EXAMINATION

Code: FE-01

Program: Bachelor of Hospitality, Sport and Tourism Management **Course Code: MAT1092** **Course Title: Advanced Mathematics** **Level:** **Time allowed:** **Date: 29/08/2019** **Time: 120 min**	***Lecturer's Signature*** *Date: ………………………………*
	Department's Signature *Date: ………………………………*

Instructions to students:

1. Closed/Opened book examination: Closed book examination
2. Students are permitted to retain this examination paper

This exam paper contains 2 pages, including the cover page.

CODE: FE-01

Problem 1. (1.0 pt) An individual saves $1000 in a bank account at the beginning of each year. The bank offers a return of 8% compounded annually. Determine the amount saved after 10 years.

Problem 2. (1.5 pts) A person borrows $100 000 at the beginning of a year and agrees to repay the loan in ten equal installments at the end of each year. Interest is charged at a rate of 6% compounded annually.

a) Find the annual repayment.

b) Work out the total amount of interest paid.

Problem 3. (1.5 pts) Let the average cost function of a good

$$AC = 2Q + 9 + \frac{15}{Q}.$$

a) Find an expression for the total cost TC and write down an expression for the marginal cost function.

b) If the current output Q is 10, estimate the effect on TC of a 2 unit increase in Q.

Problem 4. (1.5 pts) Given the demand function

$$P = 100 - 2Q$$

find the elasticity when the price is 20. Is the demand inelastic, unit elastic or elastic at this price?

Problem 5. (1.5 pts) Let the following matrices

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

a) Find $A + 2B, AB.$

b) Find the inverse matrix of A if it exists.

Problem 6. (1.0 pt) Find the consumer's surplus at $P = 5$ for the demand functions $P = 25 - 2Q.$

Problem 7. (2.0 pts) An electronics firm decides to launch two models of a tablet, TAB1 and TAB2. The cost of making each device of type TAB1 is $120 and the cost for TAB2 is $160. The firm recognises that this is a risky venture so decides to limit the total weekly production costs to $4000. Also, due to a shortage of skilled labour, the total number of tablets that the firm can produce in a week is at most 30. The profit made on each device is $60 for TAB1 and $70 for TAB2. How should the firm arrange production to maximise profit?

EXAMINATION

Code: FE-02

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Course Code: MAT1092

Course Title: Advanced Mathematics

Level:

Time allowed:

Date: 29/08/2019

Time: 120 min</td><td>Lecturer’s Signature

Date: ………………………………</td></tr>
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Date: ………………………………</td></tr>
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Instructions to students:

1. Closed/Opened book examination: Closed book examination
2. Students are permitted to retain this examination paper

This exam paper contains 2 pages, including the cover page.

CODE: FE-02

Problem 1. (1.0 pt) An individual saves \$1500 in a bank account at the beginning of each year. The bank offers a return of 6% compounded annually. Determine the amount saved after 12 years.

Problem 2. (1.5 pts) A person borrows \$150 000 at the beginning of a year and agrees to repay the loan in ten equal installments at the end of each year. Interest is charged at a rate of 8% compounded annually.

a) Find the annual repayment.

b) Work out the total amount of interest paid.

Problem 3. (1.5 pts) Let the average cost function of a good

$$AC = 3Q + 6 + \frac{12}{Q}.$$

a) Find an expression for the total cost TC and write down an expression for the marginal cost function.

b) If the current output Q is 10, estimate the effect on TC of a 2 unit decrease in Q.

Problem 4. (1.5 pts) Given the demand function

$$P = 120 - 3Q$$

find the elasticity when the price is 20. Is the demand inelastic, unit elastic or elastic at this price?

Problem 5. (1.5 pts) Let the following matrices

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

a) Find $2A + B, BA.$

b) Find the inverse matrix of B if it exists.

Problem 6. (1.0 pt) Find the consumer's surplus at $P = 8$ for the demand functions $P = 30 - 2Q.$

Problem 7. (2.0 pts) An electronics firm decides to launch two models of a tablet, TAB1 and TAB2. The cost of making each device of type TAB1 is \$130 and the cost for TAB2 is \$150. The firm recognises that this is a risky venture so decides to limit the total weekly production costs to \$5500. Also, due to a shortage of skilled labour, the total number of tablets that the firm can produce in a week is at most 40. The profit made on each device is \$70 for TAB1 and \$80 for TAB2. How should the firm arrange production to maximise profit?

KEY

Code: FE-01

Program: Bachelor of Hospitality, Sport and Tourism Management **Course Code: MAT1092** **Course Title: Advanced Mathematics** **Date: 29/11/2018** **Time: 120 min**	***Lecturer's Signature*** *Date: ………………………………*

Code: FE-01

Problem 1. (1.0 pt) $a = 1000, q = 1 + 8\% = 1.08, n = 10$. The amount saved after 10 years is

$$aq\frac{q^n - 1}{q - 1} = 1000 \times 1.08 \times \frac{1.08^{10} - 1}{1.08 - 1} = 15645.49$$

Problem 2. (1.5 pts) $L = 100000, r = 6\%, q = 1 + 6\% = 1.06, n = 10$.

a) The annual repayment:

$$a = Lq^n : \frac{q^n - 1}{q - 1} = 100000 \times 1.06^{10} : \frac{1.06^{10} - 1}{1.06 - 1} = 13586.80$$

b) The total amount of interest paid is:

$$10 \times 13586.80 - 100000 = 35868$$

Problem 3. (1.5 pts)

a) The total cost $TC = AC \times Q = (2Q + 9 + \frac{15}{Q})Q = 2Q^2 + 9Q + 15,$

Then $MC = \frac{d(TC)}{dQ} = 4Q + 9$

b) $MC(10) = 4 \times 10 + 9 = 49$

$\Delta(TC) \cong MC \times \Delta Q = 49 \times 2 = 98$, so TC increases by 98 units approximately.

