A Method for Converting a Random PSD to a Sine Tone
Revision A

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Variables

m	Mass
c	Viscous damping coefficient
k	Stiffness
$\ddot{x}$	Acceleration of mass
$\ddot{y}$	Base acceleration
R	Overall GRMS response to the base input
S	Sine base input peak amplitude (G peak)
Q	Amplification Factor
α	Standard deviation scale factor

Introduction

Consider a single-degree-of-freedom system subject to base excitation. Assume that the natural frequency and amplification factor are both known.

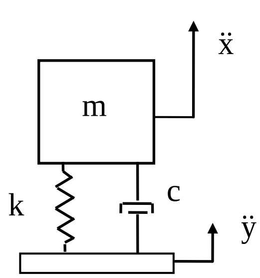

Figure 1.

Assume a case where the base input is a PSD which must be converted to a sine tone at the SDOF system's natural frequency. The conversion will be made in terms of the acceleration response of the mass to each input.

The conversion formula is

$$\alpha R = QS \tag{1}$$

$$S = \frac{\alpha R}{Q} \tag{2}$$

Note that the R value is calculated from the PSD using a separate equation, as taken from Reference 1.

The coefficient for electronic equipment is $\alpha = 1.9$ per Reference 2. Round this value up to 2.0 for conservatism.

This means that the sine response will be equal to the random response 2.0-sigma level. Note that the 1-sigma level is equal the RMS level assuming zero mean.

Example

An example is given in Appendix A. The example shows that equation (2) is satisfactory in terms of a sine tone which envelops a random vibration input in terms of relative fatigue damage.

References

1. T. Irvine, An Introduction to the Vibration Response Spectrum, Revision D, Vibrationdata, 2009.

2. T. Irvine, Extending Steinberg's Fatigue Analysis of Electronics Equipment Methodology to a Full Relative Displacement vs. Cycles Curve, Revision C, Vibrationdata, 2013.

3. David O. Smallwood, An Improved Recursive Formula for Calculating Shock Response Spectra, Shock and Vibration Bulletin, No. 51, May 1981.

4. T. Irvine, Derivation of the Filter Coefficients for the Ramp Invariant Method as Applied to Base Excitation of a Single-degree-of-Freedom System, Revision B, Vibrationdata, 2013.

5. ASTM E 1049-85 (2005) Rainflow Counting Method, 1987.

6. Dave Steinberg, Vibration Analysis for Electronic Equipment, Second Edition, Wiley-Interscience, New York, 1988.

7. Test Methods and Control, Martin Marietta, M-67-45 (Rev 4), Denver, Colorado, January 1989. See Paragraph 8.20.4.3.

8. T. Irvine, Miner's Cumulative Damage via Rainflow Cycle Counting, Revision F, Vibrationdata, 2013.

9. T. Irvine, A Method for Converting a Sine Tone to a Narrowband PSD Revision E, Vibrationdata, 2013.

APPENDIX A

Example

An SDOF system has a natural frequency of 300 Hz and amplification factor of Q=10. It is exposed to the NAVMAT P-9492 base input in Figure A-1 for 60 seconds. Derive an equivalent sine tone at the natural frequency with the same duration. Assume a coefficient of α=2.0. Also assume a fatigue exponent = 6.4. The fatigue exponent will be used for verification only.

