DaNel.Hogan@schools.pima.gov
Director of The STEMAZing Project

Binary Math "Magic" Build Your Own Trick Cards

This lesson will demonstrate to students how to make the binary math magic cards. A common question when engaging students with the Binary Math "Magic" Lesson is: "Why are those numbers on the cards?" In this lesson, they will start with blank cards and put the numbers on the cards so they understand the numbers are not random. The numbers on each card are the numbers, which have that position turned on to a 1 in the binary system.

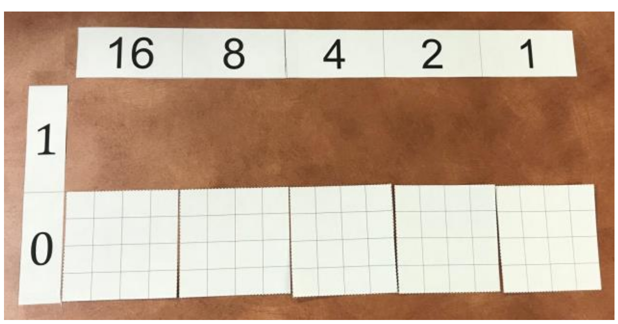
Materials:
A set of blank cards, the Binary Numbers 0-31 worksheet, and the 0, 1 and 1, 2, 4, 8, 16 number lines for each student.
Marker
Scissors

Engage:
You may have already done the Binary Math "Magic" Lesson, in which case, students should be ready to tackle the "why" behind how that trick works. You could also start with this lesson and then do the other. At any rate, you should remind students about the binary number system as well as positional notation. It might be helpful to watch the video explaining the binary system again. Show this video until 3:45 when it becomes an advertisement for the sponsor: http://bit.ly/BinaryNumbersVideo

Binary Numbers – a numeral base system which uses only two numbers – 1 and 0

Numeral Base Systems – systems which uses only the digits 0-9 or a subset of those digits to represent different values – examples are base 10 (which we use every day), base 2 or binary which uses just two values, but there are also base three, base four, and so on.

Explore:
You should have students cut out the blank set of cards along the dotted lines and set up the number lines as shown in the picture below. It helps to tape the 0, 1 and 1, 2, 4, 8, 16 number lines down to the table. The blank cards should be left free to move.



DaNel.Hogan@schools.pima.gov
Director of The STEMAZing Project

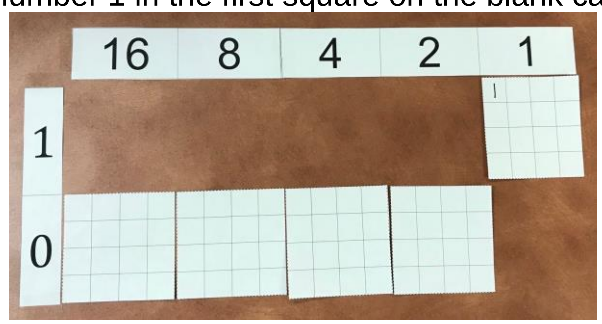
On the Binary Numbers 0-31 worksheet, students should write 00000 next to the 0 = indicating that 00000 is used to represent the value of zero in binary. Students can also be reminded about positional notation here. They should note that in our base ten system, the first ten positions would the ones, tens, hundreds, thousands, and ten thousands. They would be listed with the ones on the right and each new position to the right like this:

10,000s	1000s	100s	10s	1s

In the binary number system, because we are only using two symbols, 0 and 1, and not all ten digits, the positions now represent the following values and increase in value from right to left, just as they do the in base 10 system we are so used to using.

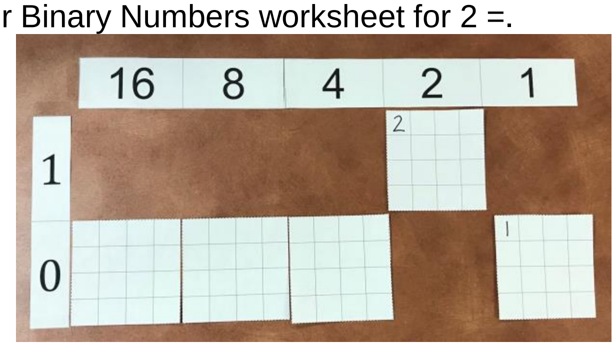
16	8	4	2	1

In order to represent a value of 1, we need to turn the first position in the binary number system on to a 1. So, students should shift the blank card on the right to the “on” or 1 location and write the number 1 in the first square on the blank card as shown below.



