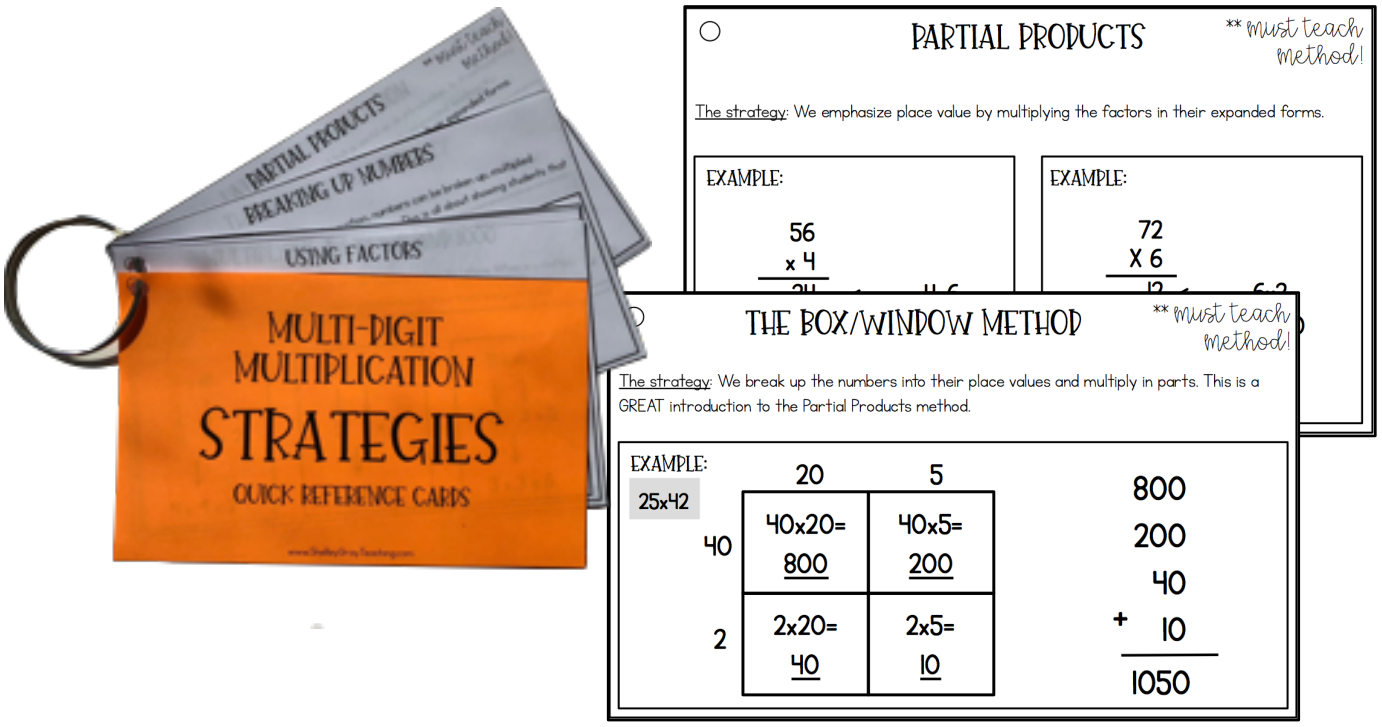
shelley grays

MULTI-DIGIT MULTIPLICATION & DIVISION STRATEGY GUIDE



introduction

Hi there! I'm Shelley Gray, and I want to challenge you to focus intensively on math facts and strategies this school year. Let's stop seeing math facts as an isolated math unit, and begin integrating them wherever possible into our math and daily routines.

progressing to multi-digit

Sometimes we think that just because we teach a certain grade level, we need to be working on only the curriculum expectations from that grade. However we know that students will not be successful with multi-digit multiplication and division until they know their basic facts. If your students are still struggling with basic facts, consider taking some time to work on basic multiplication and division before diving into the strategies outlined in this guide.

big goals

We want to teach our students to be flexible thinkers when it comes to solving an equation. This means that they are able to manipulate the numbers in different ways in order to solve a problem. The steps that one student takes to solve a problem might be very different than the steps that another student takes. We want to celebrate this flexible thinking!

Math fact fluency should not be based on the ability to perform a memorized series of steps. It is so much more than that.

Throughout your math fact instruction and practice this year, try to keep three main words in mind when it comes to how your students are solving a problem or equation: EFFECTIVE, EFFICIENT, FLEXIBLE. Is the strategy effective and efficient (is it quick and works well)? Are they able to think flexibly with the numbers?

how to use this guide

This guide is intended as a reference guide for the various mental math strategies that are best-suited to your particular grade level.

