Exceedance Probability in Catastrophe Modeling

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Abstract

This article explores two of the most important notions in Catastrophic Modeling: the Occurrence Exceedance Probability (OEP) and the Aggregate Exceedance Probability (AEP) curves. Construction of each curve is discussed and comparisons are made. Several numerical and theoretical examples demonstrate introduced metrics and techniques. A separate discussion is dedicated to a connection between the distribution of loss severities and the OEP depending on the distribution of claim counts. The article is concluded with demonstration of OEP and AEP curves for the deadliest, costliest, and most intense US tropical cyclones based on the 2011 National Oceanic and Atmospheric Administration (NOAA) report.

Keywords. Aggregate Exceedance Probability, Average Annual Loss, Catastrophe Modeling, Collective Risk Model, Exceedance Probability, Loss Return Period, Monte Carlo Simulation, Occurrence Exceedance Probability.

1 Introduction

Catastrophe Modeling is a type of estimation technique used in the Property and Casualty (P&C) industry to predict and evaluate damage caused by *natural* catastrophes such as hurricanes, earthquakes, tornados, hail, winter storms, floods and wild fires, as well as *man-made* catastrophes such as terrorism, [1].

Catastrophe models are widely used in ratemaking, portfolio management and optimization, underwriting and risk selection, loss mitigation strategies, allocation of cost of capital, cost of reinsurance, reinsurance and risk transfer analysis, enterprise risk management, as well as financial and capital adequacy analysis utilized by rating agencies, [1].

The Occurrence Exceedance Probability (OEP) and the Aggregate Exceedance Probability (AEP) are two primary metrics used in catastrophe modeling that give an insurer immediate feedback on the financial nature of a disaster.

This paper explores the notions of OEP and AEP and demonstrates their use through several numerical, as well as theoretical examples.

2 Exceedance Probability

Exceedance Probability (EP) is one of the most commonly used metrics in catastrophe modeling. It is the probability that a certain loss value will be exceeded in a predefined future time period. Exceedance probability is used in planning for potential hazards such as river and stream flooding, hurricane storm surges and droughts, reserving for reservoir storage levels and providing homeowners and community members with risk assessment.

To define exceedance probability, let $D_1, D_2, \cdots$ be a set of natural disasters. Let p_i and X_i be an annual probability of occurrence and a corresponding total loss associated with a natural disaster D_i. Thus, D_i is a Bernoulli random variable with

$$\mathbf{P}(D_i \; occurs) = p_i$$
$$\mathbf{P}(D_i \; does \; not \; occur) = 1 - p_i$$

If an event D_i does not occur, the loss is zero. The expected loss for a given event D_i in a given year is $\mathbf{E}[X] = p_i X_i$.

The overall expected loss for the entire set of events is known as the **average annual loss** (AAL) and is defined as the sum of the expected losses of each of the individual events for a given year:

$$\text{AAL} = \sum_{i=1}^{\infty} p_i X_i$$

The Exceedance Probability (EP) is the probability that a loss random variable exceeds a certain amount of loss. This probability is sometimes denoted as $EP(x)$ and is called the **Exceedance Probability Curve**. Let X be a loss random variable. Then

$$\mathbf{EP}(x) = \mathbf{P}(X > x) = 1 - \mathbf{P}(X \leq x)$$

Using probabilistic terminology, $EP(x)$ is the survival function of X.

In particular, if $x = X_i$, which is a loss associated with a disaster D_i, then

$$\mathbf{EP}(X_i) = \mathbf{P}(X > X_i) = 1 - \mathbf{P}(X \leq X_i) = 1 - \prod_{j=1}^{i}(1 - p_j),$$

where $D_1, D_2, \cdots, D_i$ are the events with higher level of losses such that $X_1 \geq X_2 \geq \cdots \geq X_i$.

The probability that all the other events with possible losses above the value X_i have not occurred is

$$\mathbf{P}(X \leq X_i) = \prod_{j=1}^{i}(1 - p_j)$$

and is sometimes called the **Non-Exceedance Probability** (NEP).

A characteristic sometimes associated with the Exceedance Probability is the **Return Period** or the **Loss Return Period** of a natural disaster. It is calculated as a reciprocal of the EP:

$$RP = \frac{1}{EP}.$$

2.1 Example of an Exceedance Probability Curve

Suppose that during a given year no more than one hurricane can occur. The following table shows the probability of each category of hurricane and the associated loss that would incurred.

Event (D_i)	Description	Annual probability of occurrence (p_i)	Loss (X_i)
1	Category 5 Hurricane	0.003	15,000,000
2	Category 4 Hurricane	0.006	8,000,000
3	Category 3 Hurricane	0.011	5,000,000
4	Category 2 Hurricane	0.030	3,000,000
5	Category 1 Hurricane	0.040	1,000,000

Table 1: Event Loss Data

Note that the Saffir/Simpson Hurricane Wind Scale, [6], provides specific wind values for each hurricane category:

Scale Number (Category)	Winds Max 1-min (mph)
1	74 – 95
2	96 – 110
3	111 – 130
4	131 – 155
5	> 155

Table 2: The Saffir/Simpson Hurricane Wind Scale, 1974

Calculating the Exceedance Probability at each level of loss and the Expected Loss for each level of disaster, we obtain

Event (D_i)	Annual probability of occurrence (p_i)	Loss (X_i)	Exceedance Probability $1-(1-p_1)(1-p_2)\cdots$	$\mathbf{E}[X] = p_iX_i$
1	0.003	15,000,000	0.0030	45,000
2	0.006	8,000,000	0.0090	48,000
3	0.011	5,000,000	0.0199	55,000
4	0.030	3,000,000	0.0493	90,000
5	0.040	1,000,000	0.0873	40,000

Table 3: Exceedance Probability and Expected Loss Results

Note that the probability that no hurricane occurs is

$$\mathbf{P}(\text{No Disaster}) = 1 - \sum_{i=1}^{5} p_i = 1 - 0.09 = 0.91.$$

The Average Annual Loss is

$$\text{AAL} = \sum_{i=1}^{\infty} p_iX_i = 45{,}000 + 48{,}000 + 55{,}000 + 90{,}000 + 40{,}000 = 278{,}000.$$

The Exceedance Probability Curve in this example is

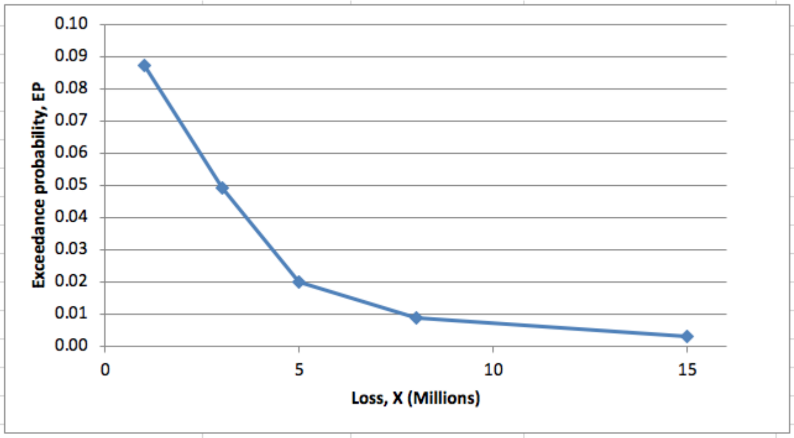

Figure 1: Exceedance Probability Curve in Example 2.1

The probabilities of non-occurrence and non-exceedance are shown in connection with exceedance probability as follows:

Event (D_i)	Annual probability of occurrence p_i	Probability of Non-Occurrence $1-p_i$	Probability of Non-Exceedance $(1-p_1)(1-p_2)\cdots$	Exceedance Probability $1-(1-p_1)(1-p_2)\cdots$
1	0.003	0.997	0.997	0.0030
2	0.006	0.994	0.991	0.0090
3	0.011	0.989	0.980	0.0199
4	0.030	0.970	0.951	0.0493
5	0.040	0.960	0.913	0.0873

Table 4: Non-Occurrence and Non-Exceedance Probabilities

Calculating the Return Period of each event, we have

Event (D_i)	Description	Annual probability of occurrence (p_i)	Exceedance Probability $1-(1-p_1)(1-p_2)\cdots$	Return Period (years) $=1/EP$
1	Category 5 Hurricane	0.003	0.0030	333.33
2	Category 4 Hurricane	0.006	0.0090	111.33
3	Category 3 Hurricane	0.011	0.0199	50.29
4	Category 2 Hurricane	0.030	0.0493	20.29
5	Category 1 Hurricane	0.040	0.0873	11.45

Table 5: Return Period of the Event

The return period is illustrated in the following chart:

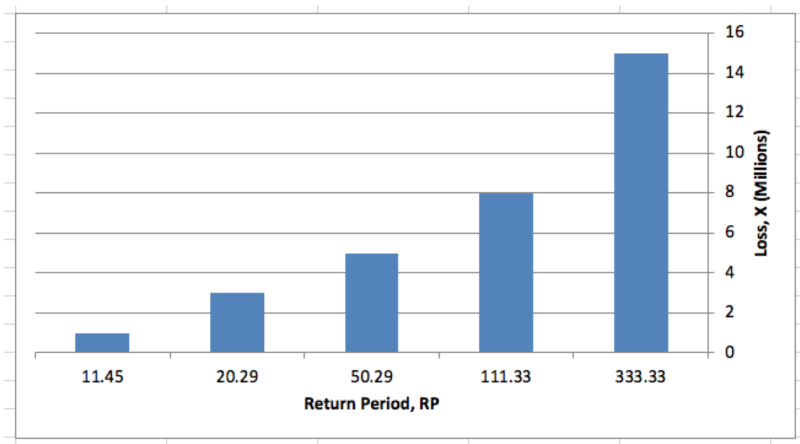

Figure 2: Return Period of the Event in Example 2.1

The exceedance probability can be further broken down into the **occurrence** exceedance probability, OEP, and the **aggregate** exceedance probability, AEP.

3 Occurrence Exceedance Probability

The Occurrence Exceedance Probability (OEP) is the probability that the *largest* loss in a year exceeds a certain amount of loss. This probability is sometimes denoted as $O(x)$ and is called the **Occurrence Exceedance Probability Curve**.

Let $X_1, X_2, \cdots, X_N$ be losses in a given year. Then

$$O(x) = \mathbf{P}(max_{1\leq i\leq N}(X_i) > x) = 1 - \mathbf{P}(max_{1\leq i\leq N}(X_i) \leq x) = 1 - \prod_{i=1}^{N} \mathbf{P}\left(X_i \leq x\right)$$

Using probabilistic terminology, if $X_{(1)}, X_{(2)}, \cdots, X_{(N)}$ is the ordered statistic with $X_{(N)} = \max_{1\leq i\leq N} X_{(i)}$, then $O(x)$ is the survival function of $X_{(N)}$.

