

ICTCM - Scottsdale
Kierland 1A

Saturday - Mar. 16, 2019
9:00 to 9:30

Is it Magic?
No. It's Mathematics.

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Rochester Community and Technical College

Today's Presentation

- * Predicting an outcome for scrambled cards
- * Predicting the sum of "random" numbers
- * Mind reading with cards
- * Calculator magic
- * Fitch's card trick

Predicting the Outcome for Scrambled Cards

(a self-working card trick)

Demonstration



Performing This Trick

- **Determine your desired outcome (prediction) ahead of time. Select those cards from the deck and place them, randomly, as the bottom “half” in the deck.**
- **Arrange two cards in the middle of the deck to be the point for cutting the deck. Fan open the deck to show that the cards are “randomly” shuffled. Cut the deck at the predetermined position. Hand the top half of the deck to the audience member.**
- **Each person randomly chooses cards from his/her deck and places them on the table. Each person then takes the cards that the other person placed on the table, turns them over, and places them in his/her deck. Each person shuffles his/her deck.**
- **Repeat the previous step as many times as you wish (3 times is fairly convincing.)**
- **At the end, turn over your entire deck and place it on top of the audience member’s deck. Ask the audience member to spread out the cards. The prediction is now face up.**

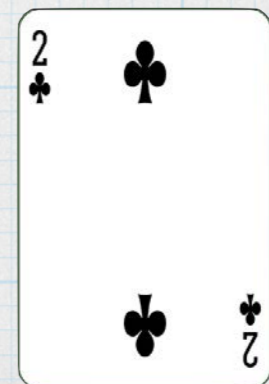
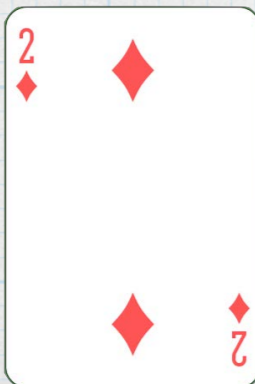
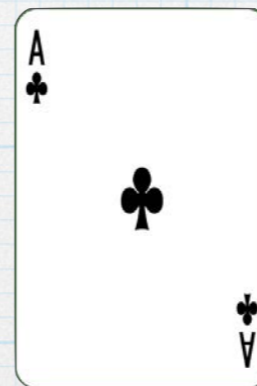
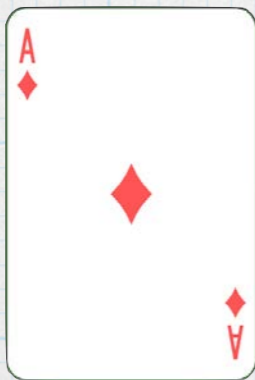
Why it Works

Consider a six-card deck split into black and red cards.