It can be really confusing to teach math strategies. How do you integrate them? When do you move on to the next one? How do you differentiate to the different ability levels?

My hope is that this guide gives you a starting point for reinforcing the strategies. Begin with the first strategy, allow your students to master it, and then move along to the next one.

If you are not in our 30-Day Math Fact Challenge private Facebook group yet, be sure to join so that you collaborate with other teachers who have the same goals as you. Join here: https://www.facebook.com/groups/1246720380226 27/

resources

You do not need to purchase any resources to reinforce these strategies. You simply need a commitment to teaching and reinforcing them throughout the year.

However, if you would like a complete system to help you do this, here is a link to The Multi-Digit Multiplication Station and The Long Division Station, which will reinforce all of the strategies that are outlined in this guide. The entire Multiplication and Division Station programs are self-paced so that students will move through the strategies as they feel ready.

Multi-Digit Multiplication Station:
https://www.teacherspayteachers.com/Product/The-Multi-Digit-Multiplication-Station-self-paced-student-centered-3157826

Long Division Station:
https://www.teacherspayteachers.com/Product/The-Long-Division-Station-self-paced-student-centered-3552960

what's included?

QUICK REFERENCE CARDS

The Quick Reference Cards can be laminated and put on a ring for quick and easy reference to the strategies that are best suited for this grade level.

They can also be used for oral assessments. I highly recommend oral assessments to assess math strategy knowledge. When you SEE a student solve an equation, you get a far different perspective than you do when you simply mark a written solution.

Oral assessments enable you to see which facts/strategies a student struggles with, which ones are quicker than the rest, and which strategies are used to solve a problem.

Oral assessment is the assessment method that is used in all of my math stations. Although this might seem like a huge task, it only takes about 1-2 minutes, and many teachers report that it is their favorite part of using the stations.

If you would like to try oral assessments, you can use the Quick Reference Cards from the previous few pages as a guide. Look for the following:

- Is he using an effective strategy to solve the equation?
- Is his strategy efficient? (meaning that he can solve the equation in 1-3 seconds)
- Can you see flexibility in his thinking? (is he able to manipulate the numbers in a flexible way to make the strategy work for him?)

MULTI-DIGIT MULTIPLICATION

PAGES 7-11

MULTI-DIGIT MULTIPLICATION

STRATEGIES

QUICK REFERENCE CARDS

www.ShelleyGrayTeaching.com

USING FACTORS

<u>The strategy</u>: A factor can be split into two smaller factors.

This will help students see how equations can be manipulated to make them easier to solve.

EXAMPLES:

4 x 18

↓ ↓

4 9 x 2

↓

4 x 9 x 2

5 x 22

↓ ↓

5 11 x 2

↓

5 x 11 x 2

3 x 15

↓ ↓

3 3 x 5

↓

3 x 3 x 5

MULTIPLYING BY 10, 100, AND 1000

<u>The strategy</u>: When we multiply by 10, we increase the place values by 1 place. When we multiply by 100, we increase the place values by 2 places. When we multiply by 1000, we increase the place values by 3 places.

Once students possess place value understanding, we can teach them the "adding zeros" trick, where we add 1 zero when we multiply by 10, 2 zeros when we multiply by 100, or 3 zeros when we multiply by 1000.

EXAMPLE:

6×100 → 6 → 600

Let's look at the other number (not the 100). In this case that is 6.

We INCREASE the place value by TWO places. This means that we add TWO zeros to the end.

Now the 6 is in the hundreds place instead of the ones place. To increase the place values we added two zeros. Our number is now 600.

MULTIPLYING 1-DIGIT NUMBERS BY MULTIPLES OF 10, 100, AND 1000

We can start by factoring and grouping the numbers differently:

EXAMPLE:

5×3000

$5 \times 3 \times 1000$ ← Let's break the 3000 into 3x1000.

$(5 \times 3) \times 1000$ ← Now let's group the numbers differently, so that we multiply by 1000.

$15 \times 1000 = \underline{15{,}000}$ ← Now we can use what we know about multiplying by 1000!

Then later we can teach the shortcut!

EXAMPLE:

$5 \times 3000 = \underline{15{,}000}$

1. Multiply 5x3.
2. Then add THREE zeros, since there are 3 zeros in the equation.

MULTIPLYING 2-DIGIT NUMBERS BY MULTIPLES OF 10, 100, AND 1000

We can start by factoring and grouping the numbers differently:

EXAMPLE:

40 x 60

4x10 x 6x10 ← We can break up both numbers.