Let $F(x)$ be the cumulative distribution function (**CDF**) of X. Then for a fixed N the OEP is

$$O(x) = 1 - \left(F_X(x)\right)^N.$$

If N is the random claim count with the probability mass function (**p.m.f.**) P_N, then by the law of total probability,

$$
\begin{aligned}
O(x) &= \sum_{n=0}^{\infty} \mathbf{P}(max_{1\le i\le n}(X_i) > x|N=n)\mathbf{P}(N=n) = \\
&= 1-\sum_{n=0}^{\infty} \mathbf{P}(max_{1\le i\le n}(X_i) \le x|N=n)\mathbf{P}(N=n) = \\
&= 1-\sum_{n=0}^{\infty} \left(\prod_{i=1}^{n} \mathbf{P}\left(X_i \le x\right)\right)\mathbf{P}(N=n) = 1-\sum_{n=0}^{\infty} \left(F_X(x)\right)^n \mathbf{P}(N=n) = \\
&= 1-\mathbf{E}_N\left(\left(F_X(x)\right)^N\right) = 1-\mathbf{PGF}\left(F_X(x)\right),
\end{aligned}
$$

where $\mathbf{PGF}(x)$ is the probability generating function for N defined as

$$
\mathbf{PGF}(t) = \mathbf{E}\left(t^N\right) = \sum_{n=0}^{\infty} t^n \cdot \mathbf{P}\left(N=n\right).
$$

Thus,

$$
O(x) = 1-\mathbf{PGF}\left(F_X(x)\right). \tag{3.1}
$$

The expected value of $X_{(N)}$ is by definition

$$
\mathbf{E}\left[X_{(N)}\right] = \int_0^{\infty} O(x)\, dx.
$$

In catastrophe modeling the Occurrence Exceedance Probability is used for occurrence based reinsurance structures such as quota share or working excess.

3.1 Example of an Occurrence Exceedance Probability Curve

Following is a simplified example that demonstrates construction of an Occurrence Exceedance Probability Curve outlined in [3]. Data is simulated over ten years assuming a fixed number of losses per year. Severities are assumed to be Pareto-distributed, with parameters $\alpha = 3$ and $\theta = 1000$. Recall that for a two-parameter Pareto distribution, the cumulative distribution function is of the form

$$
F(x) = 1-\left(\frac{\theta}{x+\theta}\right)^{\alpha}.
$$

Using the *inversion method* of the *Monte Carlo Simulation* (MCS) technique, we calculate the inverse function of $F(x)$ as

$$u = 1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha} \Leftrightarrow 1 - u = \left(\frac{\theta}{x+\theta}\right)^{\alpha} \Leftrightarrow (1-u)^{-1/\alpha} = \frac{x}{\theta} + 1 \Leftrightarrow$$
$$x = \theta\left[(1-u)^{-1/\alpha} - 1\right] \Leftrightarrow F^{-1}(x) = \theta\left[(1-x)^{-1/\alpha} - 1\right].$$

Table 10 of Appendix A contains a 100 simulated losses. Assuming 10 losses per year, the data is simulated over 10 years. Calculating the largest loss within each year, we have

Year	$\max_{1\leq i\leq 10}(X_i)$
1	869.63
2	1,390.24
3	1,713.30
4	3,330.60
5	1,069.76
6	604.58
7	578.61
8	721.97
9	1,644.01
10	1,042.16

Table 6: Maximum Loss by Year

These amounts are highlighted in Table 10. Sorting annual losses from highest to lowest and ranking each year, we obtain

OEP	Rank	Year	$\max_{1\leq i\leq 10}(X_i)$
0.1	1	4	3,330.60
0.2	2	3	1,713.30
0.3	3	9	1,644.01
0.4	4	2	1,390.24
0.5	5	5	1,069.76
0.6	6	10	1,042.16
0.7	7	1	869.63
0.8	8	8	721.97
0.9	9	6	604.58
1.0	10	7	578.61

Table 7: Sorted and Ranked Maximum Losses by Year

The resulting Occurrence Exceedance Probability Curve is

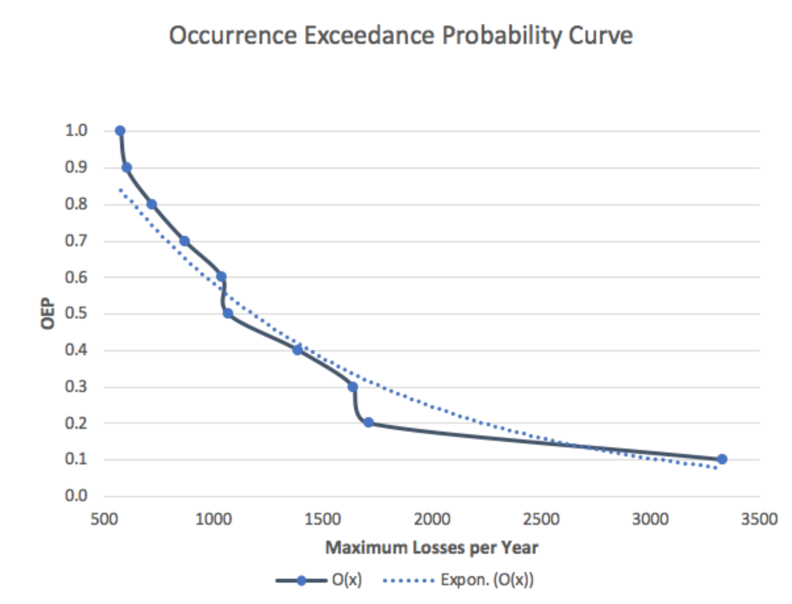

Figure 3: Occurrence Exceedance Probability Curve in Example 3.1

An exponential trend is included to demonstrate the general behavior of the function.

4 Evaluating Severity Distribution Using the OEP

It follows from the equation (3.1) that the cumulative distribution function F_X of losses X can be evaluated using the Occurrence Exceedance Probability $O(x)$ as

$$F_X(x) = \mathbf{PGF}^{-1}\left(1 - O(x)\right), \tag{4.1}$$

where $\mathbf{PGF}^{-1}(x)$ indicates the inverse function of the probability generating function for N.

The loss distribution will be consistent with the starting OEPs and the claim count assumption.

An important property of the probability generating function is outlined in the following Lemma.

Lemma 4.1 *If N and M are independent random variables, then*

$$\boldsymbol{PGF}_{N+M}(t) = \boldsymbol{PGF}_N(t) \cdot \boldsymbol{PGF}_M(t)$$

Proof. By definition,

$$\mathbf{PGF}_{N+M}(t) = \mathbf{E}\left(t^{N+M}\right) = \mathbf{E}\left(t^N \cdot t^M\right) = \mathbf{E}\left(t^N\right) \cdot \mathbf{E}\left(t^M\right) = \mathbf{PGF}_N(t) \cdot \mathbf{PGF}_M(t).$$

Following is the derivation of the cumulative distribution function F_X of losses X for a few standard discrete distributions of claim counts.

4.1 Poisson Distribution of Claim Counts

Suppose claim counts N have a Poisson distribution with mean parameter λ. This is a common assumption when modeling a number of catastrophes. The probability mass function is defined as

$$p_n = \mathbf{P}\left(N = n\right) = e^{-\lambda}\,\frac{\lambda^n}{n!}.$$

Calculating the **PGF**, we obtain

$$PGF(t) = \sum_{n=0}^{\infty} t^n \cdot \mathbf{P}\left(N = n\right) = \sum_{n=0}^{\infty} t^n \cdot e^{-\lambda}\,\frac{\lambda^n}{n!} = e^{-\lambda} \sum_{n=0}^{\infty} t^n\,\frac{\lambda^n}{n!} =$$

$$= e^{-\lambda} \sum_{n=0}^{\infty} \frac{(t\lambda)^n}{n!} = e^{-\lambda} \cdot e^{t\lambda} = e^{\lambda(t-1)}.$$

Then the inverse function is

$$y = e^{\lambda(t-1)} \Leftrightarrow \lambda(t-1) = \ln y \Leftrightarrow t = \frac{\ln y}{\lambda} + 1 \Leftrightarrow \mathbf{PGF}^{-1}(x) = \frac{\ln x}{\lambda} + 1$$

Using (4.1), cumulative distribution function F_X is

$$F_X(x) = \mathbf{PGF}^{-1}\left(1 - O(x)\right) = \frac{\ln\left(1 - O(x)\right)}{\lambda} + 1$$

4.2 Bernoulli Distribution of Claim Counts

Suppose claim counts N have a Bernoulli distribution with parameter q. The probability mass function is defined as

$$p_0 = \mathbf{P}\left(N = 0\right) = 1 - q, p_1 = \mathbf{P}\left(N = 1\right) = q$$

Calculating the **PGF**, we obtain

$$\mathbf{PGF}(t) = \sum_{n=0}^{1} t^n \cdot \mathbf{P}\left(N = n\right) = (1-q) + qt \tag{4.2}$$

Then the inverse function is

$$y = (1-q) + qt \Leftrightarrow t = \frac{y-1+q}{q} = \frac{y-1}{q} + 1 \Leftrightarrow$$
$$\mathbf{PGF}^{-1}(x) = \frac{x-1}{q} + 1$$

Using (4.1), cumulative distribution function F_X is

$$F_X(x) = \mathbf{PGF}^{-1}\left(1 - O(x)\right) = \frac{1 - O(x) - 1}{q} + 1 = \frac{O(x)}{q} + 1$$

4.3 Binomial Distribution of Claim Counts

Suppose claim counts N have a binomial distribution with parameters q and m. The probability mass function is defined as

$$p_n = \mathbf{P}\left(N = n\right) = \binom{m}{n} q^n (1-q)^{m-n}.$$

Calculating the **PGF**, we obtain

$$\mathbf{PGF}(t) = \sum_{n=0}^{m} t^n \cdot \mathbf{P}\left(N = n\right) = \sum_{n=0}^{m} t^n \cdot \binom{m}{n} q^n (1-q)^{m-n} = \sum_{n=0}^{m} \binom{m}{n} (qt)^n (1-q)^{m-n} =$$
$$= \left((1-q) + qt\right)^m = \left(1 + q(t-1)\right)^m$$

Note that the same **PGF** can be obtained using one of the properties of a probability generating function. Since a Binomial (q, m) random variable N can be expressed as a sum of m i.i.d. Bernoulli (q),

$$N = N_1 + N_2 + \cdots + N_m,$$

by Lemma (4.1), using (4.2), its **PGF** is

$$\mathbf{PGF}_N(t) = \prod_{i=1}^{m} \mathbf{PGF}_{N_i}(t) = \left((1-q) + qt\right)^m$$

The inverse function is

$$y = ((1-q)+qt)^m \Leftrightarrow (1-q)+qt = y^{1/m} \Leftrightarrow t = \frac{y^{1/m}-1+q}{q} = \frac{y^{1/m}-1}{q}+1 \Leftrightarrow$$
$$\mathbf{PGF}^{-1}(x) = \frac{x^{1/m}-1}{q}+1$$

