Stat 274
Theory of Interest

Chapter 6: Bonds

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Bonds

A bond is a security issued by a government or a corporation which promises payments at future dates.

- Maturity (or redemption) date: the time of the last payment
- Issue date: time which the investor loans the money
- Term: time period between issue and maturity
- Noncallable bond: maturity date is fixed
- Zero-coupon (discount) bond: Only payment is at maturity
- Coupon period: time between coupon payments
- Indenture: legal document specifying details of bond

Bond Terminology

Letter	Meaning
N	number of years in the bond term
m	number of coupons per year
n	number of coupons, $n = Nm$
α	nominal coupon rate, annual coupons total $F\alpha$
r	coupon rate per coupon period, each coupon is Fr
g	modified coupon rate, coupon amount is Cg
I	nominal yield rate convertible m times per year
j	effective yield rate for the coupon period
i	annual effective yield rate
F	face (or par) value, coupon amount is Fr
C	redemption amount
P	price at issue
K	present value of the redemption amount

Bond basics

The coupon amount can be defined many different ways

$$Fr = \frac{F\alpha}{m} = gC$$

The price (or time 0 value) of a bond is [the basic price formula]

$$\begin{aligned} P &= (Fr)a_{\overline{n}|j} + Cv_j^n \\ &= (Fr)a_{\overline{n}|j} + K \end{aligned}$$

Examples

An eight-year 3,000 10% bond with semiannual coupons and redemption value 2,800 is bought at a price to give the investor a yield rate of 12% convertible semiannually. Find the price of the bond. [2,618.09]

A twelve-year 2,000 8% bond with quarterly coupons is bought for 2,200 and redeems at par. Find the effective yield rate per coupon period j. [1.69395%]

A nine-year 5,000 7% bond with semiannual coupons is purchased for 4,986. The yield rate is 6% convertible semiannually. Calculate the redemption amount. [4,390.80]

The Premium-Discount Formula

The premium-discount formula can be written three ways:

$$P = C(g - j)a_{\overline{n}|j} + C \qquad \text{(price)}$$

$$P - C = C(g - j)a_{\overline{n}|j} \qquad \text{(premium)}$$

$$C - P = C(j - g)a_{\overline{n}|j} \qquad \text{(discount)}$$

Examples

A bond with a face value of 6,000 and an annual coupon rate of 12% convertible semiannually (this is another way to say semiannual coupons and $\alpha = 0.12$) will mature in ten years at par. If the bond is priced using a nominal yield rate of 6% convertible semiannually, what is the amount of premium or discount on this bond? (Use both the Frank and Premium/Discount formulas) [2,677.95]

A 7.5 year 14% bond with a face value of 2,500 has semiannual coupons and is sold to yield 7.2% convertible semiannually. The discount on this bond is 283.12. Find the price of this bond. [5,265.69]

Examples

An eight-year bond has annual coupons and a redemption value of 2,338. It is purchased to yield 9% and each coupon is for 63. Calculate the price of this bond. [1,522.06]

A 2,000 bond with coupon rate of 10% payable quarterly is redeemable after an unspecified number of periods for 2,250. The bond is bought to yield 8% convertible quarterly. If the present value of the redemption is 869.71, find the purchase price. [2,403.37]

Bond Amortization

- The balance of the debt at time t is denoted B_t (book value)
- By definition, $B_0 = P$ and $B_n = C$.
- The amount of interest due in the t^{th} coupon is $I_t = jB_{t-1}$
- The change in principal is $P_t = B_{t-1} - B_t$
- $I_t + P_t = Cg$
- $B_t = (1 + j)B_{t-1} - Cg$
- If the bond sells at a premium, $P_t > 0$
- If the bond sells at a discount, $P_t < 0$
- If the bond sells at par, $P_t = 0$

Examples

A six-year 1,800 8.5% bond with semiannual coupons is bought for 1,918 and is redeemable for 1,860. Find the amortization of premium in the 5th coupon and the interest due at that time. [4.53; 71.97]

A 10,000 par-value twenty-year 14% bond with annual coupons is bought for 9,562 and redeems at par. Find the accumulation of discount in the third coupon and the interest due at that time. [5.84; 1405.84]

(Difficult, use the premium/discount formula) A twelve-year bond with semiannual level coupons is bought at a premium to yield 7.5% convertible semiannually. If the amortization of premium in the fourth to the last payment is 8.02, find the premium and the total amortization of premium in the first three years. [145.38; 25.32]

Callable Bonds

A callable bond allows the issuer to repay the bond (or a portion of the bond) before the maturity date.

- Call dates: times when the bond can be repaid, each with its own redemption value.
- Lockout period: time before the first call date.
- Call premium: Amount over the redemption value at maturity paid at each call date
- European option: Single call date
- Bermuda option: multiple call dates
- American option: callable at any date after the lockout

Examples

You purchase a 1,000 8% bond with seminannual coupons. It is a five-year bond with call dates at the end of years 2 and 3. The redemption value at each of these dates and at maturity is 1,060 and the price is 1,022. Find the possible nominal semiannual yields to the investor. [10 payments: 8.44%, 6: 8.94%, 4: 9.56%]

Assume instead that the price of the above bond is 1,150. Calculate the possible yields. [10 payments: 5.58%, 6: 4.49%, 4: 3.13%]

Assume further (keeping the price at 1,150) that they wanted to guarantee the investor a 5.58% (nominal semiannual) yield on this bond. Calculate the call premiums required for both call dates. [at payment 6: 38.99, 4: 56.94]

SOA #10

A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%. Calculate the interest portion of the 7th coupon. [641.58]

Note: most bonds redeem at par. I will explicitly tell you if I want you to assume it redeems at par, but the SOA, and maybe your boss, will want you to assume it redeems at par unless they say otherwise

SOA #22

You have decided to invest in Bond X, an n-year bond with semi-annual coupons and the following characteristics:

- Par value is 1000.
- The ratio of the semi-annual coupon rate, r, to the desired semi-annual yield rate, i, is 1.03125.
- The present value of the redemption value is 381.50.

Given $(1+i)^{-n} = 0.5889$, calculate the price of bond X. [1055.11]

SOA #47

Bill buys a 10-year 1000 par value bond with semi-annual coupons paid at an annual rate of 6%. The price assumes an annual nominal yield of 6%, compounded semi-annually. As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of i. At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond.

Calculate i. [0.0975]

SOA #54

Matt purchased a 20-year par value bond with an annual nominal coupon rate of 8% payable semiannually at a price of 1722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The lowest yield rate that Matt can possibly receive is a nominal annual interest rate of 6% convertible semiannually.

Calculate X. [1440]

SOA #62

A 40-year bond is purchased at a discount. The bond pays annual coupons. The amount for accumulation of discount in the 15th coupon is 194.82. The amount for accumulation of discount in the 20th coupon is 306.69.

Calculate the amount of discount in the purchase price of this bond. [21,135]