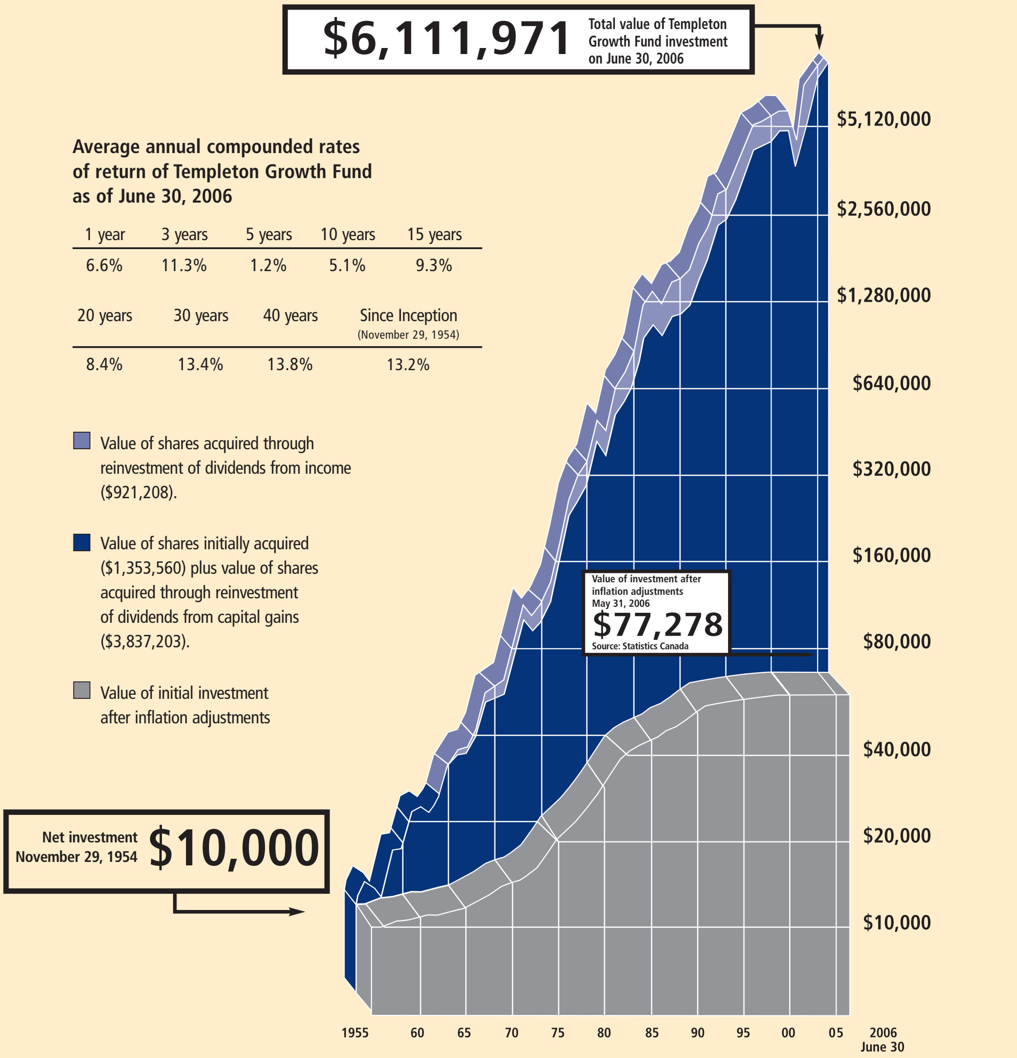
- **a.** What was the percent change in share price over the entire three years?
- **b.** How much (in dollars and cents) did the share price drop in the second year?

•13. Gabriel received $200 of income from an investment during the past year. This represents an income yield of 4%. If the capital gain yield for the year was 10%, what was the value of the investment (not including income) at the end of the year?

CASE Mountains of Money

One of the oldest mutual funds in Canada is the Templeton Growth Fund, Ltd. Shares in the fund were first sold at the end of November 1954. A common graphic for dramatically illustrating the long-term performance of a mutual fund is known as a "mountain chart." The vertical scale of the chart is a logarithmic scale. On a logarithmic scale, each doubling of the investment produces the same interval on the vertical scale.

The time axis covers 51 years and 7 months (from November 29, 1954 to June 30, 2006). The chart indicates that an investment of $10,000 on November 29, 1954 was worth $6,111,971 on June 30, 2006. (This outcome assumes that all dividends paid on shares of the fund were reinvested in additional shares.)



Average annual compounded rates of return of Templeton Growth Fund as of June 30, 2006

1 year	3 years	5 years	10 years	15 years
6.6%	11.3%	1.2%	5.1%	9.3%

20 years	30 years	40 years	Since Inception (November 29, 1954)
8.4%	13.4%	13.8%	13.2%

SOURCE: Reproduced with permission of Franklin Templeton Investments.

Based on increases in the Consumer Price Index, the chart also indicates that you needed $77,278 in June of 2006 to have the same purchasing power as $10,000 in November of 1954.

Questions

1. Calculate the fund's equivalent compound annual rate of return over the entire $51\frac{7}{12}$ years.
2. What was the equivalent compound annual rate of inflation during the entire period?
3. What was the fund's *real* compound annual rate of return over the entire period?
4. From the chart, we can estimate that the original $10,000 investment was worth about $350,000 in November 1979. Compare the fund's performance before this date to its performance after this date. (That is, compare the fund's compound annual rates of return before and after November 30, 1979.)

APPENDIX 9A

THE TEXAS INSTRUMENTS BA II PLUS INTEREST CONVERSION WORKSHEET

Notice the letters **ICONV** above the 2 key. This means that the Interest Conversion Worksheet is the second function of the 2 key. You can access the worksheet by pressing 2nd 2 in sequence. Hereafter, we will represent these keystrokes as 2nd ICONV. The calculator's display then shows:

NOM = n.nn

where the n's represent numerical digits. (Your display may show more or fewer n's.)

You should think of a worksheet as a single column of items that you can view one-at-a-time in the display. The Interest Conversion Worksheet's column consists of the following three items:

NOM = n.nn
EFF = n.nn
C/Y = n

The solid line around the first item indicates that the calculator's display currently provides a "window" to the first item in the column. You can use the scroll keys ↓ and ↑ to move down or up the list. The three worksheet symbols are defined as follows:

NOM = Nominal annual interest rate, *j*
EFF = Effective interest rate, *f*
C/Y = Number of compoundings per year, *m*

The Interest Conversion Worksheet allows you to enter values for any two of these three variables and then compute the value of the remaining third variable. Close the worksheet by pressing 2nd QUIT. (By the QUIT key, we mean the key showing **QUIT** as its second function.)

Let us use the worksheet to answer Example 9.3A which asks us to calculate the effective interest rate corresponding to 10.5% compounded monthly.

2nd ICONV	⇒ Open the Interest Conversion Worksheet.
10.5 ENTER	⇒ Enter and save the value for *NOM*.
↓ ↓ 12 ENTER	⇒ Scroll down to *C/Y*. Enter and save its value.
↑ CPT	⇒ Scroll back up to *EFF*. Compute its value.

The effective interest rate appearing in the display is 11.02%.