Using (4.1), cumulative distribution function F_X is

$$F_X(x) = \mathbf{PGF}^{-1}\left(1-O(x)\right) = \frac{(1-O(x))^{1/m}-1}{q}+1$$

4.4 Geometric Distribution of Claim Counts

Suppose claim counts N have a geometric distribution with success probability $0 < p < 1$. The probability mass function is defined as

$$p_n = \mathbf{P}\left(N = n\right) = (1-p)^n p.$$

Calculating the **PGF**, we obtain

$$\begin{aligned}\mathbf{PGF}(t) &= \sum_{n=0}^{\infty} t^n \cdot \mathbf{P}\left(N=n\right) = \sum_{n=0}^{\infty} t^n \cdot (1-p)^n p = p\sum_{n=0}^{\infty}(t(1-p))^n = \\ &= \frac{p}{1-t(1-p)}\end{aligned} \tag{4.3}$$

Then the inverse function is

$$y = \frac{p}{1-t(1-p)} \Leftrightarrow y - yt(1-p) = p \Leftrightarrow yt(1-p) = y-p \Leftrightarrow t = \frac{y-p}{y(1-p)} \Leftrightarrow$$
$$\mathbf{PGF}^{-1}(x) = \frac{x-p}{x(1-p)}$$

Using (4.1), cumulative distribution function F_X is

$$F_X(x) = \mathbf{PGF}^{-1}\left(1-O(x)\right) = \frac{1-O(x)-p}{(1-O(x))(1-p)}$$

4.5 Negative Binomial Distribution of Claim Counts

Suppose claim counts N have a negative binomial distribution with parameters p and r. The probability mass function is defined as

$$p_n = \mathbf{P}\left(N = n\right) = \binom{n+r-1}{n} p^r (1-p)^n.$$

For an integer r, since a Negative Binomial (p, r) random variable N can be expressed as a sum of r i.i.d. geometric (p),

$$N = N_1 + N_2 + \cdots + N_r,$$

by Lemma (4.1), using (4.3), its **PGF** is

$$\mathbf{PGF}_N(t) = \prod_{i=1}^{m} \mathbf{PGF}_{N_i}(t) = \left(\frac{p}{1-t(1-p)}\right)^r$$

Then the inverse function is

$$y = \left(\frac{p}{1-t(1-p)}\right)^r \Leftrightarrow \frac{p}{1-t(1-p)} = y^{1/r} \Leftrightarrow y^{1/r} - y^{1/r}t(1-p) = p \Leftrightarrow$$
$$y^{1/r}t(1-p) = y^{1/r} - p \Leftrightarrow t = \frac{y^{1/r}-p}{y^{1/r}(1-p)} \Leftrightarrow \mathbf{PGF}^{-1}(x) = \frac{x^{1/r}-p}{x^{1/r}(1-p)}$$

Using (4.1), cumulative distribution function F_X is

$$F_X(x) = \mathbf{PGF}^{-1}\left(1 - O(x)\right) = \frac{(1-O(x))^{1/r}-p}{(1-O(x))^{1/r}(1-p)}$$

5 Aggregate Exceedance Probability

The Aggregate Exceedance Probability (AEP) is the probability that the *sum* of losses in a year exceeds a certain amount of loss. This probability is sometimes denoted as $A(x)$ and is called the **Aggregate Exceedance Probability Curve**.

Let $X_1, X_2, \cdots, X_N$ be losses in a given year. Then

$$A(x) = \mathbf{P}(X_1 + X_2 + \cdots + X_N > x) = 1 - \mathbf{P}(X_1 + X_2 + \cdots + X_N \leq x)$$

Using the terminology of the aggregate loss models, if S is the collective risk model, defined as $S = \sum_{i=1}^{N} X_i$, then $A(x)$ is the survival function of S.

For a fixed N this probability is

$$A(x) = 1 - F_X^{(N)}(x),$$

where $F_X^{(N)}$ is an N-fold convolution of $F_X(x)$, defined as

$$F_X^{(N)}(x) = \int_0^x F_X^{(N-1)}(x-y) f_X(y)\, dy \quad \text{for } N = 2, 3, \cdots .$$

For $N = 1$ this equation reduces to $F_X^{(1)}(x) = F_X(x)$, [5].

If N is the random claim count with the probability mass function (**p.m.f.**) P_N, then by the law of total probability,

$$\begin{aligned} A(x) &= \sum_{n=0}^{\infty} \mathbf{P}(S > x | N = n)\mathbf{P}(N = n) = \\ &= 1 - \sum_{n=0}^{\infty} \mathbf{P}(S \leq x | N = n)\mathbf{P}(N = n) = \\ &= 1 - \sum_{n=0}^{\infty} F_X^{(n)}(x)\mathbf{P}(N = n) = 1 - \mathbf{E}_N\left(F_X^{(N)}\right) \end{aligned}$$

The expected value of S is by definition

$$\mathbf{E}[S] = \int_0^{\infty} A(x)\, dx = \mathbf{E}[X]\,\mathbf{E}[N].$$

In catastrophe modeling the Aggregate Exceedance Probability is used for aggregate based reinsurance structures such as stop loss and reinstatements.

5.1 Example of an Aggregate Exceedance Probability Curve

Following is a simplified example that demonstrates construction of an Aggregate Exceedance Probability Curve outlined in [3]. We use the same data as in Example 3.1.

In that example data was simulated over ten years assuming a fixed number of losses per year. Severities were assumed to be Pareto-distributed, with parameters $\alpha = 3$ and $\theta = 1000$. Losses were simulated using the inversion method of the Monte Carlo Simulation (MCS) technique. Table 10 of Appendix A contains a 100 simulated losses. Assuming 10 losses per year, the data is simulated over 10 years.

Calculating the sum of losses within each year, we have

Year	$\sum_{i=1}^{10} X_i$
1	2,936.52
2	3,867.36
3	4,589.80
4	7,092.26
5	4,125.27
6	2,831.38
7	2,589.09
8	1,832.78
9	5,400.46
10	3,087.66

Table 8: Sum of Losses by Year

These amounts are highlighted in Table 10. Sorting annual losses from highest to lowest and ranking each year, we obtain

AEP	Rank	Year	$\sum_{i=1}^{10} X_i$
0.1	1	4	7,092.26
0.2	2	9	5,400.46
0.3	3	3	4,589.80
0.4	4	5	4,125.27
0.5	5	2	3,867.36
0.6	6	10	3,087.66
0.7	7	1	2,936.52
0.8	8	6	2,831.38
0.9	9	7	2,589.09
1.0	10	8	1,832.78

Table 9: Sorted and Ranked Sum of Losses by Year

The resulting Aggregate Exceedance Probability Curve is

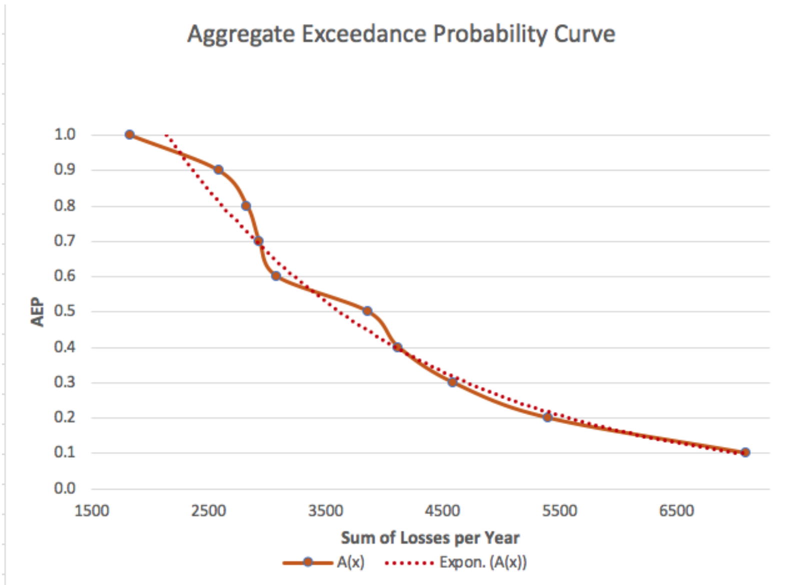

Figure 4: Aggregate Exceedance Probability Curve in Example 5.1

An exponential trend is included to demonstrate the general behavior of the function.

6 Comparison of the OEP and the AEP

In the simplified examples 3.1 and 5.1 we constructed the Occurrence and the Aggregate Exceedance Probability curves using the Monte Carlo simulation technique. These curves are shown in the following Figure.

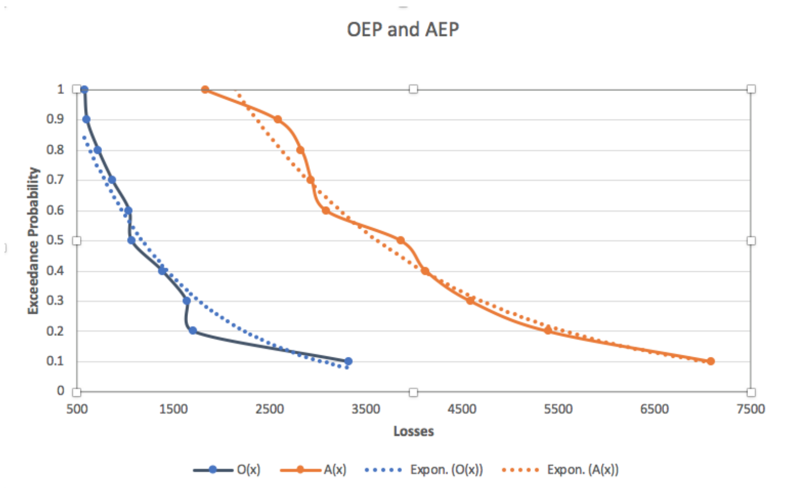

Figure 5: Occurrence and Aggregate EP Curves

In this graph the curves appear to be parallel-shifted due to the nature of the simplified assumption on the fixed number of losses per year.

A more typical visualization of the $O(x)$ and $A(x)$ curves is

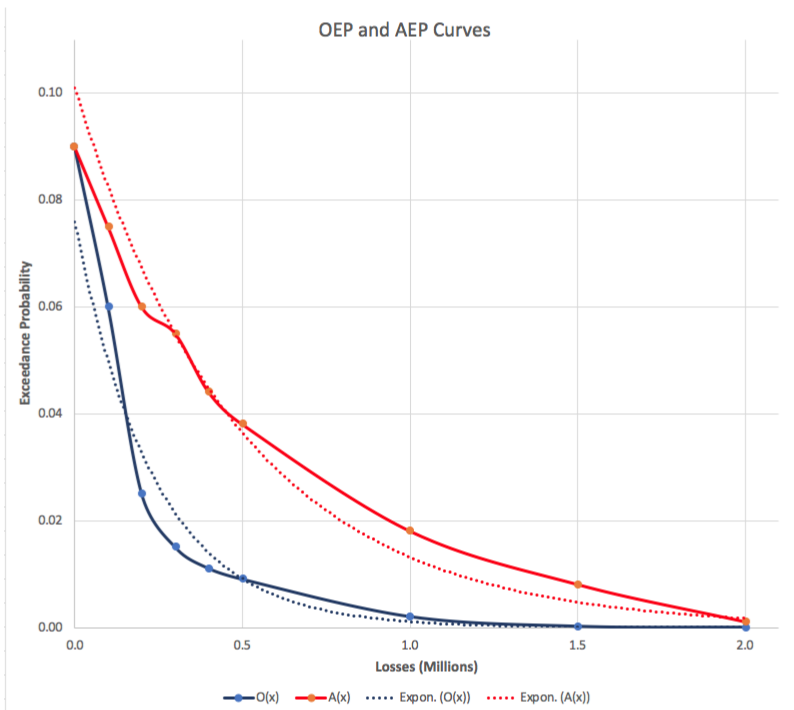

Figure 6: A Standard Visualization of the Occurrence and Aggregate EP Curves

Homer and Li, [2], address a question of when the OEP and the AEP are alike.