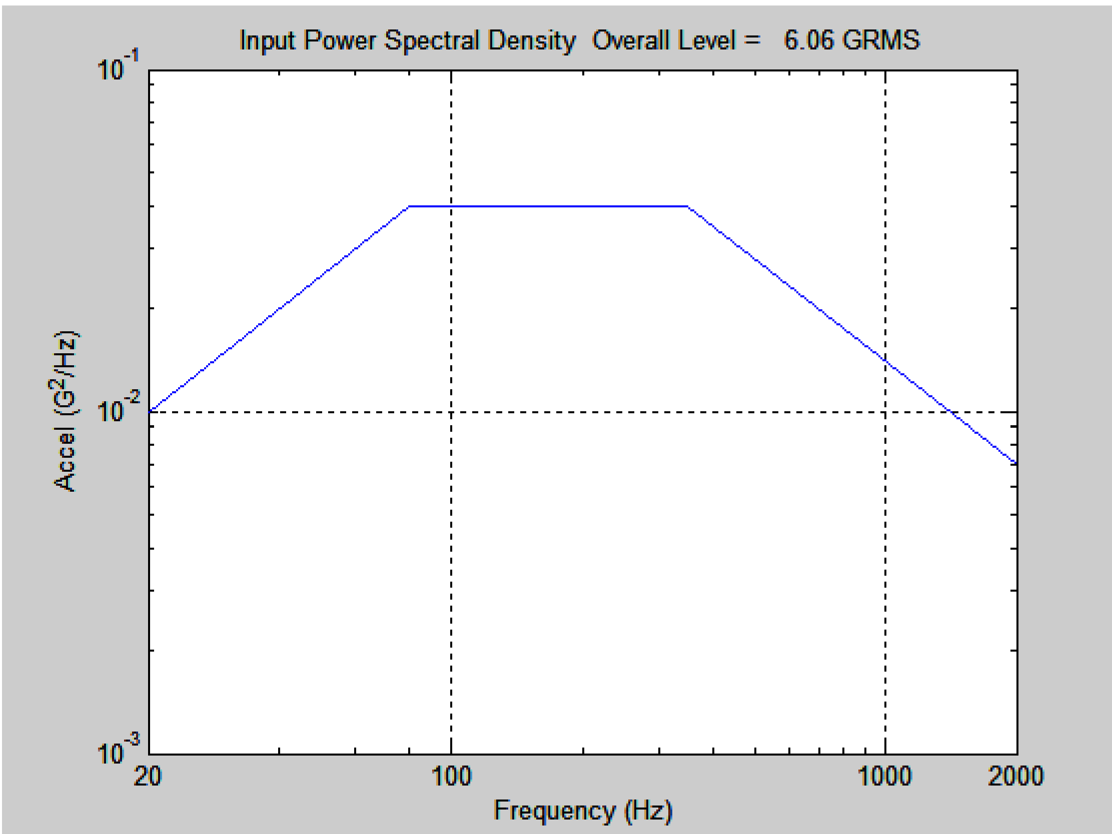

Figure A-1.