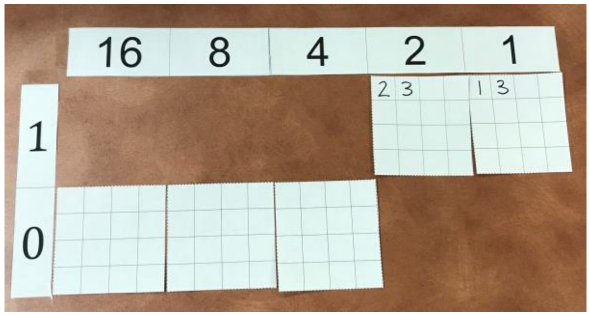
Students should also record the binary number 00001 on their Binary Numbers 0-31 worksheet.

To represent a value of 2, the card in position 1 needs to be turned “off” or moved back to the 0 location and the card in position 2 should be turned “on” or moved to the 1 location, as shown in the image below. The number 2 should be written in the first square on the blank card in position 2 as shown. Students should record the binary number 00010 on their Binary Numbers worksheet for 2 =.

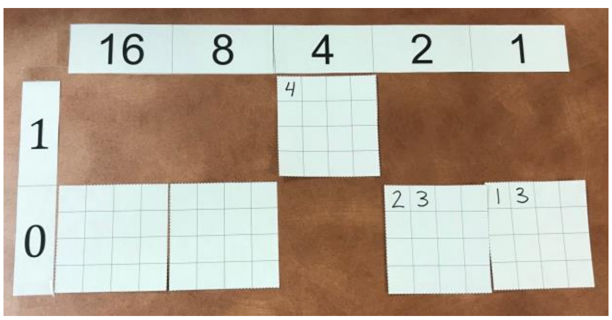


DaNel.Hogan@schools.pima.gov
Director of The STEMAZing Project

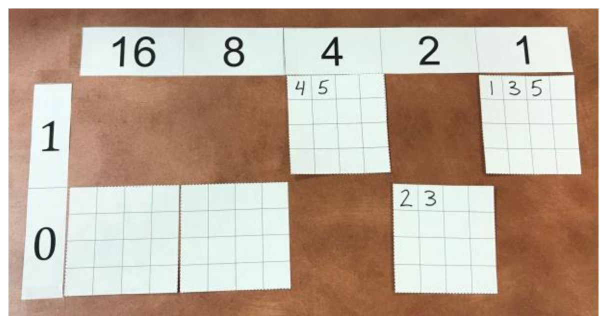
To represent a value of 3, the cards in positions 1 + 2 need to be turned "on" or moved to the 1 location and the number 3 should be written in the next blank square on BOTH cards as shown below. Students should record the binary number 00011 on their Binary Numbers worksheet for 3 =.



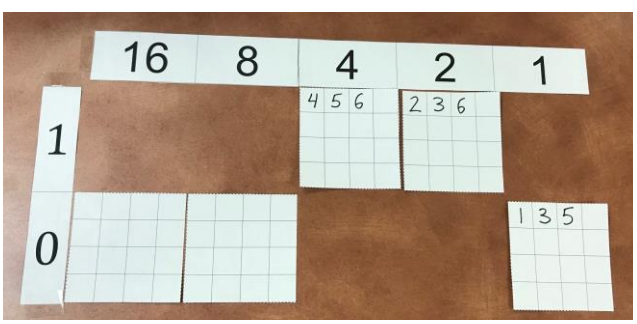
And so it should continue with students working their way through the values 0-31 until the cards are completely filled. Examples up to the value of 10 are shown below along with the Binary Numbers worksheet filled out for the first 10 numbers.