Problem 4. (1.5 pts) Since $P = 100 - 2Q \Rightarrow Q = 50 - \frac{1}{2}P$

When $P = 20$ then $Q = 50 - \frac{1}{2}20 = 40$, $\frac{dQ}{dP}(20) = -\frac{1}{2}$

The elasticity $E = -\frac{P}{Q}\frac{dQ}{dP} = -\frac{20}{40} \times (-\frac{1}{2}) = 0.25$

$E = 0.25 < 1$ so the demand is inelastic.

Problem 5. (1.5 pts)

a) Find $A + 2B = \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 4 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 0 & 2 \end{bmatrix}$.

$$AB = \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -2 & -8 \end{bmatrix}$$

b) $A = 4 - 2 = 2 \neq 0$ so A is inversible.

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1/2 \\ 1 & 1/2 \end{bmatrix}$$

Problem 6. (1.0 pt) The demand functions $P = 25 - 2Q$. If $P = 5$ then $Q = 10$.

$$CS = \int_0^{10} (25 - 2Q)dQ - 5 \times 10 = \left[25Q - Q^2\right]_0^{10} - 50 = 100$$

Problem 7. (2.0 pts) Let x and y be the number of TAB1, and TAB2 to be made each week, respectively.

We go to maximize the objective function

$$c = 60x + 70y$$

subject to the constraints

$$\begin{aligned} &120x + 160y \leq 4000 \\ &x + y \leq 30 \\ &x \geq 0 \\ &y \geq 0 \end{aligned}$$

Sketch the feasible region….The intersection of the two straight lines is (20,10).

4 corners of the feasible region are: (0,0), (0,25), (20,10), (30,0).

Corner	Objective function
(0,0)	0
(0,25)	1750
(20,10)	1900
(30,0)	1800

Therefore the firm should produce 20 tablets of model TAB1 and 10 of model TAB2 to achieve a maximum profit of $1900.

KEY

Code: FE-02

Program: Bachelor of Hospitality, Sport and Tourism Management **Course Code: MAT1092** **Course Title: Advanced Mathematics** **Date: 29/11/2018** **Time: 120 min**	***Lecturer's Signature*** *Date: …………………………………*

Code: FE-02

Problem 1. (1.0 pt) $a = 1500, q = 1 + 6\% = 1.06, n = 12$. The amount saved after 12 years is

$$aq\frac{q^n - 1}{q - 1} = 1500 \times 1.06 \times \frac{1.06^{12} - 1}{1.06 - 1} = 26823.2$$

Problem 2. (1.5 pts) $L = 150000, r = 8\%, q = 1 + 8\% = 1.08, n = 10$.

a) The annual repayment:

$$a = Lq^n : \frac{q^n - 1}{q - 1} = 150000 \times 1.08^{10} : \frac{1.08^{10} - 1}{1.08 - 1} = 22354.42$$

b) The total amount of interest paid is:

$$10 \times 22354.42 - 150000 = 73544.23$$

Problem 3. (1.5 pts)

a) The total cost $TC = AC \times Q = (3Q + 6 + \frac{12}{Q})Q = 3Q^2 + 6Q + 12,$

Then $MC = \frac{d(TC)}{dQ} = 6Q + 6$

b) $MC(10) = 6 \times 10 + 6 = 66$

$\Delta(TC) \cong MC \times \Delta Q = 66 \times (-2) = -132$, so TC decreases by 132 units approximately.

Problem 4. (1.5 pts) Since $P = 120 - 3Q \Rightarrow Q = 40 - \frac{1}{3}P$

When $P = 20$ then $Q = 40 - \frac{1}{3}20 = \frac{100}{3}$, $\frac{dQ}{dP}(20) = -\frac{1}{3}$

The elasticity $E = -\frac{P}{Q}\frac{dQ}{dP} = -\frac{20}{\frac{100}{3}} \times (-\frac{1}{3}) = 0.2$

$E = 0.2 < 1$ so the demand is inelastic.

Problem 5. (1.5 pts)

a) Find $2A + B = 2\begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -3 & 7 \end{bmatrix}$.

$$BA = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 3 & -5 \end{bmatrix}$$

b) $\det(B) = -3 - 2 = -5 \neq 0$ so B is inversible.

$$B^{-1} = \frac{1}{-5} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1/5 & 2/5 \\ 1/5 & -3/5 \end{bmatrix}$$

Problem 6. (1.0 pt) The demand functions $P = 30 - 2Q$, so if $P = 8$ then $Q = 11$.

$$CS = \int_0^{11} (30 - 2Q) dQ - 8 \times 11 = \left[30Q - Q^2 \right]_0^{11} - 88 = 121$$

Problem 7. (2.0 pts) Let x and y be the number of TAB1, and TAB2 to be made each week, respectively.

We go to maximize the objective function

$$c = 70x + 80y$$

subject to the constraints

$$\begin{aligned} &130x + 150y \leq 5500 \\ &x + y \leq 40 \\ &x \geq 0 \\ &y \geq 0 \end{aligned}$$

Sketch the feasible region….The intersection of the two straight lines is (25,15).

4 corners of the feasible region are: (0,0), (0, 110/3), (25,15), (40,0).

Corner	Objective function
(0,0)	0
(0, 110/3)	2933.33
(25,15)	2950
(40,0)	2800

Therefore the firm should produce 25 tablets of model TAB1 and 15of model TAB2 to achieve a maximum profit of $2950.