SINE AND RANDOM VIBRATION EQUIVALENT DAMAGE
Revision B

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This paper includes a simple historical method for calculating sine and random vibration fatigue damage using instantaneous amplitudes. A more accurate method is rainflow cycle counting to identify peak-cycles pairs which can then be input to the Palmgren-Miner summation. See ASTM E 1049-85.

Introduction

There is no true equivalence between sine and random vibration. There are several fundamental differences between these types. Sine vibration has a bathtub-shaped histogram. On the other hand, random vibration has a bell-shaped histogram.

A single frequency is excited by sine vibration in terms of the steady-state response. A system undergoing steady-state vibration will vibrate at the base input frequency regardless of its natural frequency.

A broad spectrum of frequency components is present in random vibration, however. The differences are particularly significant if the test item is a multi-degree-of-freedom system.

Nevertheless, an occasional need arises to compare sine and random test levels or environments. The purpose of this paper is to present a comparison method, which is based on Reference 1. The method assumes that the test item is a single-degree-of-freedom system.

Time Scaling Equation

One vibration test may be substituted for another test using a time scaling equation. The following time scaling equation is taken from Reference 1, section 8.25, page 238.

$$T_1 G_1^b = T_2 G_2^b \tag{1}$$

where

T	is the duration
G	is the acceleration level
b	is an empirical constant

Equation (1) assumes that fatigue is the sole damage mechanism.

Random Damage

Random vibration is considered in terms of its 3σ and 2σ instantaneous amplitude values. Note that 1σ is equivalent to the RMS value for random vibration, assuming a zero mean.

The fatigue damage for random vibration can be expressed using equation (1) as follows:

$$\text{random damage} = (0.0433)(G_{3\sigma})^{b} + (0.271)(G_{2\sigma})^{b} \tag{2}$$

Furthermore, note the G values are response values rather than input values. These response values can be calculated from Miles equation.

Equation (1) assumes that the random vibration has a normal distribution. Let x be the instantaneous amplitude. The amplitude probabilities are shown in the following table.

Limits	Probability
$1\sigma < \lvert x \rvert < 2\sigma$	27.1%
$2\sigma < \lvert x \rvert < 3\sigma$	4.33%

Equation (2) can be simplified as follows:

$$\text{random damage} = (0.0433)(3\,G_{1\sigma})^{b} + (0.271)(2\,G_{1\sigma})^{b} \tag{3}$$

$$\text{random damage} = (0.0433)(3)^{b}(G_{1\sigma})^{b} + (0.271)(2)^{b}(G_{1\sigma})^{b} \tag{4}$$

$$\text{random damage} = [(0.0433)(3)^{b} + (0.271)(2)^{b}](G_{1\sigma})^{b} \tag{5}$$

Sine Damage

The sine fatigue damage is expressed as follows:

$$\text{sine damage} = (G_{in}\ Q)^b = (G_{out})^b \tag{6}$$

where

G_{in} and G_{out} are each in terms of a peak G level

Q is the amplification factor (typically Q=10)

Equation (6) assumes that the sine amplitude is at its peak level extremes 100% of the time. This assumption is reasonably accurate. The sine wave would behave as a rectangular wave if the assumption were completely accurate, however.

Equivalent Damage

Assume that the sine and random tests have equal duration.

$$\text{random damage} = \text{sine damage} \tag{7}$$

$$[(0.0433)(3)^b + (0.271)(2)^b](G_{1\sigma})^b = (G_{out})^b \tag{8}$$

$$[(0.0433)(3)^b + (0.271)(2)^b]^{1/b}(G_{1\sigma}) = (G_{out}) \tag{9}$$

$$(G_{out}) = [(0.0433)(3)^b + (0.271)(2)^b]^{1/b}(G_{1\sigma}) \tag{10}$$

The exponent b is taken as 6.4 in Reference 1.

