Equivalent Payments (Compound Interest)

Any amount of money that is subject to rate of interest will grow overtime. Thus, time value of money is important. The value of an original amount at any particular time is called equivalent value or dated value. The equivalent payment combines the original sum with the interest earned up to the dated value date.

When sums of money fall due or are payable at different time, they are not directly comparable. To make the sum of money comparable, a point in time - the **focal date** or **comparison date** must be chosen.

To find the Equivalent Payments in compound interest, any point can be chosen as the focal date; the choice does not affect the final answers. However, **it is always better to choose a date on which an amount is unknown.** The choice determines which formula is to be used.

If the equivalent amount is in the future or after the due date, use the future value formula,

$$FV = PV(1+i)^n$$

If the equivalent amount is in the past or before the due date, use present value formula,

$$PV = FV(1+i)^{-n}$$

Where **i = the periodic rate of interest** and **n = number of interest periods**

$i = \frac{j}{m}$ (*j* is annual interest rate compounded *m* times per year)

Example 1

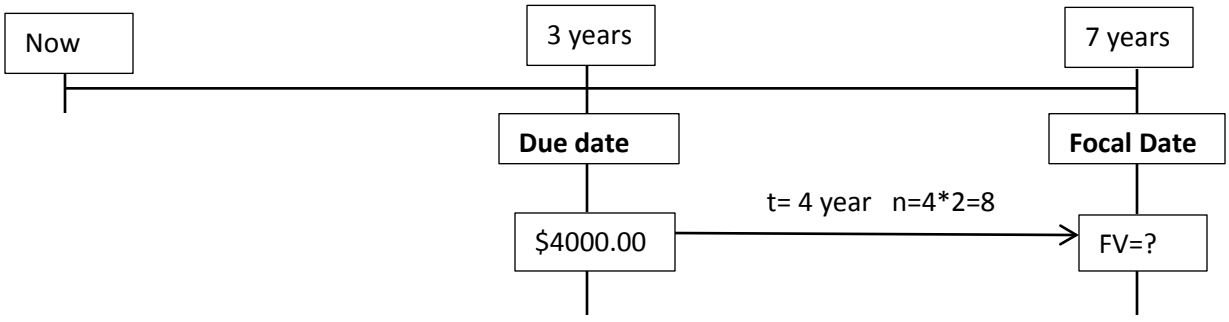
A sum of $4000 is due for payment three years from now. If money is worth 9% p.a. compounded semi-annually, determine the equivalent value

(i) Seven years from now

(ii) Now

Solution

i) Start with drawing a time line to represent the situation.



From the time line, we can see that the focal date falls **after** the due date. This tells us to use the future value formula to determine the equivalent payment amount.

Method 1 (Using formula):

PV = $4000

n = 8 semi-annual periods (4 years x 2)

i = $\frac{9\%}{2} = \frac{0.09}{2} = 0.045$

FV = ??

Substitute the above values into the future value formula, $FV = PV(1+i)^n$

$FV = 4000(1+0.045)^8$

$= 4000(1.422101)$

$= 5688.40$

Method 2 (Using a BAII Plus Calculator):

Enter the information into the calculator as follows:

Set P/Y=2 and C/Y=2

4000 ± PV

9 I/Y

8 N

Press CPT FV

= 5688.402451

Both methods show us that the equivalent payment of $4000 seven years from now is $5688.40.

(ii) Start with drawing a time line to represent the situation.

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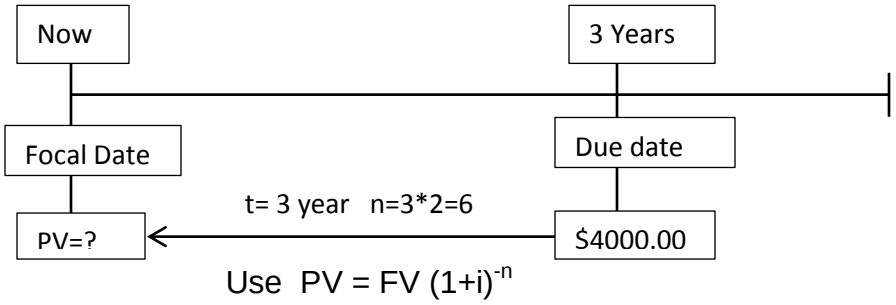

From the time line, we can see that the focal date falls **before** the due date. This tells us to use the present value formula to determine the equivalent payment amount.

Method 1 (Using formula):

FV = $4000

n = 6 semi-annual periods (3 years x 2)

i = $\frac{9\%}{2} = \frac{0.09}{2} = 0.045$

PV = ??

Substitute the above values into the present value formula, $PV = FV(1+i)^{-n}$

$PV = 4000(1+.0045)^{-6}$

$= 4000(0.767896$

= $3071.58

Method 2 (Using BAII Plus calculator):

Set P/Y=2 and C/Y=2

4000 ± FV

9 I/Y

6 N

Press CPT PV

= 3071.582953

So the Equivalent payment of $4000.00 now is $3071.58