Three Black Cards



Three Red Cards



Predicting the Sum of "Random" Numbers

Demonstration

4791

5208

3689

6310

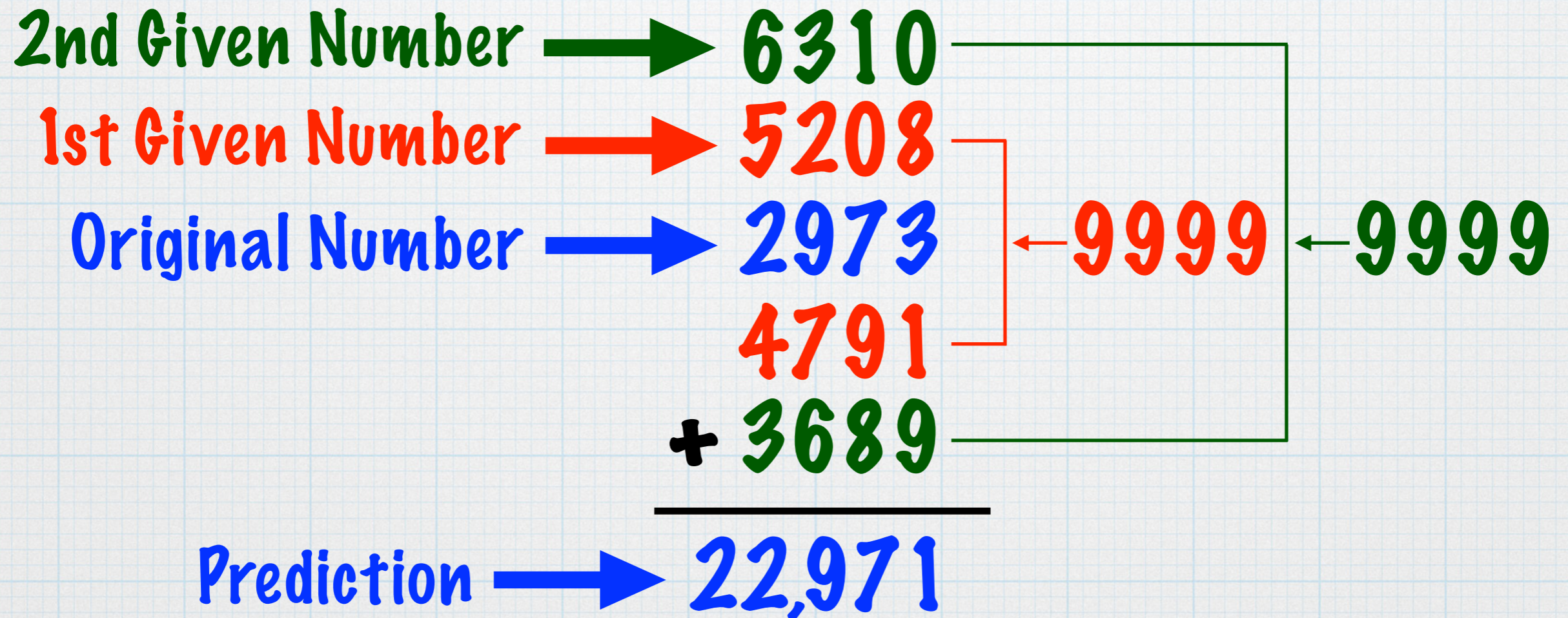
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22971

Performing This Trick

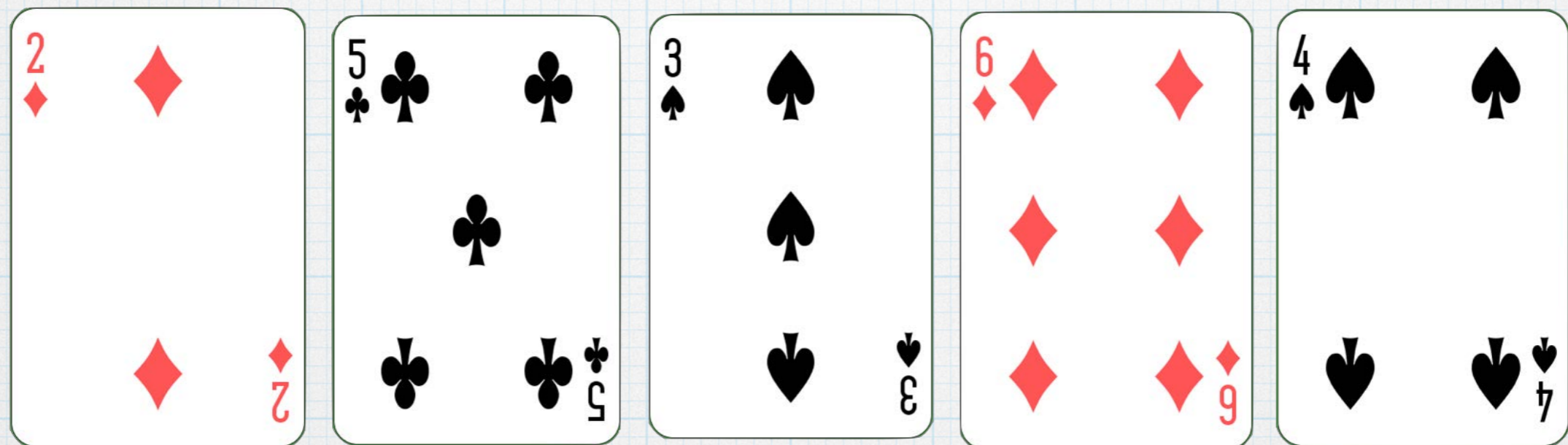
- Have an audience member name a four-digit number. Asking for four different digits is not necessary, but will help to avoid giving away the mechanism for this trick.
- After this original number is named, write down your prediction on a piece of paper. Your prediction is (the given four-digit number) + 20,000 - 2.
- Have a different audience member name a four-digit number. Write it down at the top of the list (above the original number.) Now write down your own four-digit number at the bottom of the list (below the original number.) Your number is constructed so that each digit of your number sums to 9 with corresponding digits of the number written at the top of the list.
- Repeat the previous step one more time.
- By constructing your numbers in the specified way, you have simply added 19,998 to the original number.