Proposition 6.1 *Let X be the severity of loss random variable and N be the number of claims random variable. Suppose that X and N are mutually independent. Then for any $\epsilon > 0$ there exists a $\delta > 0$ such that*

$$\textit{If } \sum_{n=2}^{\infty} \boldsymbol{P}_N(n) < \delta \textit{ then } |A(x) - O(x)| < \epsilon$$

Proof. Let $X_1, X_2, \cdots, X_N$ be losses in a given year. By definition,

$$O(x) = \mathbf{P}\left(\max_{1 \le i \le N} (X_i) > x\right) \text{ and } A(x) = \mathbf{P}\left(\sum_{i=1}^{N} X_i > x\right)$$

We have shown in Sections 3 and 5 that

$$O(x) = 1 - \sum_{n=0}^{\infty} (F_X(x))^n \, \mathbf{P}(N = n) \text{ and}$$
$$A(x) = 1 - \sum_{n=0}^{\infty} F_X^{(n)}(x) \mathbf{P}(N = n),$$

where $F_X^{(n)}$ is an n-fold convolution of $F_X(x)$.

If $P_N(n) = \mathbf{P}(N = n) = 0$ for $n > 1$, then $A(x) = O(x)$. Otherwise, let $\epsilon > 0$. Choose $\delta = \epsilon/2$. Suppose that

$$\sum_{n=2}^{\infty} P_N(n) < \delta$$

Then,

$$|A(x) - O(x)| \le \sum_{n=2}^{\infty} \mathbf{P}(N = n) \left| F_X^{(n)}(x) - (F_X(x))^n \right| \le 2 \sum_{n=2}^{\infty} P_N(n) < 2\delta = \epsilon.$$

The following inequality is always true:

$$\max_{1 \le i \le N} (X_i) \le \sum_{i=1}^{N} X_i.$$

In addition, the following proposition shows connection between the OEP and the AEP with the survival function of the loss severity random variable.

Proposition 6.2 *Let $X_1, X_2, \cdots, X_N$ be losses in a given year, $F_X(x)$ and $S_X(x)$ be the cumulative distribution and survival functions of a loss random variable X. Then*

$$O(x) \geq 1 - F_X(x) = S_X(x) \text{ and}$$
$$A(x) \geq 1 - F_X(x) = S_X(x)$$

Proof. For any N, we have:

$$O(x) = 1 - (F_X(x))^N \geq 1 - F_X(x) = S_X(x)$$
$$A(x) = 1 - F_X^{(N)}(x) = 1 - \int_0^x F_X^{(N-1)}(x-y) f_X(y)\, dy \geq$$
$$\geq 1 - \int_0^x f_X(y)\, dy = 1 - F_X(x) = S_X(x)$$

7 The Deadliest, Costliest, and Most Intense US Tropical Cyclones

In this section we consider the information reported by the National Oceanic and Atmospheric Administration (NOAA) in 2011, [6], on the the deadliest, costliest, and most intense US tropical cyclones from 1851 to 2010 and construct the corresponding OEP and AEP curves for each category.

7.1 Ranking Tropical Cyclones by Deaths

Table 11 of Appendix B lists the tropical cyclones that have caused at least 25 deaths on the U.S. mainland during the period 1851-2010, [6].

Based on this table, the Galveston Hurricane of 1900 was responsible for at least 8000 deaths and remains first on the list. Hurricane Katrina of 2005 remains the third deadliest hurricane to strike the United States. Although these systems are spread out over most of the coast, there is a clustering of tracks on the coasts of Texas, southeastern Louisiana, south Florida, North Carolina and New England.

The following Figure 7, curtesy of [6], shows the paths of these deadly cyclones.

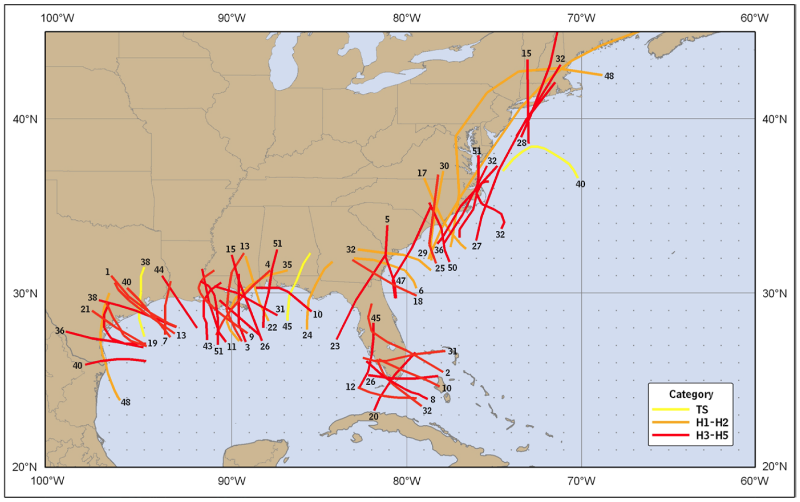

Figure 7: Mainland United States tropical cyclones causing 25 or more deaths, 1851-2010. The black numbers are the ranks of a given storm on Table 11 (e.g. 1 is the deadliest all-time). The colors are the intensity of the tropical cyclone at its maximum impact on the United States.

Table 12 provides maximum deaths and the sum of deaths by year with multiple hurricane years being highlighted. In addition, tables 13 and 14 show maximum number of deaths and the sum of the number of deaths sorted from highest to lowest resulting in the following Occurrence and Aggregate Exceedance Probability Curves

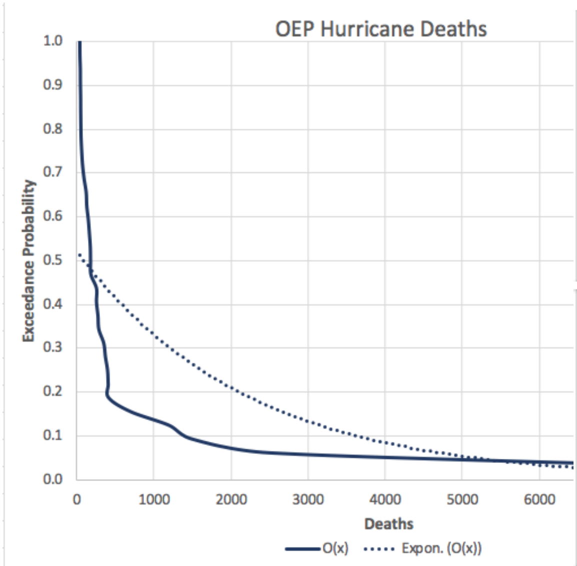

Figure 8: Occurrence EP Curve TC Deaths

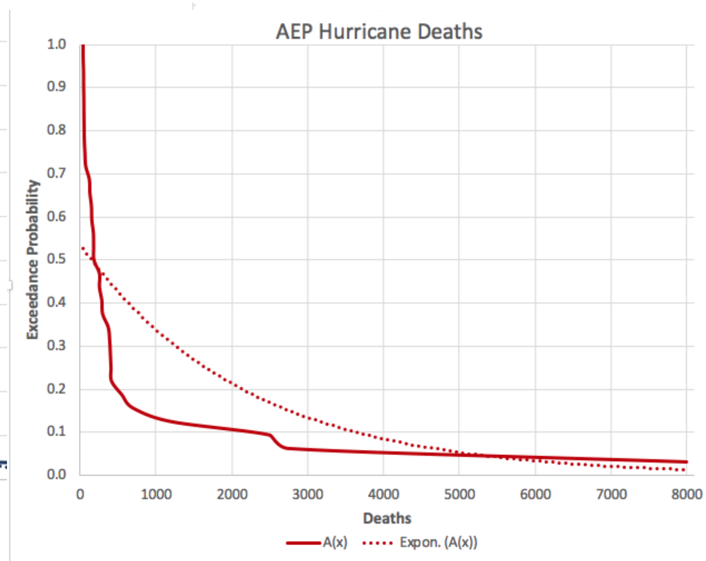

Figure 9: Aggregate EP Curve TC Deaths

An exponential trend is included to demonstrate the general behavior of the functions.

Since there are only a few years with multiple hurricanes, the comparison between the $O(x)$ and the $A(x)$ shown in the following graph is subtle

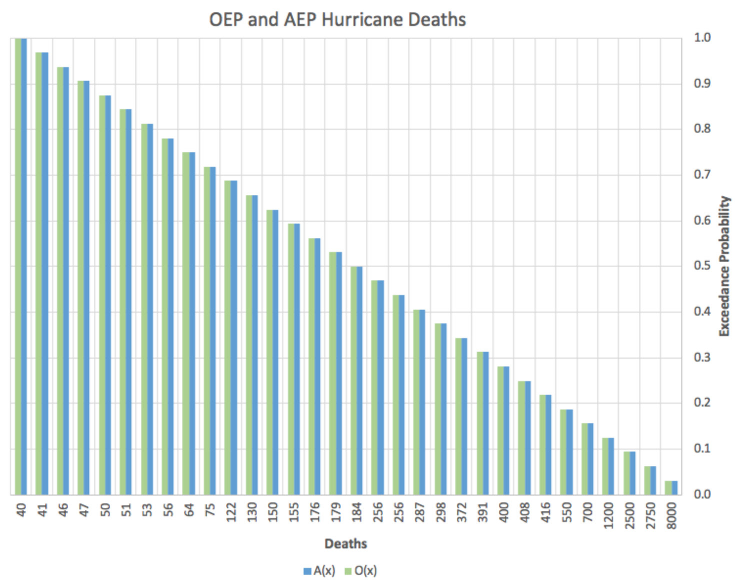

Figure 10: The Deadliest US Tropical Cyclones: Occurrence and Aggregate EP

7.2 Ranking Tropical Cyclones by Costs

Table 15 of Appendix C lists the 30 costliest mainland United States tropical cyclones, 1900-2010, not adjusted for inflation, [6].

Based on this table, hurricane Ike of 2008 was the second-costliest hurricane on record. Hurricane Katrina of 2005 was responsible for at least $108 billion of property damage and is by far the costliest hurricane to ever strike the United States. It is of note that the last ten hurricane seasons have produced 14 out of the 30 costliest systems to affect the United States.