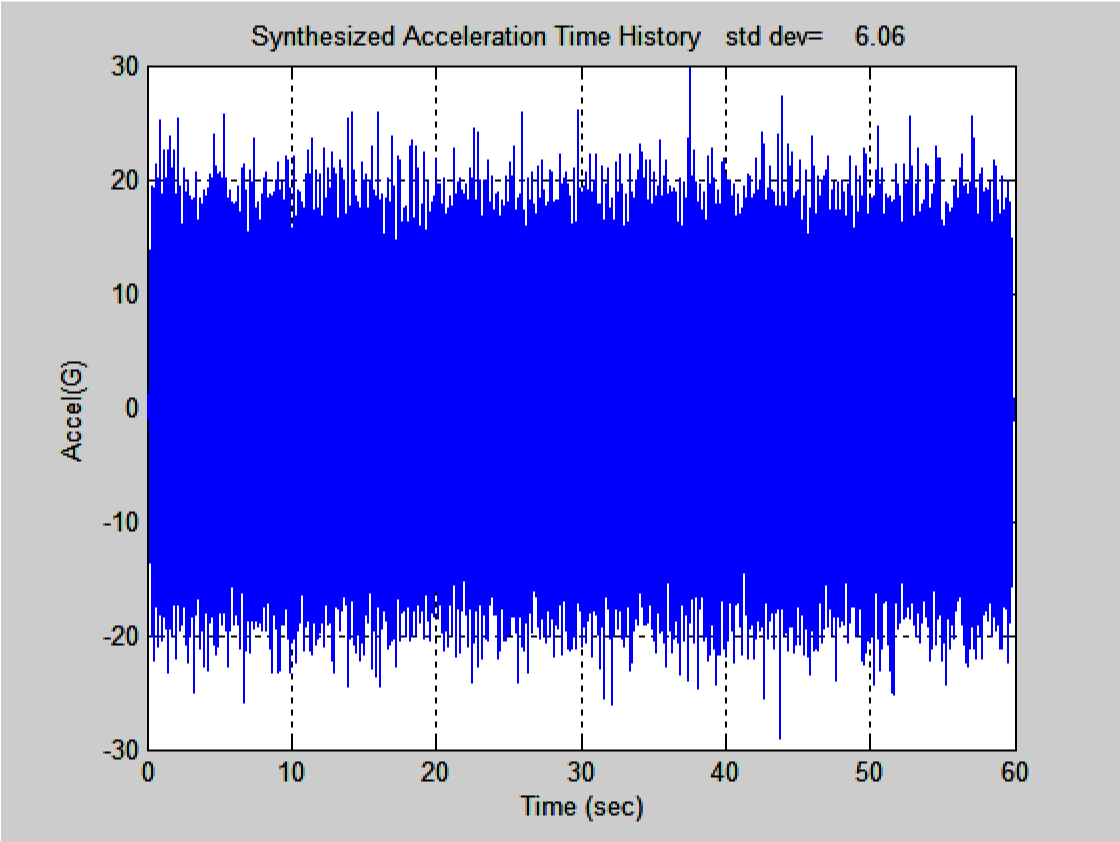

Figure A-2.

A time history was synthesized to meet the PSD in Figure A-1. This is for verification purposes only.