How it Works



“Mind Reading” with Cards

Demonstration



Performing This Trick

- Using only A - 8 from each suit, arrange the 32 cards in a cyclic pattern (described later.)
- Line up five audience members and hand the cards to the rightmost person. Have each person, in turn from right to left, cut the cards 0, 1, or 2 times. Then have each person, in turn from left to right, select the top card from the deck.
- Pretend to read their minds but complain about brain wave "noise" in the room. Ask the participants who have red cards to raise their hands.
- Refer to the cheat sheet (described later) for the selected card order - from left to right.

How it Works...

The de Bruijn Sequence

Dutch mathematician, Nicolaas Govert de Bruijn, proved that for any size- k alphabet A there is a cyclic sequence in which every possible length- n string on A occurs exactly once as a substring. Such string has length k^n .

Here's one for size-2 alphabet and $k = 3$

00010111

000, 001, 010, 101, 011, 111, 110, 100

A de Bruijn Sequence for size-2 alphabet, $k = 5$

00000100101100111110001101110101

The key to this trick:

If we lop off any number of digits from the end of this string and paste them on the beginning of the string, the cyclic sequence remains intact.

Why is this Important?

Every card from Ace to 8 in any suit can be uniquely identified with 5 bits.

First Two bits

00 = Clubs
01 = Spades
10 = Diamonds
11 = Hearts

Last Three bits

001 = Ace	101 = 5
010 = 2	110 = 6
011 = 3	111 = 7
100 = 4	000 = 8

Note that the first bit identifies the card color.

A de Bruijn Sequence for size-2 alphabet, $k = 5$

00000100101100111110001101110101
RBRRR

8C,AC,2C,4C,AS,2D,5C,3S,6D,4S,AH,3D,7C,7S,7H,6H,
4H,8H,AD,3C,6C,5S,3H,7D,6S,5H,2H,5D,2S,4D,8S,8D.



If I know the five cards are arranged as **RBRRR**, then
I know that the cards are **7D, 6S, 5H, 2H, and 5D**.