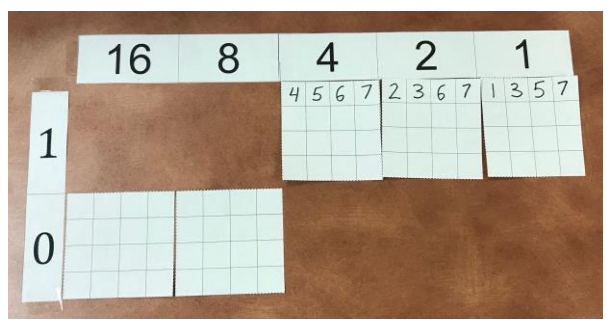
4 = 00100



4 + 1 = 5 = 00101

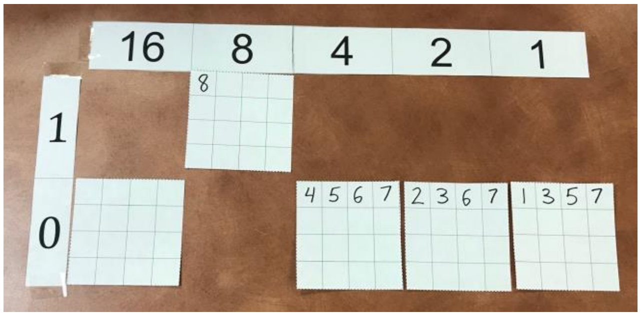


4 + 2 = 6 = 00110

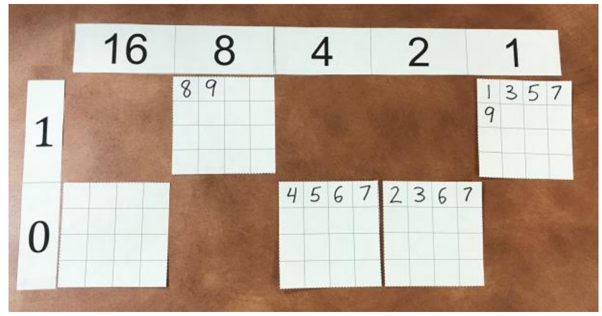


4 + 2 + 1 = 7 = 00101

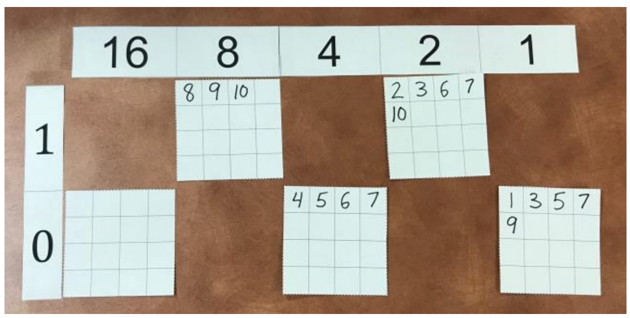
DaNel.Hogan@schools.pima.gov
Director of The STEMAZing Project