(4x6)x(10x10) ← Now we can group the numbers differently.

24x100=2400 ← Now we can use what we know about multiplying by 100!

Then later we can teach the shortcut!

EXAMPLE:

40x60=2400

1. Take off the 2 zeros and multiply 4x6.
2. Write the product (24) and add the two zeros back.

BREAKING UP NUMBERS

The strategy: When faced with a difficult equation, numbers can be broken up, multiplied separately, and then the products can be added together. This is all about showing students that there are many ways to solve an equation.

EXAMPLE:

11 x 30

10x30 1x30 ← We can break the 11 up into a 10 and a 1, and multiply both of those numbers by 30.

300 30

330 ← Then we can add the products together.

101 x 20

100x20 1x20

2000 20

2020

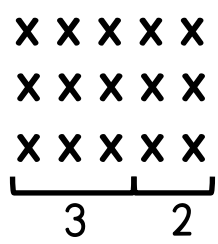
THE DISTRIBUTIVE PROPERTY

The strategy: When one factor is written as the sum of two numbers, the product does not change. We can illustrate this using an array.

EXAMPLE:

This array represents 3x5.

We could also break it up. Now it represents 3x(3+2). The 3+2 is simply the 5, broken into two numbers.



EXAMPLE:

5x34 → 5x(30+4) → (5x30)+(5x4) → 150+20=170

THE BOX/WINDOW METHOD

** must teach method!

The strategy: We break up the numbers into their place values and multiply in parts. This is a GREAT introduction to the Partial Products method.

EXAMPLE:

25x42

	20	5
40	40x20= 800	40x5= 200
2	2x20= 40	2x5= 10

800
200
40
\+ 10
1050

PARTIAL PRODUCTS

** must teach method!

The strategy: We emphasize place value by multiplying the factors in their expanded forms.

EXAMPLE:

$$\begin{array}{r} 56 \\ \times\ 4 \\ \hline 24 \\ +\ 200 \\ \hline 224 \end{array} \quad \begin{array}{l} \leftarrow 4\times6 \\ \leftarrow 4\times50 \end{array}$$

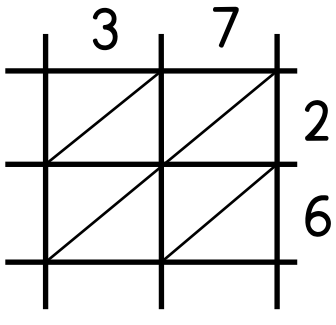
EXAMPLE:

$$\begin{array}{r} 72 \\ \times\ 6 \\ \hline 12 \\ +\ 420 \\ \hline 432 \end{array} \quad \begin{array}{l} \leftarrow 6\times2 \\ \leftarrow 6\times70 \end{array}$$

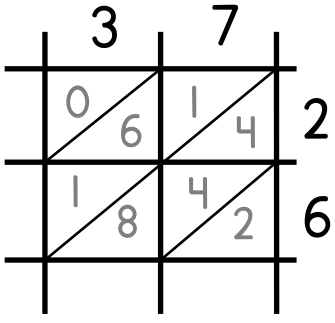
LATTICE MULTIPLICATION

The strategy: This method is not mental math based, but is useful as an introduction to traditional long multiplication (if you will be teaching the traditional method). I do not recommend teaching this method unless your students have a solid understanding of partial products.

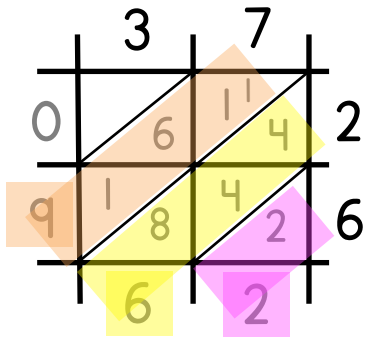
Step 1: Draw a lattice and write the factors along the top and right-hand side.



Step 2: Multiply the numbers and write the products in the grid. For example, in the bottom right square, you would multiply 7x6 to make 42.



Step 3: Add the numbers in each diagonal row. If you need to carry a number, it gets carried to the next diagonal row.



Step 4: The product is the numbers that you wrote along the side and bottom, in this case 962. So 37x26=962.