The following Figure 11, curtesy of [6], displays the near-landfall portion of these tropical cyclone tracks and shows concentrations of costly hurricanes along the central Gulf Coast, south Florida and the Carolinas.

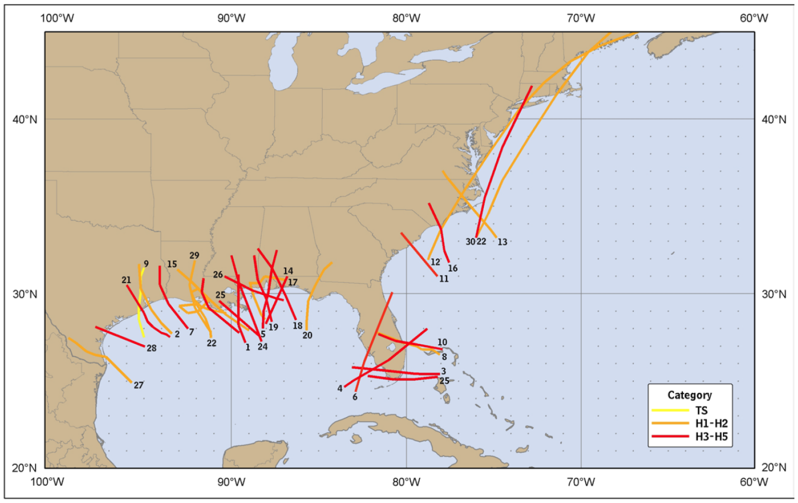

Figure 11: The 30 costliest tropical cyclones to strike the United States, 1900-2010. The black numbers are the ranks of a given storm on Table 15 (e.g. 1 is the costliest all-time). The colors are the intensity of the tropical cyclone at its maximum impact on the United States.

Table 16 re-orders Table 15 and the historical database after adjusting to 2010 dollars, which adds several other hurricanes. After this normalization to todays societal vulnerability, the last decade still accounts for eight of the top 30 tropical cyclones.

The following Figure 12, curtesy of [6], displays the near-landfall portion of these tropical cyclone tracks and shows concentrations of costly hurricanes along the central Gulf Coast, south Florida and the Carolinas.

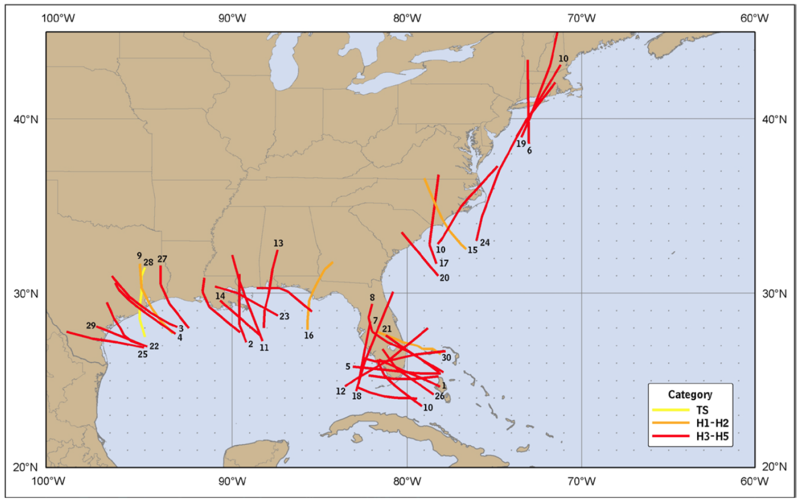

Figure 12: The 30 costliest tropical cyclones to strike the United States, ranked by normalization for inflation, population and wealth, 1900-2010. The black numbers are the ranks of a given storm on Table 16. The colors are the intensity of the tropical cyclone at its maximum impact on the United States.

Table 17 provides maximum costs and the sum of costs by year with multiple hurricane years being highlighted. In addition, tables 18 and 19 show maximum costs and the sum of costs sorted from highest to lowest resulting in the following Occurrence and Aggregate Exceedance Probability Curves

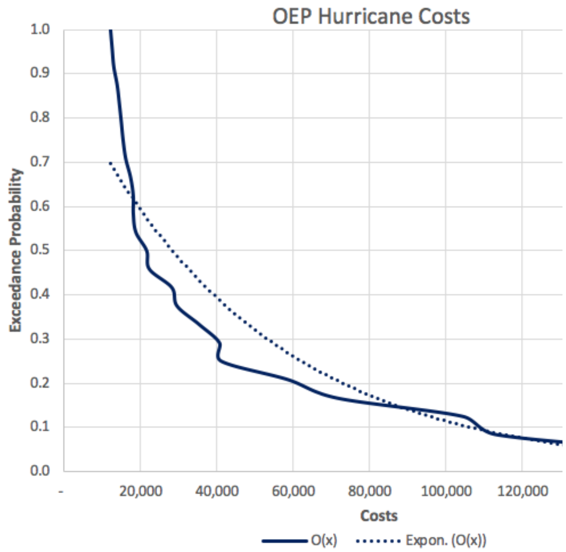

Figure 13: Occurrence EP Curve TC Costs

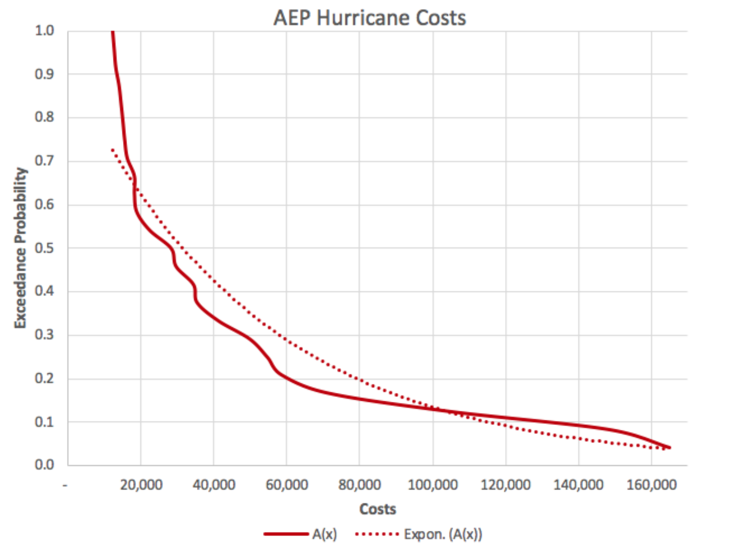

Figure 14: Aggregate EP Curve TC Costs

An exponential trend is included to demonstrate the general behavior of the functions.

Since there are only a few years with multiple hurricanes, the comparison between the $O(x)$ and the $A(x)$ shown in the following graph is subtle

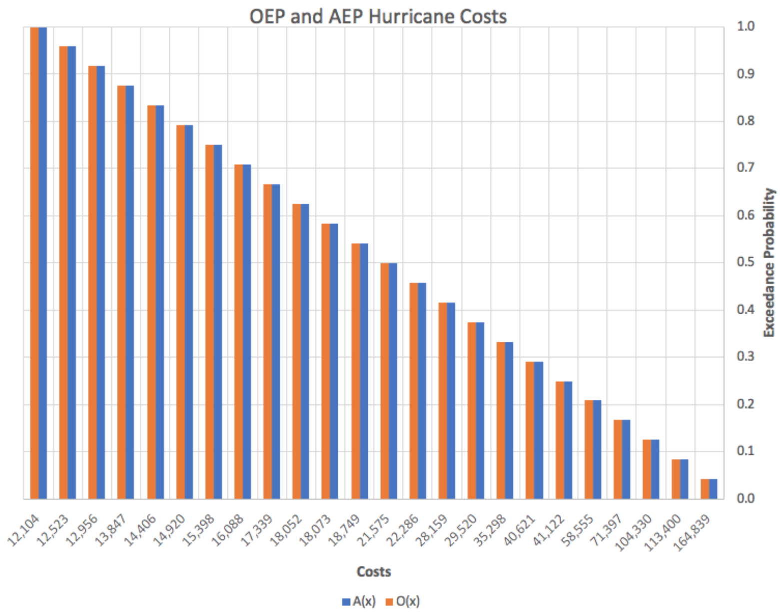

Figure 15: The Costliest US Tropical Cyclones Occurrence and Aggregate EP

7.3 Ranking Tropical Cyclones by Intensity

Table 20 of Appendix D lists the most intense major hurricanes to strike the U.S. mainland during the period 1851– 2010, [6]. In this study, the major hurricanes have been ranked by estimating *central pressure* at time of landfall. Central pressure is used as a proxy for intensity due to the uncertainties in maximum wind speed estimates for many historical hurricanes.

Based on this table, Hurricane Katrina had the third lowest pressure ever noted at landfall, behind the 1935 Florida Keys hurricane and Hurricane Camille in 1969.

The following Figure 16, curtesy of [6], shows where these major hurricanes struck the coast.

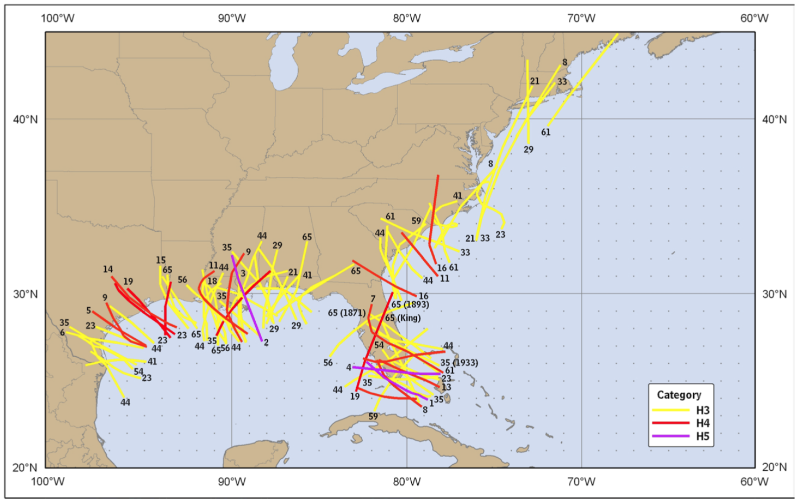

Figure 16: The most intense United States major hurricanes, ranked by pressure at landfall, 1851-2010. The black numbers are the ranks of a given storm on Table 20 (e.g. 1 has the lowest pressure all-time). The colors are the intensity of the tropical cyclone at its maximum impact on the United States.

Table 21 provides minimum and maximum intensities by year with multiple hurricane years being highlighted.

Using the definition of a hurricane intensity, adopted in [6], the most intense tropical storm is the one with the *lowest* central pressure. Thus, the usual definition of exceedance probability must be modified. Let I be an intensity random variable. Then

$$\mathbf{EP}_I(x) = \mathbf{P}(I < x)$$

Using probabilistic terminology, the $EP_I(x)$ is the cumulative distribution function of I.

