MATHEMATICS
Module 4
Trigonometry

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REFERENCES

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TERMINAL OBJECTIVE

1.0 Given a calculator and a list of formulas, **APPLY** the laws of trigonometry to solve for unknown values.

ENABLING OBJECTIVES

1.1 Given a problem, **APPLY** the Pythagorean theorem to solve for the unknown values of a right triangle.

1.2 Given the following trigonometric terms, **IDENTIFY** the related function:

a. Sine
b. Cosine
c. Tangent
d. Cotangent
e. Secant
f. Cosecant

1.3 Given a problem, **APPLY** the trigonometric functions to solve for the unknown.

1.4 **STATE** the definition of a radian.

PYTHAGOREAN THEOREM

This chapter covers right triangles and solving for unknowns using the Pythagorean theorem.

EO 1.1 Given a problem, APPLY the Pythagorean theorem to solve for the unknown values of a right triangle.

Trigonometry is the branch of mathematics that is the study of angles and the relationship between angles and the lines that form them. Trigonometry is used in Classical Physics and Electrical Science to analyze many physical phenomena. Engineers and operators use this branch of mathematics to solve problems encountered in the classroom and on the job. The most important application of trigonometry is the solution of problems involving triangles, particularly right triangles.

Trigonometry is one of the most useful branches of mathematics. It is used to indirectly measure distances which are difficult to measure directly. For example, the height of a flagpole or the distance across a river can be measured using trigonometry.

As shown in Figure 1 below, a triangle is a plane figure formed using straight line segments (*AB*, *BC*, *CA*) to connect three points (*A*, *B*, *C*) that are not in a straight line. The sum of the measures of the three interior angles (a', b', c') is 180°, and the sum of the lengths of any two sides is always greater than or equal to the third.

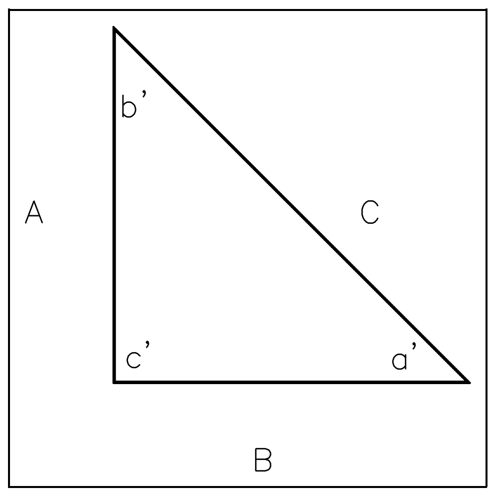

Figure 1 Triangle

Pythagorean Theorem

The Pythagorean theorem is a tool that can be used to solve for unknown values on right triangles. In order to use the Pythagorean theorem, a term must be defined. The term *hypotenuse* is used to describe the side of a right triangle opposite the right angle. Line segment C is the hypotenuse of the triangle in Figure 1.

The Pythagorean theorem states that in any right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.

This may be written as $c^2 = a^2 + b^2$ or $c = \sqrt{a^2 + b^2}$. (4-1)

Example:

The two legs of a right triangle are 5 ft and 12 ft. How long is the hypotenuse?

Let the hypotenuse be c ft.

$a^2 + b^2 = c^2$

$12^2 + 5^2 = c^2$

$144 + 25 = c^2$

$169 = c^2$

$\sqrt{169} = c$

$13 \text{ ft} = c$

Using the Pythagorean theorem, one can determine the value of the unknown side of a right triangle when given the value of the other two sides.

Example:

Given that the hypotenuse of a right triangle is 18" and the length of one side is 11", what is the length of the other side?

$a^2 + b^2 = c^2$

$11^2 + b^2 = 18^2$

$b^2 = 18^2 - 11^2$

$b^2 = 324 - 121$

$b = \sqrt{203}$

$b = 14.2 \text{ in}$

Summary

The important information in this chapter is summarized below.

Pythagorean Theorem Summary

- The Pythagorean theorem states that in any right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.

  This may be written as $c^2 = a^2 + b^2$ or $c = \sqrt{a^2 + b^2}$.

TRIGONOMETRIC FUNCTIONS

This chapter covers the six trigonometric functions and solving right triangles.

EO 1.2 Given the following trigonometric terms, IDENTIFY the related function:

a. Sine
b. Cosine
c. Tangent
d. Cotangent
e. Secant
f. Cosecant

EO 1.3 Given a problem, APPLY the trigonometric functions to solve for the unknown.