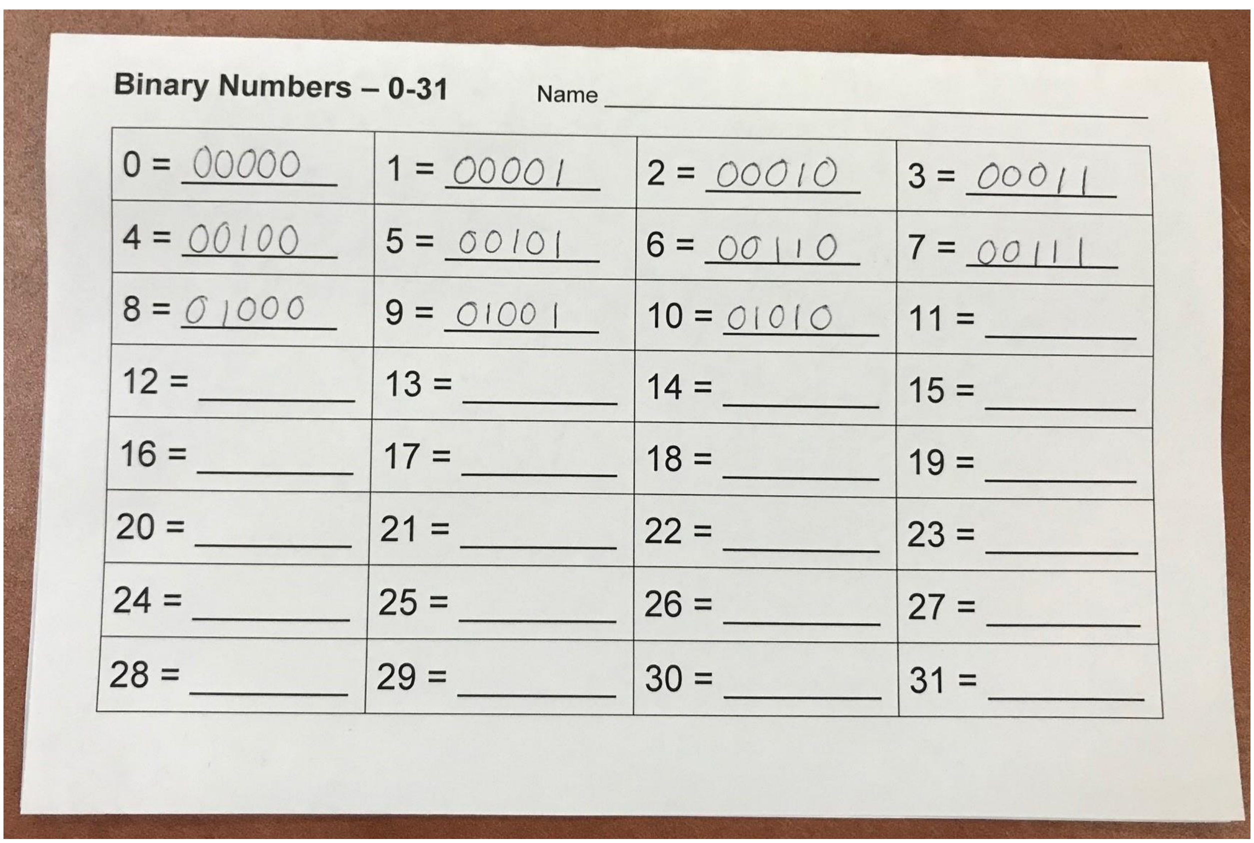
8 = 01000



8 + 1 = 9 = 01001



8 + 2 = 10 = 01010



Binary Numbers – 0-31 Name ____________________

0 = 00000	1 = 00001	2 = 00010	3 = 00011
4 = 00100	5 = 00101	6 = 00110	7 = 00111
8 = 01000	9 = 01001	10 = 01010	11 = ________
12 = ________	13 = ________	14 = ________	15 = ________
16 = ________	17 = ________	18 = ________	19 = ________
20 = ________	21 = ________	22 = ________	23 = ________
24 = ________	25 = ________	26 = ________	27 = ________
28 = ________	29 = ________	30 = ________	31 = ________

PIMA COUNTY
SCHOOL SUPERINTENDENT

DaNel.Hogan@schools.pima.gov
Director of The STEMAZing Project

Students should continue filling out the cards until they are all full of numbers. The last number to go in the bottom right square of each blank card should be 31.

Explain:
The students should now be able to explain why all the numbers are on each card. In a nutshell, those are the numbers for which that position in the binary number system is turned "on" or a 1 to make that value. For instance, 13 shows up on the 8, 4, and 1 position cards because 8 + 4 + 1 = 13. This leads directly into the math "magic" card trick.

Use the Binary "Math" Magic Lesson to have students notice and wonder about the patterns they see on the cards. Then, show them the trick you can do using the 0-31 cards they built or printing out the ready-made cards.

Explore:
For further exploration, you can challenge students to come up with a simple rule about how to increase a binary number by 1. There is a simple rule, which can be expressed in one sentence, to explain what you would do to increase the number by 1. This involves looking at patterns to figure out what this simple rule might be.

SPOILER ALERT: The rule is this – To increase a binary by one, simply start at the first position on the right and switch cards from 0 to 1 and 1 to 0 until you reach a position which was initially at 0 and you moved it up to 1.

Students can also be challenged with creating the cards 0-63 and 0-127. Templates for the setup and blank cards for these sets are included below. These represent the first six positions in binary and the first seven positions in binary.

Other ways to engage students with binary numbers include:

- Can you figure out a simple way to explain what happens when you add two binary numbers together?
- Can you figure out a simple way to explain what happens when you subtract two binary numbers?
- What happens when you multiply two binary numbers? Is there a simple way to explain how to get the product?
- What happens when you divide two binary numbers? Is there a simple way to explain how to get the quotient?