Tables 22 and 23 show minimum and maximum intensities sorted from lowest to highest resulting in the following Min and Max Exceedance Probability Curves

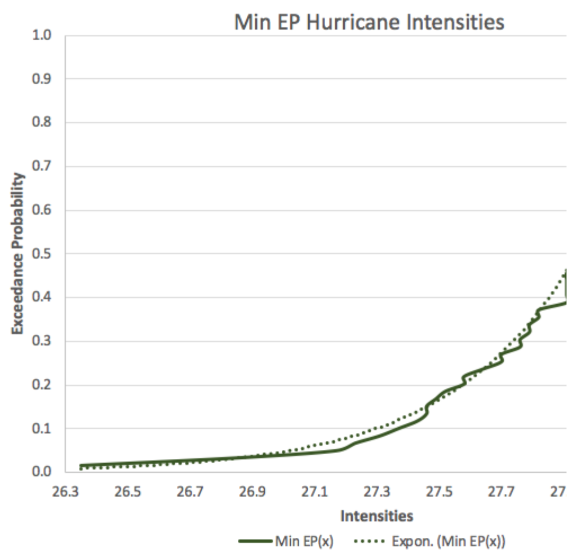

Figure 17: Occurrence EP Curve TC Intensities

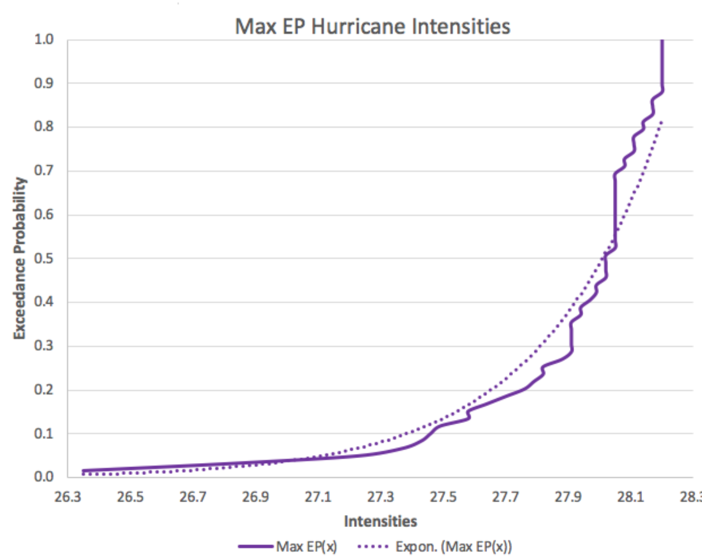

Figure 18: Aggregate EP Curve TC Intensities

An exponential trend is included to demonstrate the general behavior of the functions.

Since there are only a few years with multiple hurricanes, the comparison between the Min EP and the Max EP shown in the following graph is subtle

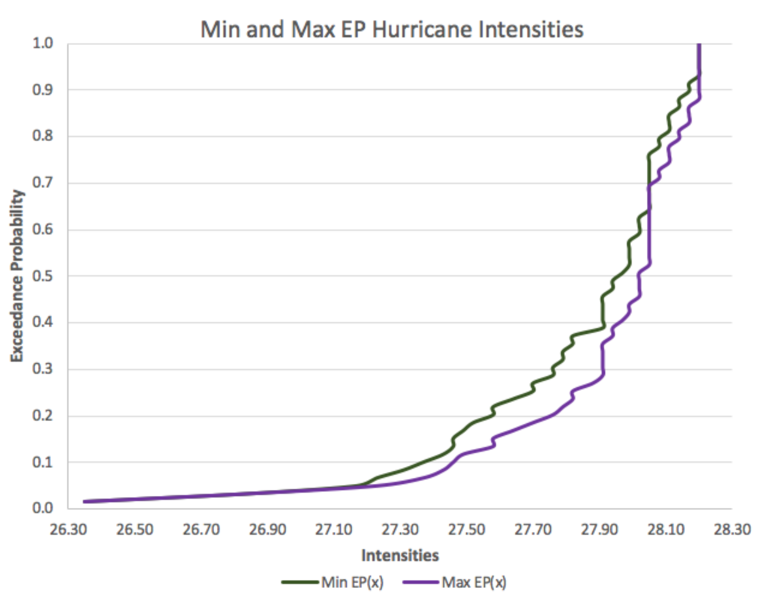

Figure 19: The Most Intense US Tropical Cyclones Occurrence and Aggregate EP Curves

Following [4], the difference between the aggregate and occurrence EP curves would vary depending on:

1. Peril, such as hurricane, earthquake, flood, severe convective storm, etc;
2. Geographic Scope that includes all of the US, by state, by county, by ZIP or by region such as California vs. East Coast vs. Gulf Coast vs. Midwest, etc;
3. Portfolio composition such as construction, occupancy, year built, building height, etc;
4. Insurance structure such as deductibles, endorsements, exclusions, etc.

8 Conclusion

In this paper we explored two of the most important notions in Catastrophic Modeling, the Occurrence Exceedance Probability (OEP) and the Aggregate Exceedance Probability (AEP). We discussed construction of each curve and compared these two metrics in several numeric and theoretical examples. In particular, we discussed a connection between the distribution of loss severities and the OEP depending on the distribution of claim counts. One of the examples involved Monte Carlo Simulation, an important technique that allows to account for risk in quantitative analysis and decision making. Finally, we produced the OEP and AEP curves for the deadliest, costliest, and most intense US tropical cyclones based on the 2011 National Oceanic and Atmospheric Administration (NOAA) report.

A OEP and AEP Curves Simulation

Table 10: Simulated Losses for $O(x)$ and $A(x)$

No	u	X_i	$\max_{1\leq i\leq 10}(X_i)$	$\sum_{i=1}^{10} X_i$
1	0.1244	45.27	**869.63**	**2,936.52**
2	0.2997	126.11	869.63	3,325.47
3	0.8470	869.63	869.63	3,610.52
4	0.4592	227.44	594.64	2,905.16
5	0.3690	165.89	1,390.24	4,067.96
6	0.0547	18.92	1,390.24	4,370.74
7	0.1723	65.05	1,390.24	4,466.30
8	0.4739	238.72	1,390.24	5,122.50
9	0.7534	594.64	1,390.24	4,984.59
10	0.7488	584.86	1,390.24	4,427.47
11	0.6610	434.22	**1,390.24**	**3,867.36**
12	0.6441	411.16	1,390.24	3,434.54
13	0.3664	164.26	1,390.24	3,295.06
14	0.9268	1,390.24	1,713.30	4,844.10
15	0.6843	468.66	1,713.30	4,418.00
16	0.2776	114.49	1,713.30	4,132.96
17	0.8039	721.24	1,713.30	4,811.47
18	0.2503	100.81	1,713.30	4,245.80
19	0.1046	37.52	1,713.30	4,225.72
20	0.0707	24.75	1,713.30	4,334.76
21	0.0042	1.40	**1,713.30**	**4,589.80**
22	0.5137	271.67	1,713.30	5,326.43
23	0.9499	1,713.30	1,713.30	5,126.44
24	0.8680	964.14	964.14	3,548.89
25	0.3969	183.62	793.00	2,796.58
26	0.8265	793.00	793.00	2,959.52
27	0.3519	155.57	1,290.22	3,456.74

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Table 10 – *Continued from previous page*

No	u	X_i	$\max_{1\le i\le 10}(X_i)$	$\sum_{i=1}^{10} X_i$
28	0.2078	80.74	3,330.60	6,631.78
29	0.3365	146.55	3,330.60	6,597.18
30	0.5229	279.79	3,330.60	6,533.40
31	0.8095	738.03	**3,330.60**	**7,092.26**
32	0.1875	71.68	3,330.60	7,311.69
33	0.3174	135.76	3,330.60	7,286.00
34	0.4381	211.83	3,330.60	7,624.17
35	0.5904	346.57	3,330.60	8,482.10
36	0.9168	1,290.22	3,330.60	8,340.17
37	0.9877	3,330.60	3,330.60	7,954.98
38	0.1265	46.13	1,069.76	4,786.62
39	0.2123	82.78	1,069.76	4,783.34
40	0.8391	838.65	1,069.76	4,705.71
41	0.8667	957.46	**1,069.76**	**4,125.27**
42	0.1262	46.00	1,069.76	3,484.08
43	0.6877	473.92	1,069.76	3,713.75
44	0.8872	1,069.76	1,069.76	3,482.34
45	0.4279	204.63	905.04	2,696.83
46	0.8554	905.04	905.04	2,802.84
47	0.3631	162.25	316.27	1,945.86
48	0.1183	42.84	316.27	1,855.80
49	0.0153	5.16	568.24	2,381.19
50	0.4980	258.21	604.58	2,980.62
51	0.5615	316.27	**604.58**	**2,831.38**
52	0.5183	275.67	604.58	2,660.73
53	0.4787	242.51	604.58	2,543.30
54	0.5279	284.25	604.58	2,497.96
55	0.5558	310.64	604.58	2,422.68
56	0.1313	48.06	604.58	2,152.96

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Table 10 – *Continued from previous page*

No	u	X_i	$\max_{1\le i\le 10}(X_i)$	$\sum_{i=1}^{10} X_i$
57	0.1887	72.18	604.58	2,609.00
58	0.7407	568.24	604.58	2,661.87
59	0.7579	604.58	604.58	2,341.09
60	0.2668	108.98	504.10	2,119.46
61	0.3349	145.61	**578.61**	**2,589.09**
62	0.3564	158.24	578.61	2,473.83
63	0.4172	197.17	578.61	2,465.94
64	0.4341	208.98	721.97	2,990.75
65	0.1133	40.92	721.97	2,826.52
66	0.7061	504.10	721.97	2,988.58
67	0.2978	125.04	721.97	2,689.37
68	0.4849	247.47	721.97	2,607.54
69	0.6219	382.95	721.97	2,590.02
70	0.7458	578.61	721.97	2,312.96
71	0.0858	30.35	**721.97**	**1,832.78**
72	0.3431	150.36	721.97	2,374.31
73	0.8042	721.97	721.97	2,251.83
74	0.1231	44.75	571.89	2,012.57
75	0.4256	202.97	1,254.22	3,222.04
76	0.4283	204.90	1,644.01	4,663.08
77	0.1192	43.22	1,644.01	4,960.95
78	0.4625	229.95	1,644.01	5,008.84
79	0.2606	105.89	1,644.01	4,940.07
80	0.2454	98.42	1,644.01	5,268.69
81	0.7425	571.89	**1,644.01**	**5,400.46**
82	0.0792	27.87	1,644.01	5,155.09
83	0.6932	482.72	1,644.01	5,132.26
84	0.9127	1,254.22	1,644.01	5,691.70
85	0.9459	1,644.01	1,644.01	4,479.08

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Table 10 – *Continued from previous page*