Thus, the peak sine response value is

$$(G_{out})_{peak} = 1.95\ (G_{1\sigma}) \quad , \quad \text{for } b = 6.4 \tag{11}$$

The RMS sine response value is

$$(G_{out})_{RMS} = 1.38 \; (G_{1\sigma}) \quad , \quad \text{for } b = 6.4 \tag{12}$$

The peak sine input value is

$$(G_{in})_{peak} = \frac{1.95 \; (G_{1\sigma})}{Q} \quad , \quad \text{for } b = 6.4 \tag{13}$$

Again, $G_{1\sigma}$ is the single-degree-of-freedom response to the random vibration base input. It is equal to the GRMS response, assuming a zero mean. This response value can be calculated using the methods in References 2 and 3.

An approach using Reference 3 is given in Appendix A.

The equivalence method is extended with additional terms in Appendix B.

References

1. Dave S. Steinberg, Vibration Analysis for Electronic Equipment, Second Edition, Wiley-Interscience, New York, 1988.

2. T. Irvine, An Introduction to the Vibration Response Spectrum, Vibrationdata Publications, 2001.

3. T. Irvine, Derivation of Miles Equation, Vibrationdata Publications, 2000.

APPENDIX A

Equivalence Formula using Miles Equation

The equivalence formula from the main text is

$$(G_{in})_{peak} = \frac{1.95 \; (G_{1\sigma})}{Q} \quad , \quad \text{for } b = 6.4 \tag{A-1}$$

Miles equation from Reference 3 is

$$G_{1\sigma} = \sqrt{\left(\frac{\pi}{2}\right) f_n \, Q \, A} \tag{A-2}$$

where

fn = natural frequency in (Hz)

A = power spectral density level in (G^2/Hz)

There is a rule-of-thumb for using Miles equation. The power spectral density should be flat within one octave on either side of the natural frequency. Otherwise, the method in Reference 2 should be used instead of Miles equation.

Substitute equation (A-2) into (A-1).

$$(G_{in})_{peak} = \frac{1.95\sqrt{\left(\frac{\pi}{2}\right) f_n \, Q \, A}}{Q} \tag{A-3}$$

$$(G_{in})_{peak} = 1.95\sqrt{\left(\frac{\pi}{2Q}\right) f_n \, A} \tag{A-4}$$

Now solve for A.

$$\left[\left(G_{in}\right)_{peak}\right]^2 = 3.80\left(\frac{\pi}{2Q}\right) f_n\, A \tag{A-5}$$

$$\left[\left(G_{in}\right)_{peak}\right]^2 = 1.90\left(\frac{\pi}{Q}\right) f_n\, A \tag{A-6}$$

$$\frac{\left[\left(G_{in}\right)_{peak}\right]^2}{1.90\left(\frac{\pi}{Q}\right) f_n} = A \tag{A-7}$$

$$A = \frac{\left[\left(G_{in}\right)_{peak}\right]^2}{1.90\left(\frac{\pi}{Q}\right) f_n} \tag{A-8}$$

$$A = \frac{Q\left[\left(G_{in}\right)_{peak}\right]^2}{1.90\, \pi f_n} \tag{A-9}$$

Note that these equations assume that the sinusoidal frequency is fn.

APPENDIX B

Effect of Adding Terms

Limits	Probability
$0 < \lvert x \rvert < 1\sigma$	68.3%
$1\sigma < \lvert x \rvert < 2\sigma$	27.1%
$2\sigma < \lvert x \rvert < 3\sigma$	4.33%
$3\sigma < \lvert x \rvert < 4\sigma$	0.26%
$4\sigma < \lvert x \rvert < 5\sigma$	0.0063%

The random damage D_r is

$$D_r = (G_{1\sigma})^b \left\{ \sum_{i=1}^{5} \left(P_i\right)\left(i\right)^b \right\} \quad \text{(B-1)}$$

where P_i is the probability for level i

The equivalent peak response sine level is

$$(G_{out})_{peak} = \left\{ \sum_{i=1}^{5} \left(P_i\right)\left(i\right)^b \right\}^{1/b} (G_{1\sigma}) \quad \text{(B-2)}$$

The peak sine input value is

$$(G_{in})_{peak} = \frac{2.0 \; (G_{1\sigma})}{Q} \quad , \quad \text{for} \;\; b = 6.4 \qquad \text{(B-3)}$$

This approach gives a conservative estimate for the random vibration damage due to the upward rounding which takes place in the summation equation. For example, all the amplitudes between zero and 1σ are counted as 1σ. The sine damage also has an upward rounding to its peak value.