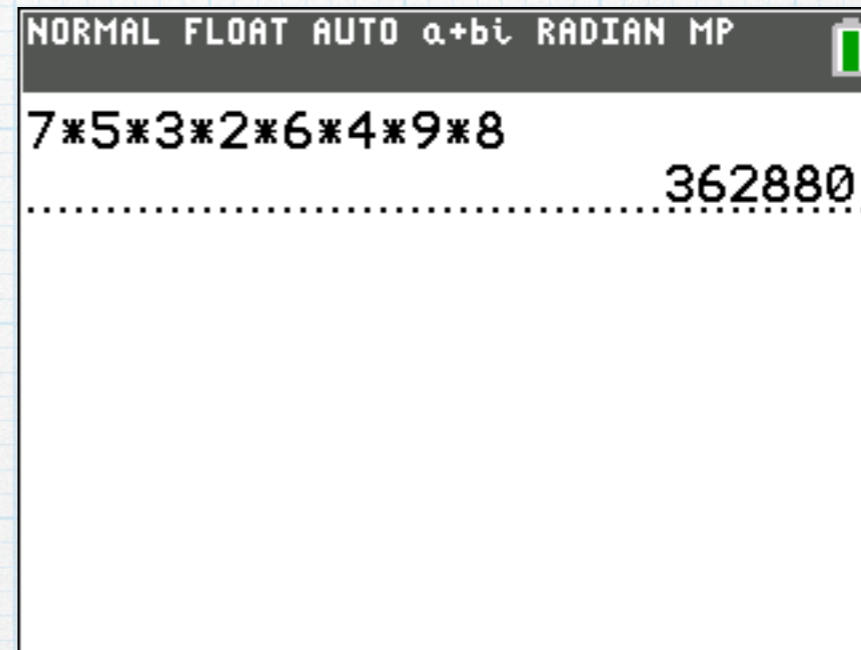
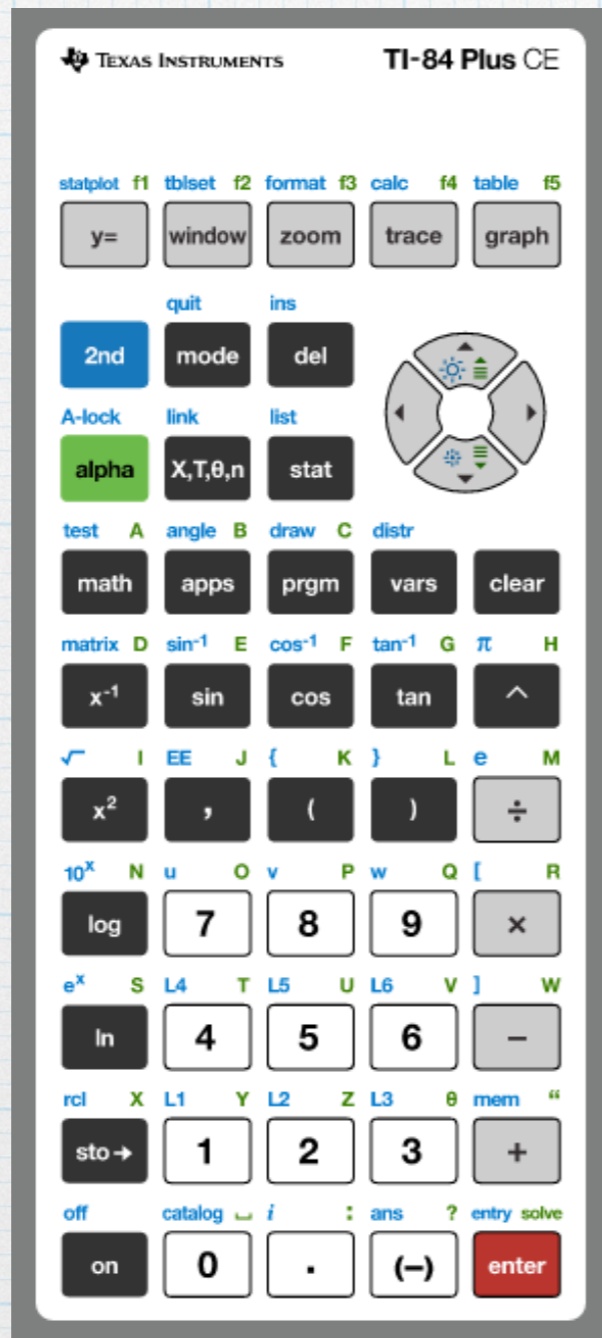
Mind Reading