No	u	X_i	$\max_{1\leq i\leq 10}(X_i)$	$\sum_{i=1}^{10} X_i$
86	0.7053	502.77	1,042.16	3,115.89
87	0.2302	91.11	1,042.16	3,180.42
88	0.3613	161.17	1,042.16	3,436.24
89	0.6612	434.51	1,042.16	3,281.85
90	0.4629	230.19	1,042.16	3,311.03
91	0.5716	326.52	**1,042.16**	**3,087.66**
92	0.0150	5.04	1,042.16	2,761.14
93	0.8826	1,042.16	1,042.16	2,756.10
94	0.1151	41.60	567.30	1,713.94
95	0.5241	280.82	567.30	1,672.34
96	0.7403	567.30	567.30	1,391.52
97	0.5908	346.93	463.69	824.23
98	0.0201	6.78	463.69	477.30
99	0.6811	463.69	463.69	470.52
100	0.0202	6.83	6.83	6.83

B The Deadliest US Tropical Cyclones

Table 11: Mainland U.S. Tropical Cyclones Deaths 1851-2010

Rank	Hurricane	Year	Category	Deaths
1	TX (Galveston)	1900	4	8,000
2	FL (SE/Lake Okeechobee)	1928	4	2,500
3	KATRINA (SE LA/MS)	2005	3	1,200
4	LA (Cheniere Caminanda)	1893	4	1,250
5	SC/GA (SeaIs lands)	1893	3	1,500
6	GA/SC	1881	2	700
7	AUDREY (SW LA N TX)	1957	4	416
8	FL (Keys)	1935	5	408
9	LA (Last Island)	1856	4	400
10	FL (Miami) IMS/AUPensacola	1926	4	372
11	LA (Grand Isle)	1909	3	350
12	FL (Keys)/S TX	1919	4	287
13	LA (New Orleans)	1915	3	275
13	TX (Galveston)	1915	4	275
15	New England	1938	3	256
15	CAMILLE (MS/SE LA/VA)	1969	5	256
17	DIANE (NE U.S.)	1955	1	184
18	GA, SC, NC	1898	4	179
19	TX	1875	3	176
20	SE FL	1906	3	164
21	TX (Indianola)	1886	4	150
22	MS/AUPensacola	1906	2	134
23	FL, GA, SC	1896	3	130
24	AGNES (FL/NE U.S.)	1972	1	122
25	HAZEL (SC/NC)	1954	4	95
26	BETSY (SE FL/SE LA)	1965	3	75
27	Northeast U.S.	1944	3	64

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Table 11 – *Continued from previous page*

Rank	Hurricane	Year	Category	Deaths
28	CAROL (NE U.S.)	1954	3	60
29	FLOYD (Mid Atlantic & NE U.S.)	1999	2	56
30	NC	1883	2	53
31	SE FL/SE LA/MS	1947	4	51
32	NC, SC	1899	3	50
32	GA/SCINC	1940	2	50
32	DONNA (FL/Eastem U.S.)	1960	4	50
35	LA	1860	2	47
36	NC, VA	1879	3	46
36	CARLA	1961	4	46
38	TX (Velasco)	1909	3	41
38	ALLISON (SE D9	2001	TS	41
40	Mid-Atlantic	1889	TS	40
40	TX (Freeport)	1932	4	40
40	S TX	1933	3	40

Table 12: Hurricane Max and Sum of Deaths By Year

Year	Max	Sum
1856	400	400
1860	47	47
1875	176	176
1879	46	46
1881	700	700
1883	53	53
1886	150	150
1889	40	40

Continued on next page

Table 12 – *Continued from previous page*

Year	Max	Sum
1893	**1,500**	**2,750**
1896	130	130
1898	179	179
1899	50	50
1900	8,000	8,000
1906	**164**	**298**
1909	**350**	**391**
1915	**275**	**550**
1919	287	287
1926	372	372
1928	2,500	2,500
1932	40	40
1933	40	40
1935	408	408
1938	256	256
1940	50	50
1944	64	64
1947	51	51
1954	95	155
1955	184	184
1957	416	416
1960	50	50
1961	46	46
1965	75	75
1969	256	256
1972	122	122
1999	56	56
2001	41	41
2005	1,200	1,200

Table 13: Hurricane Max Deaths By Year

No	O(x)	Max Deaths Sorted
1	0.031	8,000
2	0.063	2,500
3	0.094	1,500
4	0.125	1,200
5	0.156	700
6	0.188	416
7	0.219	408
8	0.250	400
9	0.281	372
10	0.313	350
11	0.344	287
12	0.375	275
13	0.406	256
14	0.438	256
15	0.469	184
16	0.500	179
17	0.531	176
18	0.563	164
19	0.594	150
20	0.625	130
21	0.656	122
22	0.688	95
23	0.719	75
24	0.750	64
25	0.781	56
26	0.813	53
27	0.844	51
28	0.875	50

Continued on next page

Table 13 – *Continued from previous page*

No	O(x)	Max Deaths Sorted
29	0.906	47
30	0.938	46
31	0.969	41
32	1.000	40

Table 14: Hurricane Sum of Deaths By Year

No	A(x)	Sum Deaths Sorted
1	0.031	8,000
2	0.063	2,750
3	0.094	2,500
4	0.125	1,200
5	0.156	700
6	0.188	550
7	0.219	416
8	0.250	408
9	0.281	400
10	0.313	391
11	0.344	372
12	0.375	298
13	0.406	287
14	0.438	256
15	0.469	256
16	0.500	184
17	0.531	179
18	0.563	176
19	0.594	155

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Table 14 – *Continued from previous page*

No	A(x)	Sum Deaths Sorted
20	0.625	150
21	0.656	130
22	0.688	122
23	0.719	75
24	0.750	64
25	0.781	56
26	0.813	53
27	0.844	51
28	0.875	50
29	0.906	47
30	0.938	46
31	0.969	41
32	1.000	40

C The Costliest US Tropical Cyclones

Table 15: The 30 costliest mainland United States tropical cyclones, 1900-2010, (not adjusted for inflation).

Rank	Hurricane	Year	Category	Damage (Millions)
1	KATRINA (SE FL, LA, MS)	2005	3	108,000
2	IKE (TX, LA)	2008	2	29,520
3	ANDREW (SE FL/LA)	1992	5	26,500
4	WILMA (S FL)	2005	3	21,007
5	IVAN (AL/NW FL)	2004	3	18,820
6	CHARLEY (SW FL)	2004	4	15,113
7	RITA (SW LA, N TX)	2005	3	12,037
8	FRANCES (FL)	2004	2	9,507
9	ALLISON (N TX)	2001	TS	9,000
10	JEANNE (FL)	2004	3	7,660
11	HUGO (SC)	1989	4	7,000
12	FLOYD (Mid-Atlantic & NE U.S.)	1999	2	6,900
13	ISABEL (Mid-Atlantic)	2003	2	5,370
14	OPAL (NW FL/AL)	1995	3	5,142
15	GUSTAV (LA)	2008	2	4,618
16	FRAN (NC)	1996	3	4,160
17	GEORGES (FL Keys, MS,AL)	1998	2	2,765
18	DENNIS (NW FL)	2005	3	2,545
19	FREDERIC (AL/MS)	1979	3	2,300
20	AGNES (FUNE U.S.)	1972	1	2,100
21	ALICIA (N TX)	1983	3	2,000
22	BOB (NC, NE U.S)	1991	2	1,500
22	JUAN (LA)	1985	1	1,500
24	CAMILLE (MS/SE LANA)	1969	5	1,421
25	BETSY (SE FL/SE LA)	1965	3	1,421
26	ELENA (MS/AL/NW FL)	1985	3	1,250

Continued on next page

Table 15 – *Continued from previous page*

Rank	Hurricane	Year	Category	Damage (Millions)
27	DOLLY (S TX)	2008	1	1,050
28	CELIA (S TX)	1970	3	930
29	LILI (SC LA)	2002	1	925
30	GLORIA (Eastern U.S.)	1985	3	900

Table 16: The 30 costliest mainland United States tropical cyclones, 1900-2010, Ranked Using 2010 Inflation, Population and Wealth Normalization.

Rank	Hurricane	Year	Category	Damage (Millions)
1	SE Florida/Alabama	1926	4	164,839
2	KATRINA (SE LA, MS, AL)	2005	3	113,400
3	N Texas (Galveston)	1900	4	104,330
4	N Texas (Galveston)	1915	4	71,397
5	ANDREW (SE FL/LA)	1992	5	58,555
6	New England	1938	3	41,122
7	SW Florida	1944	3	40,621
8	SE Florida/Lake Okeechobee	1928	4	35,298
9	IKE (N TX/SW LA)	2008	2	29,520
10	DONNA (FUEastern U.S.)	1960	4	28,159
11	CAMILLE (MS/LANA)	1969	5	22,286
12	WILMA (S FL)	2005	3	22,057
13	IVAN (NW FL, AL)	2004	3	21,575
14	BETSY (SE FL/LA)	1965	3	18,749
15	DIANE (NE U.S.)	1955	1	18,073
16	AGNES (NW FL, NE U.S.)	1972	1	18,052
17	HAZEL (SC/NC)	1954	4	17,339
18	CHARLEY (SW FL)	2004	4	17,210

Continued on next page

Table 16 – *Continued from previous page*

Rank	Hurricane	Year	Category	Damage (Millions)
19	CAROL (NE U.S.)	1954	3	16,940
20	HUGO (SC)	1989	4	16,088
21	SE Florida	1949	3	15,398
22	CARLA (N & Central TX)	1961	4	14,920
23	SE Florida/Louisiana/Alabama	1947	4	14,406
24	NE U.S.	1944	3	13,881
25	SE FL/S TX	1919	4	13,847
26	SE Florida	1945	3	12,956
27	RITA (SW LA/N TX)	2005	3	12,639
28	ALLISON (N TX)	2001	TS	12,523
29	CELIA (S TX)	1970	3	12,104
30	FRANCES (SE FL)	2004	2	10,899

Table 17: Hurricane Max and Sum of Costs By Year

Year	Max	Sum
1900	104,330	104,330
1915	71,397	71,397
1919	13,847	13,847
1926	164,839	164,839
1928	35,298	35,298
1938	41,122	41,122
1944	**40,621**	**54,502**
1945	12,956	12,956
1947	14,406	14,406
1949	15,398	15,398
1954	**17,339**	**34,279**

Continued on next page

Table 17 – *Continued from previous page*

Year	Max	Sum
1955	18,073	18,073
1960	28,159	28,159
1961	14,920	14,920
1965	18,749	18,749
1969	22,286	22,286
1970	12,104	12,104
1972	18,052	18,052
1989	16,088	16,088
1992	58,555	58,555
2001	12,523	12,523
2004	**21,575**	**49,684**
2005	**113,400**	**148,096**
2008	29,520	29,520

Table 18: Hurricane Max Costs By Year

No	O(x)	Max Costs Sorted
1	0.042	164,839
2	0.083	113,400
3	0.125	104,330
4	0.167	71,397
5	0.208	58,555
6	0.250	41,122
7	0.292	40,621
8	0.333	35,298
9	0.375	29,520
10	0.417	28,159

Continued on next page

Table 18 – *Continued from previous page*

No	O(x)	Max Costs Sorted
11	0.458	22,286
12	0.500	21,575
13	0.542	18,749
14	0.583	18,073
15	0.625	18,052
16	0.667	17,339
17	0.708	16,088
18	0.750	15,398
19	0.792	14,920
20	0.833	14,406
21	0.875	13,847
22	0.917	12,956
23	0.958	12,523
24	1.000	12,104

