Name____ Date_________________________ Period______

Worksheet 9.1—Sequences & Series: Convergence & Divergence
Show all work. No calculator except unless specifically stated.

Short Answer

1. Determine if the sequence $\left\{\frac{\ln n}{n^2}\right\}$ converges.

2. Find the *n*th term (rule of sequence) of each sequence, and use it to determine whether or not the sequence converges.

(a) $2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25}, \ldots$

(b) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

3. Use the *n*th Term Divergence Test to determine whether or not the following series converge:

(a) $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (c) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$ (d) $\sum_{n=1}^{\infty} \frac{(n+2)!}{10n!}$

4. (Calculator Permitted)

(a) What is the sum of $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3} \right)$

(b) Using your calculator, calculate S_{500} to verify that the SOPS (sum of the partial sums) is bounded by the sum you found in part (a). (Calculator entry shown at right.)

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sum(seq(1/(N+1)-
1/(N+3),N,1,500)
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5. Use the indicated test for convergence to determine if the series converges or diverges. If possible, state the value to which it converges.

(a) Geometric Series: $3+\frac{15}{4}+\frac{75}{16}+\frac{375}{64}+\cdots$

(b) Geometric Series: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\cdots$

(c) p-series: $\sum_{n=1}^{\infty} n^{-2/3}$

(d) Integral Test: $\sum_{n=1}^{\infty} \frac{3n}{2n^2+3}$

(e) Direct Comparison: $\sum_{n=1}^{\infty} \frac{e^n}{n}$

(f) Direct Comparison: $\sum_{n=1}^{\infty} \frac{3^n}{7^n + 1}$

(g) Limit Comparison: $\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$

(h) Limit Comparison: $\sum_{n=1}^{\infty} \frac{n+5}{3n\left(4^n\right)}$

(i) Ratio Test: $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

(j) Ratio Test: $\sum_{n=1}^{\infty} \frac{2}{n^2}$

(k) AST: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

(l) AST: $\sum_{n=1}^{\infty} \frac{(-1)^n (n+3)}{2n}$

(m) Direct Comparison: $\displaystyle\sum_{n=1}^{\infty}\frac{\sin n}{n^2}$

(n) Any viable method: $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\left(4^n\right)}{n!}$

6. (Calculator permitted) To five decimal places, find the interval in which the actual sum of $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$ is contained if S_5 is used to approximate it.

Determine whether or not the series converge using the appropriate convergence test (there may be more than one applicable test.) State the test used. If possible, give the sum of the series.

7. $\sum_{n=0}^{\infty}\left(\frac{2}{7}\right)^n$

8. $\sum_{n=1}^{\infty}\frac{4}{n^3}$

9. $\sum_{n=1}^{\infty}\frac{n^2}{5^n}$

10. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n^5+5}}$

11. $\sum_{n=1}^{\infty}\frac{n^n}{n!}$

12. $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\cdots$

13. $2+\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\cdots$

14. $\sum_{n=1}^{\infty}\frac{5n^2-6n+3}{n^3-7n+8}$

15. $\sum_{n=1}^{\infty}\frac{\cos n\pi}{\sqrt{n}}$

16. $\sum_{n=1}^{\infty} \frac{3^n + 4}{2^n}$

17. $\sum_{n=1}^{\infty} \frac{8n^3 - 6n^5}{12n^4 + 9n^5}$

18. $\sum_{n=1}^{\infty} \sqrt{\frac{3n+1}{n^5 + 2}}$

19. Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{3n+4}}$ converges absolutely, converges conditionally, or diverges.

20. What is the sum of the following:

(a) $\sum_{n=0}^{\infty} \frac{3}{2^n}$ (b) $\sum_{n=2}^{\infty} \left(-\frac{3}{2}\right)^{-n}$ (c) $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$ (d) $\sum_{n=1}^{\infty} \frac{3}{(2n-1)(2n+1)}$

21. (Calculator Permitted) Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

(a) Show that the series is absolutely convergent.

(b) Calculate S_6, the sum of the first six terms. Round your answer to three decimal places.

(c) Find the number of terms necessary to approximate the sum of the series with an error less than 0.001

22. If the series $\sum_{n=1}^{\infty} a_n$ is conditionally convergent, determine which of the following series must diverge Justify each answer as to why or why not.

(a) $\sum_{n=1}^{\infty} a_n^2$ (b) $\sum_{n=1}^{\infty} |a_n|$ (c) $\sum_{n=1}^{\infty} (-1)^{2n} a_n$ (d) $\sum_{n=1}^{\infty} (-a_n)$

23. Classify any of the following convergent series as absolutely or conditionally convergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)}{n\sqrt{n}}$ (b) $\sum_{n=0}^{\infty} (-1)^n e^{-n}$ (c) $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$ (d) $\sum_{n=1}^{\infty} \left(-\frac{\pi}{e}\right)^{-n}$ (e) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+3}$

Multiple Choice:

24. S_2 is used to approximate $S=\sum_{n=1}^{\infty}\frac{4}{n^2}$. Which interval gives an upper and lower bound for this sum?

(A) $\frac{41}{9}\le S\le\frac{49}{9}$ (B) $\frac{53}{9}\le S\le\frac{58}{9}$ (C) $\frac{49}{9}\le S\le\frac{53}{9}$ (D) $\frac{58}{9}\le S\le\frac{62}{9}$ (E) Diverges

25. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}$ II. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty}\frac{1}{n}$

(A) None (B) II only (C) III only (D) I and II only (E) I and III only

26. If $\lim_{b\to\infty}\int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges (B) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges (C) $\sum_{n=1}^{\infty}\frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty}\frac{1}{n^{p-1}}$ converges (E) $\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ diverges

27. $\lim_{x \to 1} \dfrac{\int_1^x e^{t^2}\,dt}{x^2 - 1}$ is

(A) 0 (B) 1 (C) $\dfrac{e}{2}$ (D) e (E) nonexistent

28. For what integer $k,\ k > 1$, will both $\sum_{n=1}^{\infty} \dfrac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\dfrac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2