00000	8♣, A♣, 2♣, 4♣, A♠	01000	8♠, 8♦, 8♣, A♣, 2♣
00001	A♣, 2♣, 4♣, A♠, 2♦	01001	A♠, 2♦, 5♣, 3♠, 6♦
00010	2♣, 4♣, A♠, 2♦, 5♣	01010	2♠, 4♦, 8♠, 8♦, 8♣
00011	3♣, 6♣, 5♠, 3♥, 7♦	01011	3♠, 6♦, 4♠, A♥, 3♦
00100	4♣, A♠, 2♦, 5♣, 3♠	01100	4♠, A♥, 3♦, 7♣, 7♠
00101	5♣, 3♠, 6♦, 4♠, A♥	01101	5♠, 3♥, 7♦, 6♠, 5♥
00110	6♣, 5♠, 3♥, 7♦, 6♠	01110	6♠, 5♥, 2♥, 5♦, 2♠
00111	7♣, 7♠, 7♥, 6♥, 4♥	01111	7♠, 7♥, 6♥, 4♥, 8♥
10000	8♦, 8♣, A♣, 2♣, 4♣	11000	8♥, A♦, 3♣, 6♣, 5♠
10001	A♦, 3♣, 6♣, 5♠, 3♥	11001	A♥, 3♦, 7♣, 7♠, 7♥
10010	2♦, 5♣, 3♠, 6♦, 4♠	11010	2♥, 5♦, 2♠, 4♦, 8♠
10011	3♦, 7♣, 7♠, 7♥, 6♥	11011	3♥, 7♦, 6♠, 5♥, 2♥
10100	4♦, 8♠, 8♦, 8♣, A♣	11100	4♥, 8♥, A♦, 3♣, 6♣
10101	5♦, 2♠, 4♦, 8♠, 8♦	11101	5♥, 2♥, 5♦, 2♠, 4♦
10110	6♦, 4♠, A♥, 3♦, 7♣	11110	6♥, 4♥, 8♥, A♦, 3♣
10111	7♦, 6♠, 5♥, 2♥, 5♦	11111	7♥, 6♥, 4♥, 8♥, A♦

Source: *Magical Mathematics* by Persi Diaconis

Calculator Magic

Demonstration



Performing This Trick

- Ask a student to continuously multiply random, one-digit numbers on her/his calculator until the result is a six digit number.
- Have the student read any five of the digits to you.
- Add the digits in your head.
- Find the difference between your sum and the nearest multiple of 9 that is greater than or equal to your sum.
- The missing digit is this difference. (If the difference is 0, the missing digit is either a 0 or a 9.)