Table 19: Hurricane Sum of Costs By Year

No	A(x)	Sum Cosths Sorted
1	0.042	164,839
2	0.083	148,096
3	0.125	104,330
4	0.167	71,397
5	0.208	58,555
6	0.250	54,502
7	0.292	49,684
8	0.333	41,122
9	0.375	35,298

Continued on next page

Table 19 – *Continued from previous page*

No	A(x)	Sum Costs Sorted
10	0.417	34,279
11	0.458	29,520
12	0.500	28,159
13	0.542	22,286
14	0.583	18,749
15	0.625	18,073
16	0.667	18,052
17	0.708	16,088
18	0.750	15,398
19	0.792	14,920
20	0.833	14,406
21	0.875	13,847
22	0.917	12,956
23	0.958	12,523
24	1.000	12,104

D The Most Intense US Tropical Cyclones

Table 20: The Most Intense Mainland United States Hurricanes Ranked by Pressure, 1851-2010

Rank	Hurricane	Year	Category (at landfall)	Mimimum Millibars	Pressure (Inches)
1	FL (Keys)	1935	5	892	26.35
2	CAMILLE (MS/SE LA/VA)	1969	5	909	26.84
3	KATRINA (SE LA, MS)	2005	3	920	27.17
4	ANDREW (SE FL/SE LA)	1992	5	922	27.23
5	TX (Indianola)	1886	4	925	27.31
6	FL (Keys)/S TX	1919	4	927	27.37
7	FL (Lake Okeechobee)	1928	4	929	27.43
8	DONNA (FL/Eastern U.S.)	1960	4	930	27.46
8	FL (Miami)/MS/AUPensacola	1926	4	930	27.46
10	CARLA (N & Central TX)	1961	4	931	27.49
11	S TX	1916	4	932	27.52
12	LA (Last Island)	1856	4	934	27.58
12	HUGO (SC)	1989	4	934	27.58
14	TX (Galveston)	1900	4	936	27.64
15	RITA (SW LA/N TX)	2005	3	937	27.67
16	GA/FL (Brunswick)	1898	4	938	27.70
16	HAZEL (SC/NC)	1954	4	938	27.70
18	SE FL/SE LA/MS	1947	4	940	27.76
18	TX (Galveston)	1915	4	940	27.76
20	N TX	1932	4	941	27.79
20	CHARLEY (SW FL)	2004	4	941	27.79
22	GLORIA (Eastern U.S.)	1985	3	942	27.82
22	OPAL (NW FL/AL)	1995	3	942	27.82
24	LA (New Orleans)	1915	3	944	27.88
25	FL (Central)	1888	3	945	27.91
25	E NC	1899	3	945	27.91

Continued on next page

Table 20 – *Continued from previous page*

Rank	Hurricane	Year	Category (at landfall)	Mimimum Millibars	Pressure (Inches)
25	AUDREY (SW LA/N TX)	1957	4	945	27.91
25	CELIA (S TX)	1970	3	945	27.91
25	ALLEN (S TX)	1980	3	945	27.91
30	New England	1938	3	946	27.94
30	FREDERIC (AL/MS)	1979	3	946	27.94
30	/VAN (AL, NW FL)	2004	3	946	27.94
30	DENNIS (NW FL)	2005	3	946	27.94
34	NE U.S.	1944	3	947	27.97
35	LA (Chenier Caminanda)	1893	4	948	27.99
35	BETSY (SE FL/SE LA)	1965	3	948	27.99
35	SE FL/NW FL	1929	3	948	27.99
35	SE FL	1933	3	948	27.99
39	NW FL	1917	3	949	28.02
39	NW FL	1882	3	949	28.02
39	DIANA (NC)	1984	3	949	28.02
39	S TX	1933	3	949	28.02
43	MS/AL	1916	3	950	28.05
43	GA/SC	1854	3	950	28.05
43	LA/MS	1855	3	950	28.05
43	LA/MS/AL	1860	3	950	28.05
43	LA	1879	3	950	28.05
43	BEULAH (S TX)	1967	3	950	28.05
43	HILDA (Central LA)	1964	3	950	28.05
43	GRACIE (SC)	1959	3	950	28.05
43	TX (Central)	1942	3	950	28.05
43	JEANNE (FL)	2004	3	950	28.05
43	WILMA (S FL)	2005	3	950	28.05
54	SE FL	1945	3	951	28.08

Continued on next page

Table 20 – *Continued from previous page*

Rank	Hurricane	Year	Category (at landfall)	Mimimum Millibars	Pressure (Inches)
54	BRET (S TX)	1999	3	951	28.08
56	LA (Grand Isle)	1909	3	952	28.11
56	FL (Tampa Bay)	1921	3	952	28.11
56	CARMEN (Central LA)	1974	3	952	28.11
59	SC/NC	1885	3	953	28.14
59	S FL	1906	3	953	28.14
61	GA/SC	1893	3	954	28.17
61	EDNA (New England)	1954	3	954	28.17
61	SE FL	1949	3	954	28.17
61	FRAN (NC)	1996	3	954	28.17
65	SE FL	1871	3	955	28.20
65	LA/TX	1886	3	955	28.20
65	SC/NC	1893	3	955	28.20
65	NW FL	1894	3	955	28.20
65	ELOISE (NW FL)	1975	3	955	28.20
65	KING (SE FL)	1950	3	955	28.20
65	Central LA	1926	3	955	28.20
65	SW LA	1918	3	955	28.20

Table 21: Hurricane Min and Max of Intensities By Year

Year	Min Pressure	Max Pressure
1854	28.05	28.05
1855	28.05	28.05
1856	27.58	27.58
1860	28.05	28.05

Continued on next page

Table 21 – *Continued from previous page*

Year	Min Pressure	Max Pressure
1871	28.20	28.20
1879	28.05	28.05
1882	28.02	28.02
1885	28.14	28.14
1886	**27.31**	**28.20**
1888	27.91	27.91
1893	**27.99**	**28.20**
1894	28.20	28.20
1898	27.70	27.70
1899	27.91	27.91
1900	27.64	27.64
1906	28.14	28.14
1909	28.11	28.11
1915	**27.76**	**27.88**
1916	**27.52**	**28.05**
1917	28.02	28.02
1918	28.20	28.20
1919	27.37	27.37
1921	28.11	28.11
1926	**27.46**	**28.20**
1928	27.43	27.43
1929	27.99	27.99
1932	27.79	27.79
1933	**27.99**	**28.02**
1935	26.35	26.35
1938	27.94	27.94
1942	28.05	28.05
1944	27.97	27.97
1945	28.08	28.08

Continued on next page

Table 21 – *Continued from previous page*

Year	Min Pressure	Max Pressure
1947	27.76	27.76
1949	28.17	28.17
1950	28.20	28.20
1954	**27.70**	**28.17**
1957	27.91	27.91
1959	28.05	28.05
1960	27.46	27.46
1961	27.49	27.49
1964	28.05	28.05
1965	27.99	27.99
1967	28.05	28.05
1969	26.84	26.84
1970	27.91	27.91
1974	28.11	28.11
1975	28.20	28.20
1979	27.94	27.94
1980	27.91	27.91
1984	28.02	28.02
1985	27.82	27.82
1989	27.58	27.58
1992	27.23	27.23
1995	27.82	27.82
1996	28.17	28.17
1999	28.08	28.08
2004	**27.79**	**28.05**
2005	**27.17**	**28.05**

Table 22: Hurricane Min Intensities By Year

No	$MinEP(x)$	Min Intensities Sorted
1	0.017	26.35
2	0.034	26.84
3	0.051	27.17
4	0.068	27.23
5	0.085	27.31
6	0.102	27.37
7	0.119	27.43
8	0.136	27.46
9	0.153	27.46
10	0.169	27.49
11	0.186	27.52
12	0.203	27.58
13	0.220	27.58
14	0.237	27.64
15	0.254	27.70
16	0.271	27.70
17	0.288	27.76
18	0.305	27.76
19	0.322	27.79
20	0.339	27.79
21	0.356	27.82
22	0.373	27.82
23	0.390	27.91
24	0.407	27.91
25	0.424	27.91
26	0.441	27.91
27	0.458	27.91
28	0.475	27.94
29	0.492	27.94

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Table 22 – *Continued from previous page*

No	$MinEP(x)$	Min Intensities Sorted
30	0.508	27.97
31	0.525	27.99
32	0.542	27.99
33	0.559	27.99
34	0.576	27.99
35	0.593	28.02
36	0.610	28.02
37	0.627	28.02
38	0.644	28.05
39	0.661	28.05
40	0.678	28.05
41	0.695	28.05
42	0.712	28.05
43	0.729	28.05
44	0.746	28.05
45	0.763	28.05
46	0.780	28.08
47	0.797	28.08
48	0.814	28.11
49	0.831	28.11
50	0.847	28.11
51	0.864	28.14
52	0.881	28.14
53	0.898	28.17
54	0.915	28.17
55	0.932	28.20
56	0.949	28.20
57	0.966	28.20
58	0.983	28.20

Continued on next page

Table 22 – *Continued from previous page*

No	$MinEP(x)$	Min Intensities Sorted
59	1.000	28.20

Table 23: Hurricane Max Intensities By Year

No	$MaxEP(x)$	Min Intensities Sorted
1	0.017	26.35
2	0.034	26.84
3	0.051	27.23
4	0.068	27.37
5	0.085	27.43
6	0.102	27.46
7	0.119	27.49
8	0.136	27.58
9	0.153	27.58
10	0.169	27.64
11	0.186	27.70
12	0.203	27.76
13	0.220	27.79
14	0.237	27.82
15	0.254	27.82
16	0.271	27.88
17	0.288	27.91
18	0.305	27.91
19	0.322	27.91
20	0.339	27.91
21	0.356	27.91
22	0.373	27.94

Continued on next page

Table 23 – *Continued from previous page*

No	$MaxEP(x)$	Min Intensities Sorted
23	0.390	27.94
24	0.407	27.97
25	0.424	27.99
26	0.441	27.99
27	0.458	28.02
28	0.475	28.02
29	0.492	28.02
30	0.508	28.02
31	0.525	28.05
32	0.542	28.05
33	0.559	28.05
34	0.576	28.05
35	0.593	28.05
36	0.610	28.05
37	0.627	28.05
38	0.644	28.05
39	0.661	28.05
40	0.678	28.05
41	0.695	28.05
42	0.712	28.08
43	0.729	28.08
44	0.746	28.11
45	0.763	28.11
46	0.780	28.11
47	0.797	28.14
48	0.814	28.14
49	0.831	28.17
50	0.847	28.17
51	0.864	28.17

Continued on next page

Table 23 – *Continued from previous page*

No	$MaxEP(x)$	Min Intensities Sorted
52	0.881	28.20
53	0.898	28.20
54	0.915	28.20
55	0.932	28.20
56	0.949	28.20
57	0.966	28.20
58	0.983	28.20
59	1.000	28.20

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