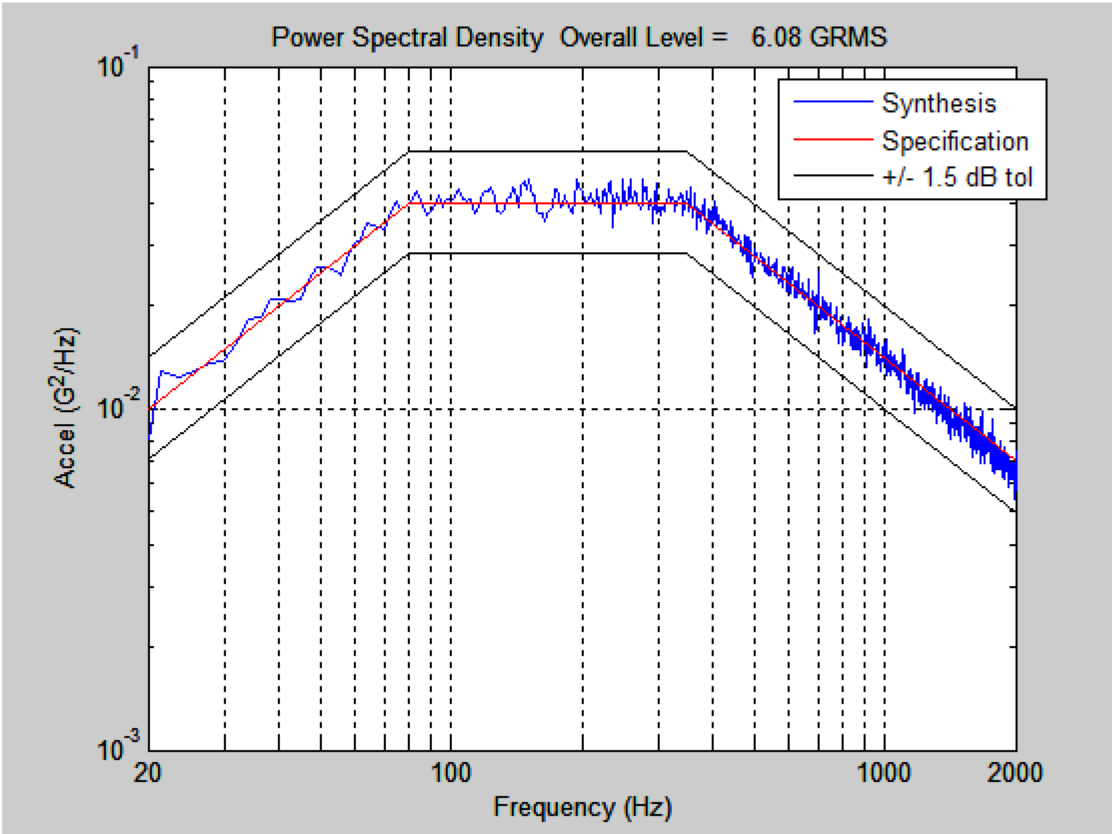

Figure A-3.

The synthesized time history satisfies the PSD specification.

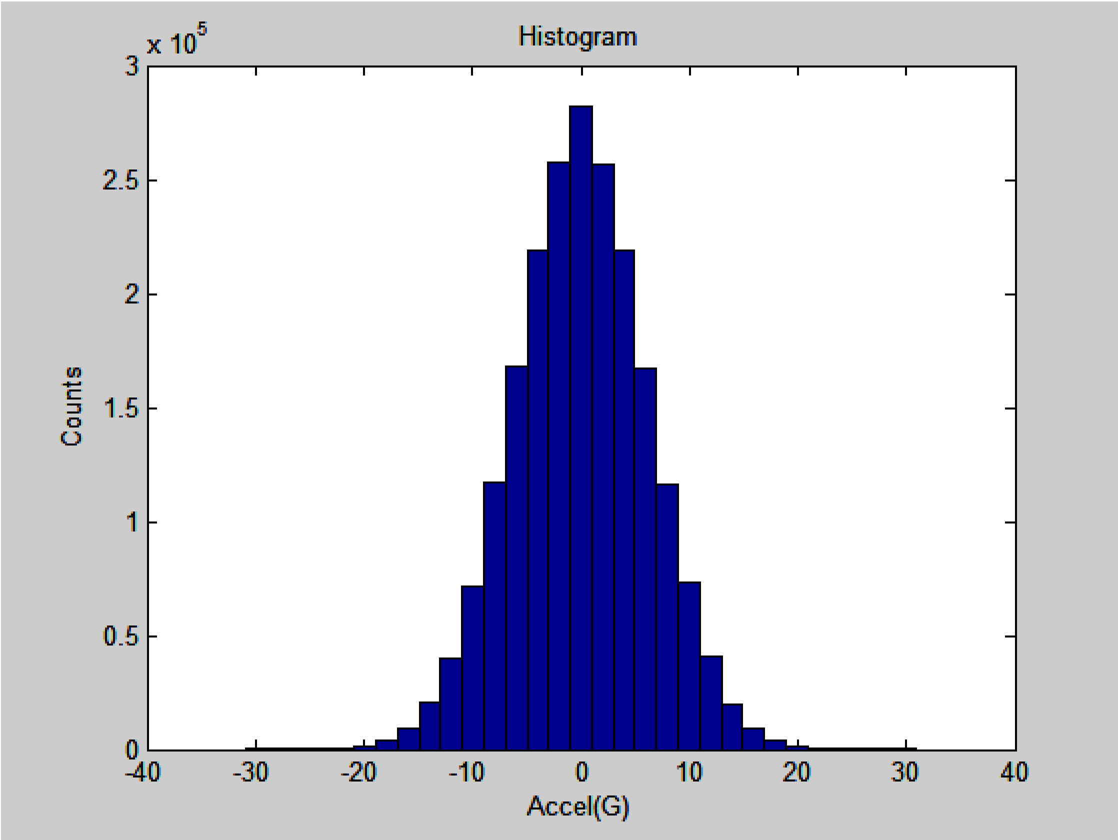

Figure A-4.