Why it Works

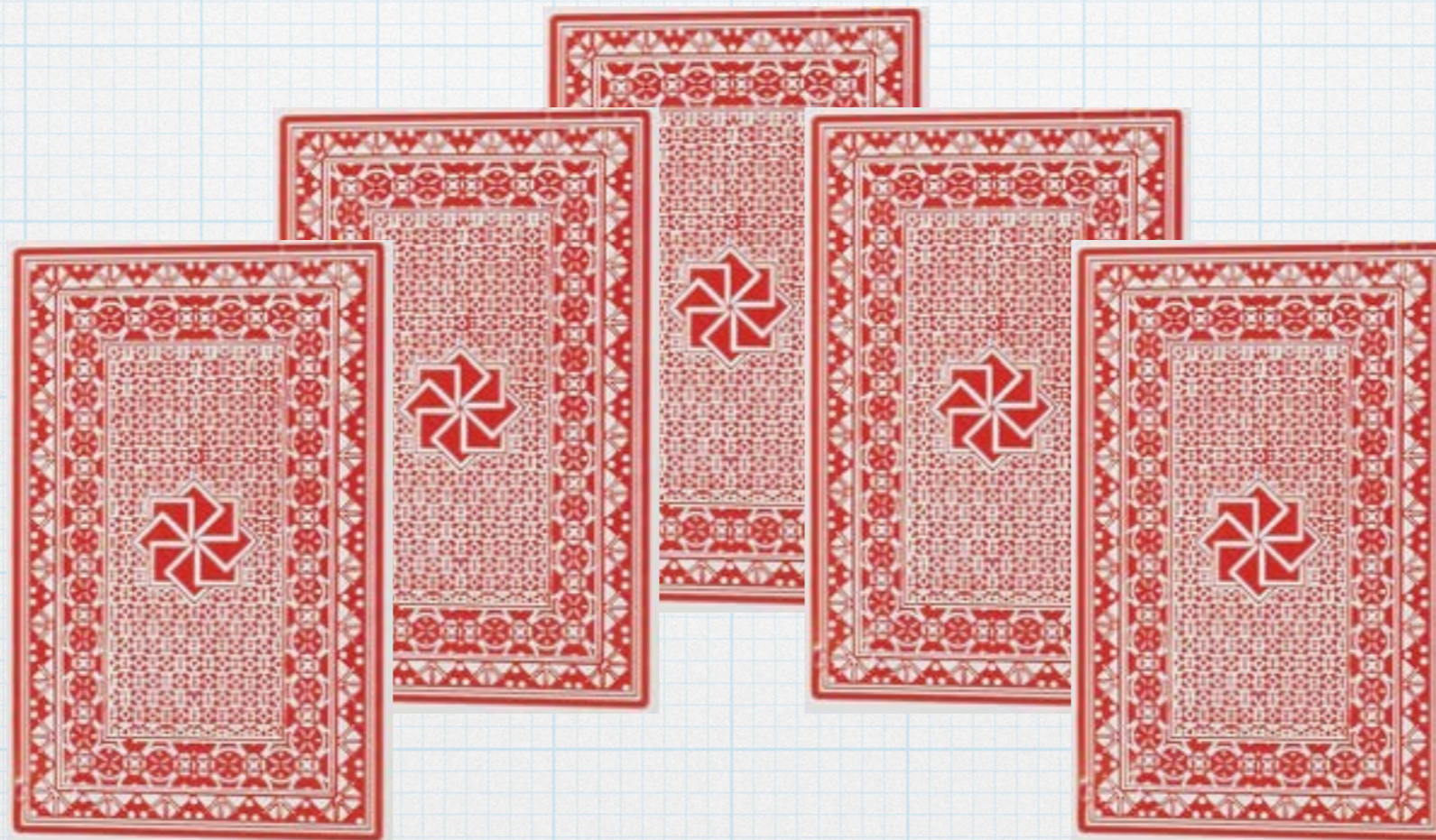
(Almost All of the Time)

- There is a very high probability that the 6-digit number resulting from multiplying a sequence of “random” digits will be a multiple of 9.
- ie. If we ever multiply $(3 \cdot 3)$, $(3 \cdot 6)$, or by 9, we get a multiple of 9.
- The digits of any number that is a multiple of 9 will sum to a multiple of 9.