Here is a good resource for the binary arithmetic above:
https://ryanstutorials.net/binary-tutorial/binary-arithmetic.php

Binary Numbers – 0-31

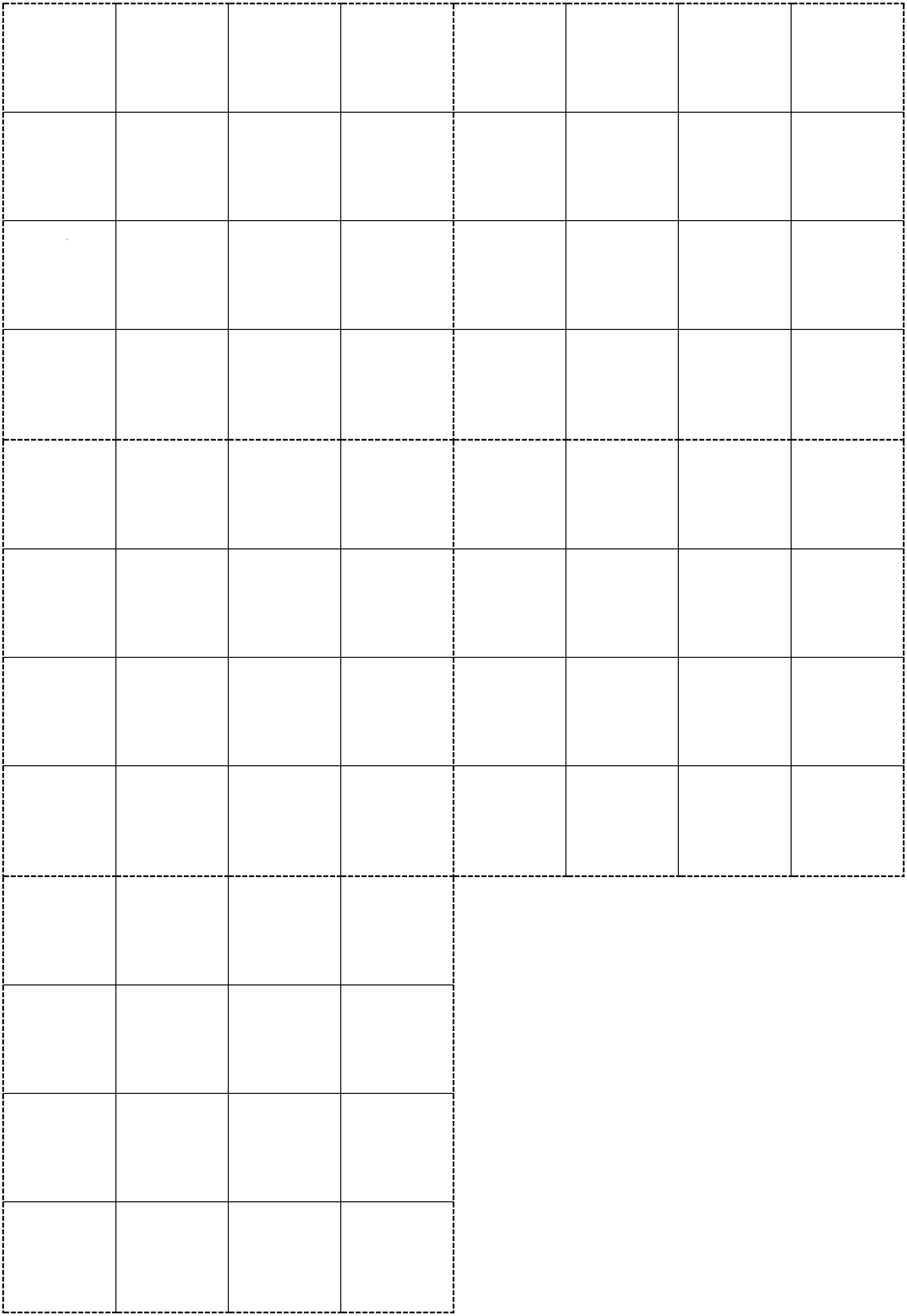
Name ______________________________

0 = __________	1 = __________	2 = __________	3 = __________
4 = __________	5 = __________	6 = __________	7 = __________
8 = __________	9 = __________	10 = __________	11 = __________
12 = __________	13 = __________	14 = __________	15 = __________
16 = __________	17 = __________	18 = __________	19 = __________
20 = __________	21 = __________	22 = __________	23 = __________
24 = __________	25 = __________	26 = __________	27 = __________
28 = __________	29 = __________	30 = __________	31 = __________