HALVING AND DOUBLING

The strategy: This can be used to make *some* equations easier. It's also a fun little trick for students to learn! This will not make all equations easier, but it works best when multiplying numbers like 5, 25, 50, 500, etc. Halving and doubling works by creating two new factors that are easier to multiply.

EXAMPLE:

24x50

HALF ↓ DOUBLE ↓

12x100=1200

EXAMPLE:

28x50

HALF ↓ DOUBLE ↓

14x100=1400

EXAMPLE:

36x500

HALF ↓ DOUBLE ↓

18x1000=18,000

TRADITIONAL LONG MULTIPLICATION

This is the traditional method for long multiplication. If you decide to teach this approach, I recommend ONLY teaching it once students have a solid understanding of the box/window or partial products methods. There is not mental math understanding involved in this method.

EXAMPLE:

434
x 2

8	6	8

Step 1: Multiply 2x4.

Step 2: Multiply 2x3.

Step 3: Multiply 2x4.

EXAMPLE:

233
x 3

6	9	9

Step 1: Multiply 3x3.

Step 2: Multiply 3x3.

Step 3: Multiply 3x2.

LONG DIVISION

PAGES 13-16

LONG DIVISION

STRATEGIES

QUICK REFERENCE CARDS

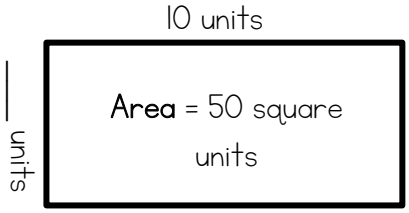
www.ShelleyGrayTeaching.com

RELATING DIVISION TO MULTIPLICATION

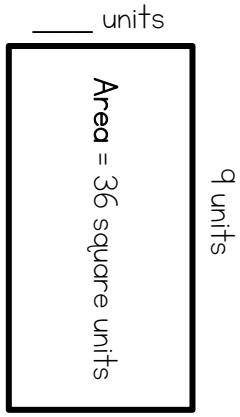
Strategy: Understand the relationship between multiplication and division. Use multiplication to solve a division equation.

EXAMPLE:

Solve the unknown.



EXAMPLE:



EXAMPLE:

Use a multiplication equation to write two division equations.

22x5=110

____÷____=____

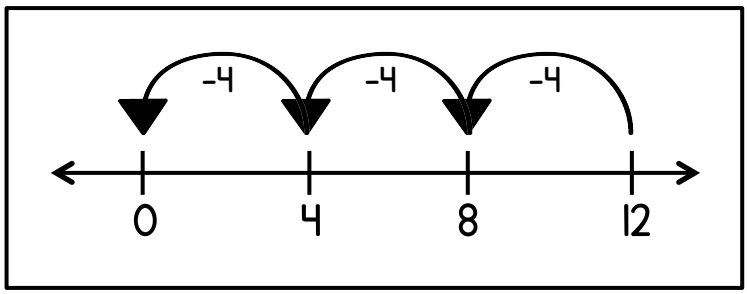
____÷____=____

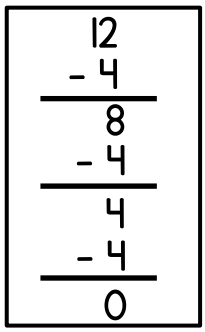
REPEATED SUBTRACTION

Strategy: Students should have already received practice with repeated subtraction when they learned basic division. The concept of repeated subtraction is important for upcoming long division concepts that students will be learning in later levels, particularly the Box Method and Partial Quotients.

EXAMPLE: 12÷4 ⇦ We can show it on a number line OR with a subtraction equation!

Number Line





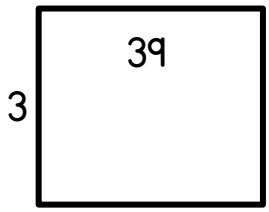
Subtraction Equation

THE BOX METHOD

** must teach method!

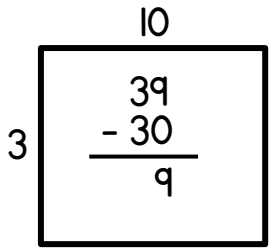
Strategy: We organize our thinking using boxes, and subtract multiples of the divisor to get down to 0.

EXAMPLE: 39÷3



STEP 1

Let's draw a box. We will write the dividend (39) inside the box and the divisor (3) on the left side.