Fitch's Card Trick

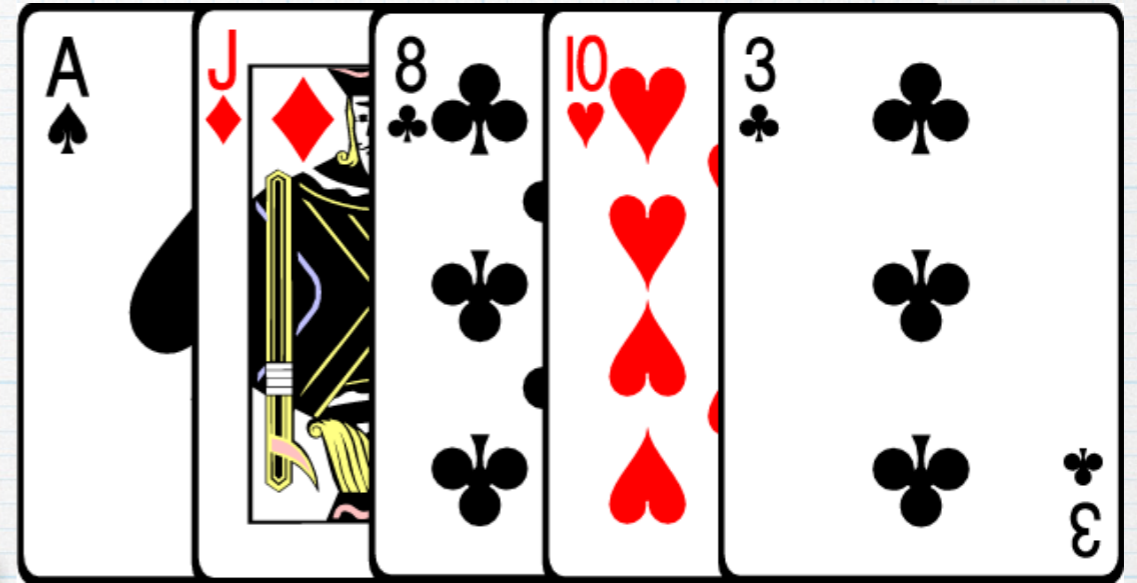
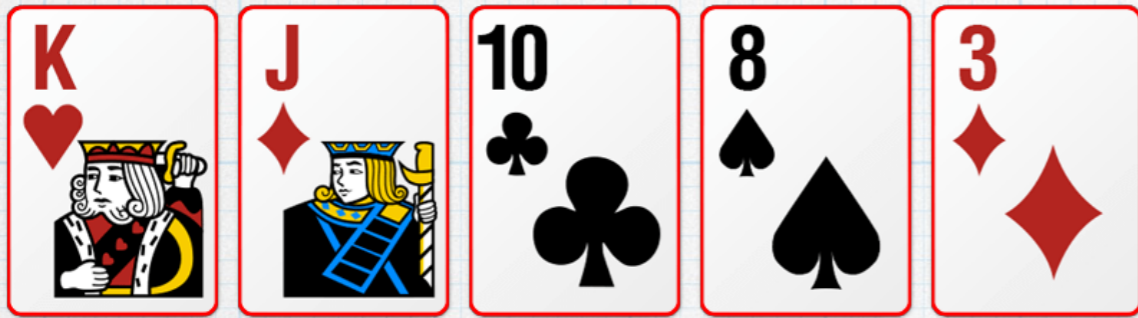
- * The pigeon hole principle
- * Simple arithmetic
- * Modular arithmetic
- * Combinatorics

Demonstration



The Pigeon Hole Principle

Given five cards, at least two must be of the same suit.



The Main Idea

No two cards in one suit are more than 6 apart.



Which Card Goes Face Down?

Suppose we have the **2** and **Jack** of **Diamonds**.

- * Locate the two cards on the wheel.
- * Count clockwise from the *target* card to the *unknown* card in 6 or fewer steps.
- * Place the *unknown* card face down on the table.
- * Remember the count from the *target* card to the *unknown* card.



Communicating the Target Card Using the Four Remaining Cards

- * Find the sum S of the face values of the cards, with Ace = 1 and Jack = Queen = King = 10.
- * Then $(S \bmod 4)$ gives the position (from left to right) of the target card (and suit).



Target Card
Jack of
Diamonds

The Agreement

Possible arrangements of 3 cards = $3 \cdot 2 \cdot 1 = 6$

Low–Medium–High (LMH) = 1

Low–High–Medium (LHM) = 2

Medium–Low–High (MLH) = 3

Medium–High–Low (MHL) = 4

High–Low–Medium (HLM) = 5

High–Medium–Low (HML) = 6

Let's Try Some



A "Difficult" Case



Math Magic for Students



I find if you put that slash through the equal sign, the number of possible correct answers vastly increases.

Thank You for Attending

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