The synthesized time history has a normal distribution.

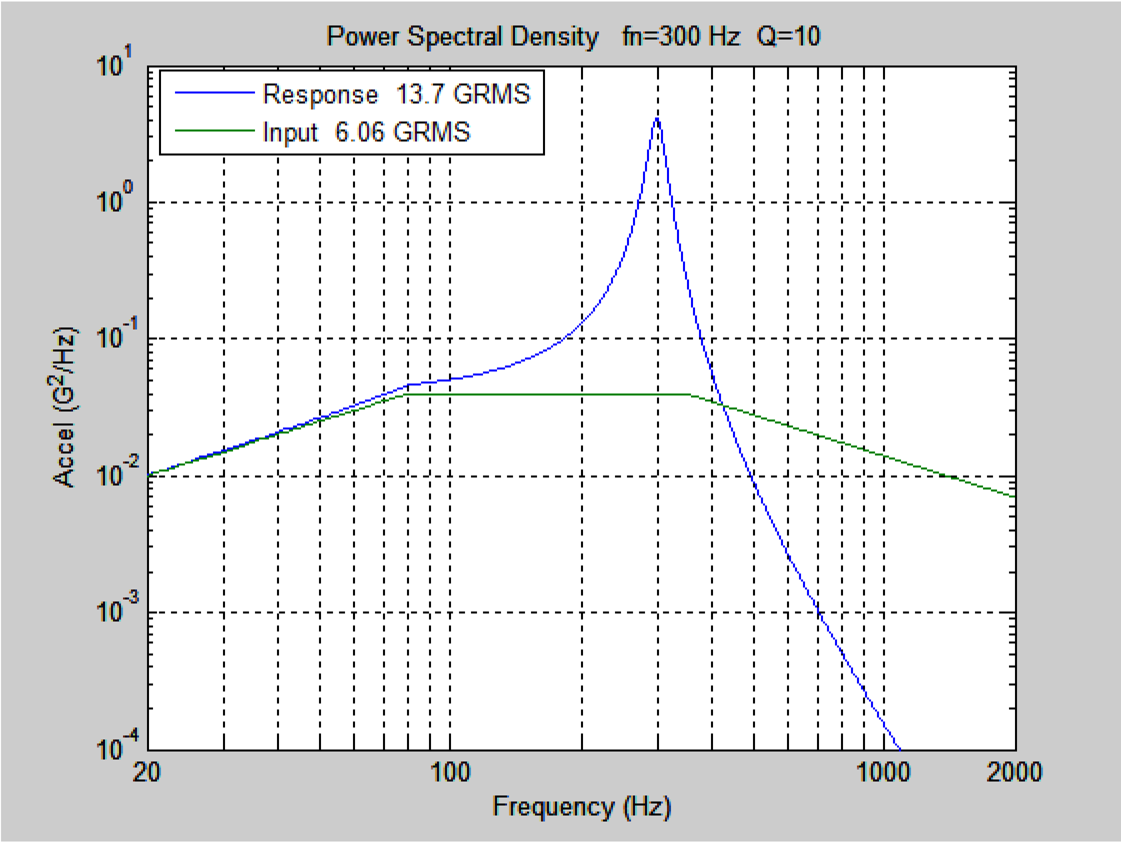

Figure A-5.

The SDOF response to the base input PSD is shown in Figure A-5.

<u>Equivalent Sine Tone</u>

The base input sine level for equivalent fatigue damage is

$$S = \frac{2.0(13.7\ \text{G})}{10} = 2.7\ \text{G} \qquad \text{(A-1)}$$

The response amplitude is

$$Q\ S = (10)(2.6\ \text{G}) = 27\ \text{G}$$

But verification is needed.