As shown in the previous chapter, the lengths of the sides of right triangles can be solved using the Pythagorean theorem. We learned that if the lengths of two sides are known, the length of the third side can then be determined using the Pythagorean theorem. One fact about triangles is that the sum of the three angles equals 180°. If right triangles have one 90° angle, then the sum of the other two angles must equal 90°. Understanding this, we can solve for the unknown angles if we know the length of two sides of a right triangle. This can be done by using the six trigonometric functions.

In right triangles, the two sides (other than the hypotenuse) are referred to as the opposite and adjacent sides. In Figure 2, side a is the opposite side of the angle θ and side b is the adjacent side of the angle θ. The terms hypotenuse, opposite side, and adjacent side are used to distinguish the relationship between an acute angle of a right triangle and its sides. This relationship is given by the six trigonometric functions listed below:

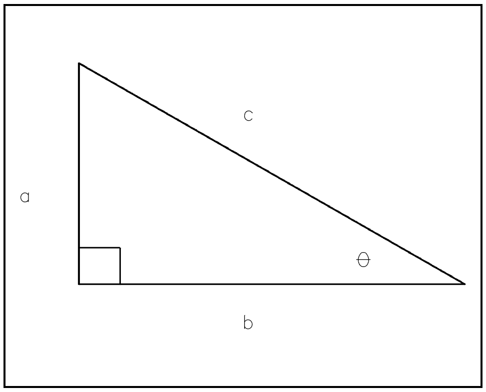

Figure 2 Right Triangle

$$\text{sine } \theta = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}} \quad (4\text{-}2)$$

$$\text{cosine } \theta = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}} \quad (4\text{-}3)$$

$$\text{tangent } \theta = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}} \tag{4-4}$$

$$\text{cosecant } \theta = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{oposite}} \tag{4-5}$$

$$\text{secant } \theta = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{adjacent}} \tag{4-6}$$

$$\text{cotangent } \theta = \frac{b}{a} = \frac{\text{adjacent}}{\text{opposite}} \tag{4-7}$$

The trigonometric value for any angle can be determined easily with the aid of a calculator. To find the sine, cosine, or tangent of any angle, enter the value of the angle into the calculator and press the desired function. Note that the secant, cosecant, and cotangent are the mathematical inverse of the sine, cosine and tangent, respectively. Therefore, to determine the cotangent, secant, or cosecant, first press the SIN, COS, or TAN key, then press the INV key.

Example:

Determine the values of the six trigonometric functions of an angle formed by the x-axis and a line connecting the origin and the point (3,4).

Solution:

To help to "see" the solution of the problem it helps to plot the points and construct the right triangle.

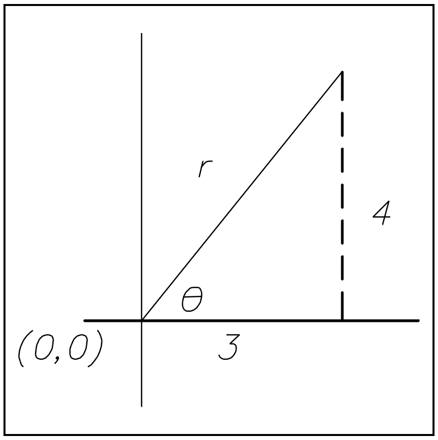

Figure 3 Example Problem

Label all the known angles and sides, as shown in Figure 3.

From the triangle, we can see that two of the sides are known. But to answer the problem, all three sides must be determined. Therefore the Pythagorean theorem must be applied to solve for the unknown side of the triangle.

$$x = 3$$

$$y = 4$$

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2}$$

$$r = \sqrt{9 + 16} = \sqrt{25} = 5$$

Having solved for all three sides of the triangle, the trigonometric functions can now be determined. Substitute the values for *x*, *y*, and *r* into the trigonometric functions and solve.

$$\sin \theta = \frac{y}{r} = \frac{4}{5} = 0.800$$

$$\cos \theta = \frac{x}{r} = \frac{3}{5} = 0.600$$

$$\tan \theta = \frac{y}{x} = \frac{4}{3} = 1.333$$

$$\csc \theta = \frac{r}{y} = \frac{5}{4} = 1.250$$

$$\sec \theta = \frac{r}{x} = \frac{5}{3} = 1.667$$

$$\cot \theta = \frac{x}{y} = \frac{3}{4} = 0.750$$

Although the trigonometric functions of angles are defined in terms of lengths of the sides of right triangles, they are really functions of the angles only. The numerical values of the trigonometric functions of any angle depend on the size of the angle and not on the length of the sides of the angle. Thus, the sine of a 30° angle is always 1/2 or 0.500.