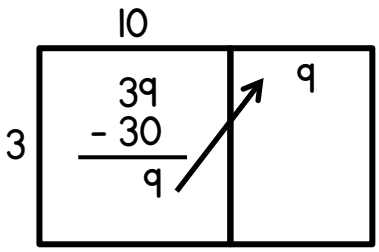


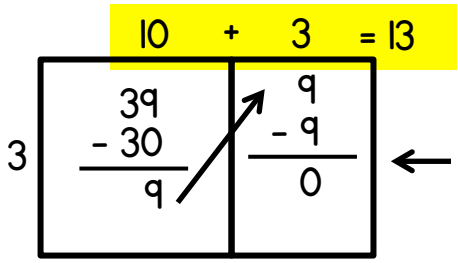
STEP 2

Now we need to subtract a multiple of 3. We can use any multiple that we know, that is less than 39. We know that 3x10 is 30, so let's subtract 30.

STEP 3

Now we have to add another box to do some more subtraction. Let's carry the 9 over to the next box.





STEP 4

We'll take away 3 groups of 3, so we will write 3 on top and subtract 9 to get to 0.

PARTIAL QUOTIENTS

** must teach method!

Strategy: The partial quotients strategy involves subtracting parts until you get to 0. This is a natural progression from the previous box strategy.

EXAMPLE #1

3 | 633	
-300	x 100
333	
-300	x 100
33	
-30	x 10
3	
-3	x 1
0	

100+100+10+1=211 So 633÷3=211.

In example #1, we took away 100 groups of 3, then another 100 groups, then 10 groups, and then 1 group.

In example #2, we took away 200 groups of 3, then 11 groups of 3.

In both examples, we come to the same answer - 211 - but it is faster in the second example because we subtracted bigger numbers.

EXAMPLE #2

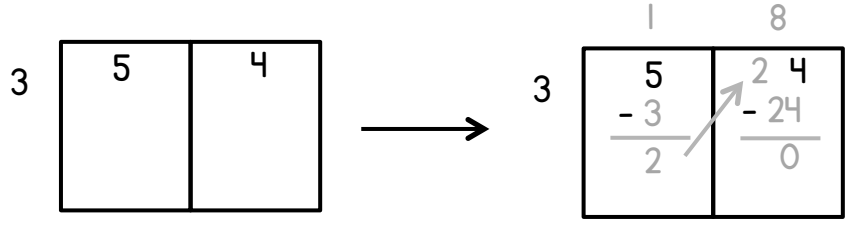
3 | 633	
-600	x 200
33	
-33	x 11
0	

200+11=211 So 633÷3=211.

THE GRID METHOD

The grid method is an introduction to traditional long division. It uses a grid to organize the numbers, which makes this method easier to understand for some students. I recommend only teaching this method once your students have mastered Partial Quotients.

EXAMPLE: 54÷3



Step #1: 3 goes into 5 once, so we write a 1 on top of the grid. Now we multiply that 1 by the 3. This makes 3, so we write "3" in the grid and subtract it from 5. This leaves 2 in the first box.

Step #2: We carry the 2 over to the second box so that our 4 becomes 24. 3 goes into 24 8 times, so we write an 8 on top of the grid 8x3=24, so we subtract 24 in the second box to leave 0. The quotient is 18.

TRADITIONAL LONG DIVISION

This is the traditional method for long division. If you decide to teach this approach, I recommend ONLY teaching it once students have a solid understanding of the box or partial quotients methods. There is not mental math understanding involved in this method.

EXAMPLE: 84÷4

$4\overline{)84}$ ← First of all, let's set up our equation.

$\begin{array}{r} 2 \\ 4\overline{)84} \end{array}$ ← Step 1: Look at the first digit of the dividend. Ask yourself, "How many times does 4 go into 8?" It goes into 8 two times, so we write a "2" on top of the 8.

$\begin{array}{r} 21 \\ 4\overline{)84} \end{array}$ ← Step 2: Look at the second digit of the dividend. Ask yourself, "How many times does 4 go into 4?" It goes into 4 one time, so we write a "1" on top of the 4.

additional resources

Are you looking for more resources to reinforce these mental math strategies in your classroom? The resources below will reinforce the strategies that are outlined in this guide.

THE MULTI-DIGIT MULTIPLICATION STATION

https://www.teacherspayteachers.com/Product/The-Multi-Digit-Multiplication-Station-self-paced-student-centered-3157826

LONG DIVISION TASK CARDS

https://www.teacherspayteachers.com/Product/Long-Division-Task-Cards-The-Big-Bundle-3580272

THE LONG DIVISION STATION

https://www.teacherspayteachers.com/Product/The-Long-Division-Station-self-paced-student-centered-3552960