Binary Numbers – 0-31

Name ______________________________

0 = __________	1 = __________	2 = __________	3 = __________
4 = __________	5 = __________	6 = __________	7 = __________
8 = __________	9 = __________	10 = __________	11 = __________
12 = __________	13 = __________	14 = __________	15 = __________
16 = __________	17 = __________	18 = __________	19 = __________
20 = __________	21 = __________	22 = __________	23 = __________
24 = __________	25 = __________	26 = __________	27 = __________
28 = __________	29 = __________	30 = __________	31 = __________



4	2	1

16	8

1	0

Binary Numbers: 0-63

Name ______________________________

0 = __________	1 = __________	2 = __________	3 = __________	4 = __________
5 = __________	6 = __________	7 = __________	8 = __________	9 = __________
10 = __________	11 = __________	12 = __________	13 = __________	14 = __________
15 = __________	16 = __________	17 = __________	18 = __________	19 = __________
20 = __________	21 = __________	22 = __________	23 = __________	24 = __________
25 = __________	26 = __________	27 = __________	28 = __________	29 = __________
30 = __________	31 = __________	32 = __________	33 = __________	34 = __________
35 = __________	36 = __________	37 = __________	38 = __________	39 = __________
40 = __________	41 = __________	42 = __________	43 = __________	44 = __________
45 = __________	46 = __________	47 = __________	48 = __________	49 = __________
50 = __________	51 = __________	52 = __________	53 = __________	54 = __________
55 = __________	56 = __________	57 = __________	58 = __________	59 = __________
60 = __________	61 = __________	62 = __________	63 = __________	64 = __________

Can you guess 64?

2	1
8	4
32	16

1	0

Binary Numbers: 0-127

Name ___________________________________

0 = ________	1= ________	2 = ________	3 = ________	4 = ________	5 = ________	6 = ________
7 = ________	8 = ________	9 = ________	10 = ________	11 = ________	12 = ________	13 = ________
14 = ________	15 = ________	16 = ________	17 = ________	18 = ________	19 = ________	20 = ________
21 = ________	22 = ________	23 = ________	24 = ________	25 = ________	26 = ________	27 = ________
28 = ________	29 = ________	30 = ________	31 = ________	32 = ________	33 = ________	34 = ________
35 = ________	36 = ________	37 = ________	38 = ________	39 = ________	40 = ________	41 = ________
42 = ________	43 = ________	44 = ________	45 = ________	46 = ________	47 = ________	48 = ________
49 = ________	50 = ________	51 = ________	52 = ________	53 = ________	54 = ________	55 = ________
56 = ________	57 = ________	58 = ________	59 = ________	60 = ________	61 = ________	62 = ________
63 = ________	64 = ________	65 = ________	66 = ________	67 = ________	68 = ________	69 = ________
70 = ________	71 = ________	72 = ________	73 = ________	74 = ________	75 = ________	76 = ________
77 = ________	78 = ________	79 = ________	80 = ________	81 = ________	82 = ________	83 = ________
84 = ________	85 = ________	86 = ________	87 = ________	88 = ________	89 = ________	90 = ________
91 = ________	92 = ________	93 = ________	94 = ________	95 = ________	96 = ________	97 = ________
98 = ________	99 = ________	100 = ________	101 = ________	102 = ________	103 = ________	104 = ________
105 = ________	106 = ________	107 = ________	108 = ________	109 = ________	110 = ________	111 = ________
112 = ________	113 = ________	114 = ________	115 = ________	116 = ________	117 = ________	118 = ________
119 = ________	120 = ________	121 = ________	122 = ________	123 = ________	124 = ________	125 = ________
126 = ________	127 = ________	128 = ________				

Can you guess 128?

4	2	1

32	16	8

64

1	0