Fatigue Damage

Calculate a relative damage index D using

$$D = \sum_{i=1}^{m} A_i^{b} n_i \qquad \text{(A-2)}$$

where

A_i is the response amplitude from the rainflow analysis

n_i is the corresponding number of cycles

b is the fatigue exponent

The response amplitude could be absolute acceleration, relative displacement or even some other metric. The response acceleration will be used for this example.

Note that the damage index is only intended to compare the effects of the two base input types.

Equation (A-2) is straightforward for pure sine vibration. The response amplitude A for sine vibration is the base input multiplied by the amplification factor Q, which is 27 for the example problem.

For the example,

D=(60 sec)(300 Hz) (27 G)^6.4 = 2.6e+13 for sine input (A-3)

Next, calculate the time domain response of the SDOF to the synthesized time history in Figure A-2, using the method in Reference 3 and 4.

Then perform a rainflow analysis per Reference 5. Finally, calculate the relative fatigue damage from Equation (A-2).

The result is: D=2.2e+13 for random input.

Thus, the sine base input of 2.7 G at 300 Hz envelops the base input PSD in terms of equivalent fatigue damage. Again, this is for the case of (fn=300 Hz, Q=10, b=6.4).

This method can be modified for other Q and b cases as needed, as show in Appendix B.

APPENDIX B

Fatigue Exponent Variation

Continue with the example in Appendix A. Vary both the fatigue exponent and the α coefficient.

Table B-1. Fatigue Exponent Trade Study, fn=300 Hz, Q=10						
Base Input	Fatigue Damage					
Type	b=4.0	b=5.0	b=6.4	b=7.0	b=8.0	b=9.0
Random, Resp=62 GRMS	4.8e+09	1.5e+11	2.2e+13	1.9e+14	7.4e+15	3.0e+17
Sine, α=2.0, S=2.7 G	**9.6e+09**	**2.6e+11**	**2.6e+13**	**1.9e+14**	5.1e+15	1.4e+17
Sine, α=2.1, S=2.9 G	**1.3e+10**	**3.7e+11**	**4.1e+13**	**3.1e+14**	**9.0e+15**	2.6e+17
Sine, α=2.2, S=3.0 G	**1.5e+10**	**4.4e+11**	**5.1e+13**	**3.9e+14**	**1.2e+16**	**3.5e+17**

The bold font indicates cases where Sine is greater than or equal to Random in terms of fatigue damage.

Note that Reference 6 gives a value of b=6.4 for electronic equipment for both sine and random vibration. But Reference 7 gives a value of b=4.0 for “electrical black boxes.”

Un-notched aluminum samples tend to have a value of b $\cong$ 9 or 10, as shown in Reference 8 for example.

Table B-1 shows that α=2.2 could be used for conservatism for calculating an equivalent sine tone for a random PSD if the fatigue exponent is unknown but is less than or equal to 9.

Conversely, a smaller α should be used for calculating an equivalent random PSD for a sine tone for conservatism, as shown in Reference 9.

Amplification Factor Variation

The effect of Q is shown in the following tables.

Table B-2a. Fatigue Exponent Trade Study, fn=300 Hz, b=6.4, Acceleration				
Base Input Type	Acceleration			
	Q=5	Q=10	Q=20	Q=50
Random, Resp (GRMS)	9.7	13.7	19.4	30.7
Sine, α=2.0, Input (G)	17.2	12.4	8.2	2.5
Sine, α=2.1, Input (G)	18.1	13.0	8.6	2.6
Sine, α=2.2, Input (G)	18.9	13.6	9.0	2.8

Table B-2b. Fatigue Exponent Trade Study, fn=300 Hz, b=6.4				
Base Input Type	Fatigue Damage			
	Q=5	Q=10	Q=20	Q=50
Random	2.5e+12	2.2e+13	2.0e+14	3.4e+15
Sine, α=2.0	**3.1e+12**	**2.6e+13**	**2.7e+14**	**5.0e+15**

The sine fatigue damage envelops the random damage for each Q case.