Inverse Trigonometric Functions

When the value of a trigonometric function of an angle is known, the size of the angle can be found. The inverse trigonometric function, also known as the arc function, defines the angle based on the value of the trigonometric function. For example, the sine of 21° equals 0.35837; thus, the arc sine of 0.35837 is 21°.

There are two notations commonly used to indicate an inverse trigonometric function.

$$\arcsin 0.35837 = 21°$$

$$\sin^{-1} 0.35837 = 21°$$

The notation *arcsin* means *the angle whose sine is.* The notation *arc* can be used as a prefix to any of the trigonometric functions. Similarly, the notation sin^{-1} means *the angle whose sine is.* It is important to remember that the -1 in this notation is not a negative exponent but merely an indication of the inverse trigonometric function.

To perform this function on a calculator, enter the numerical value, press the INV key, then the SIN, COS, or TAN key. To calculate the inverse function of cot, csc, and sec, the reciprocal key must be pressed first then the SIN, COS, or TAN key.

Examples:

Evaluate the following inverse trigonometric functions.

$$\arcsin 0.3746 = 22°$$

$$\arccos 0.3746 = 69°$$

$$\arctan 0.3839 = 21°$$

$$\text{arccot } 2.1445 = \arctan \frac{1}{2.1445} = \arctan 0.4663 = 25°$$

$$\text{arcsec } 2.6695 = \arccos \frac{1}{2.6695} = \arccos 0.3746 = 68°$$

$$\text{arccsc } 2.7904 = \arcsin \frac{1}{2.7904} = \arcsin 0.3584 = 21°$$

Summary

The important information in this chapter is summarized below.

Trigonometric Functions Summary

- The six trigonometric functions are:

$$\text{sine } \theta = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine } \theta = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent } \theta = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{cotangent } \theta = \frac{b}{a} = \frac{\text{adjacent}}{\text{opposite}}$$

$$\text{cosecant } \theta = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{secant } \theta = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

RADIANS

This chapter will cover the measure of angles in terms of radians and degrees.

EO 1.4 STATE the definition of a radian.

Radian Measure

The size of an angle is usually measured in degrees. However, in some applications the size of an angle is measured in radians. A radian is defined in terms of the length of an arc subtended by an angle at the center of a circle. An angle whose size is one radian subtends an arc whose length equals the radius of the circle. Figure 4 shows $\angle BAC$ whose size is one radian. The length of arc BC equals the radius r of the circle. The size of an angle, in radians, equals the length of the arc it subtends divided by the radius.

$$\text{Radians} = \frac{\text{Length of Arc}}{\text{Radius}} \qquad (4\text{-}8)$$

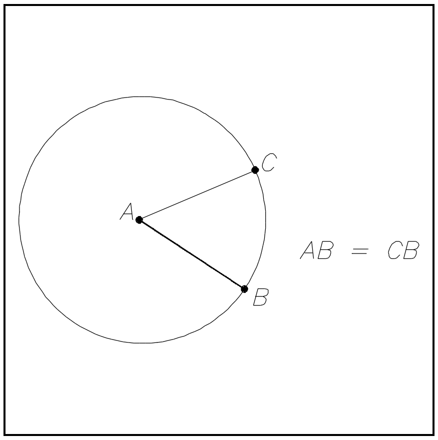

Figure 4 Radian Angle

One radian equals approximately 57.3 degrees. There are exactly 2π radians in a complete revolution. Thus 2π radians equals 360 degrees: π radians equals 180 degrees.

Although the radian is defined in terms of the length of an arc, it can be used to measure any angle. Radian measure and degree measure can be converted directly. The size of an angle in degrees is changed to radians by multiplying by $\frac{\pi}{180}$. The size of an angle in radians is changed to degrees by multiplying by $\frac{180}{\pi}$.

Example:

Change 68.6° to radians.

$$068.6° \left(\frac{\pi}{180} \right) = \frac{(68.6)\pi}{180} = 1.20 \text{ radians}$$

Example:

Change 1.508 radians to degrees.

$$(1.508 \text{ radians})\left(\frac{180}{\pi}\right) = \frac{(1.508)(180)}{\pi} = 86.4°$$

Summary

The important information in this chapter is summarized below.

Radian Measure Summary

- A radian equals approximately 57.3º and is defined as the angle subtended by an arc whose length is equal to the radius of the circle.

  $$\text{Radian} = \frac{\text{Length of arc}}{\text{Radius of circle}}$$

  π radians = 180°