Revisiting a T-Line With Any Termination

In the general case, where a transmission line is terminated in Z_L, the impedance along the line is given by:

$$Z(z) = Z_0 \frac{\left(e^{j\beta z} + \Gamma e^{-j\beta z}\right)}{\left(e^{j\beta z} - \Gamma e^{-j\beta z}\right)} = Z_0 \frac{\left(e^{j\beta z} + \frac{(Z_L - Z_0)}{(Z_L + Z_0)} e^{-j\beta z}\right)}{\left(e^{j\beta z} - \frac{(Z_L - Z_0)}{(Z_L + Z_0)} e^{-j\beta z}\right)}$$

$$= Z_0 \frac{(Z_L + Z_0)e^{j\beta z} + (Z_L - Z_0)e^{-j\beta z}}{(Z_L + Z_0)e^{j\beta z} - (Z_L - Z_0)e^{-j\beta z}} = Z_0 \frac{Z_L 2\cos\beta z + Z_0 2j\sin\beta z}{Z_L 2j\sin\beta z + Z_0 2\cos\beta z}$$

$$= Z_0 \frac{Z_L + jZ_0 \tan\beta z}{Z_0 + jZ_L \tan\beta z}$$

Note: $Z(z)=Z_0$ if $Z_L=Z_0$

The above equation shows how the input impedance to an unmatched transmission line changes with electrical length, βz. Since the electrical length changes with frequency, the input impedance to an unmatched line will be frequency dependent.

Impedance Calculations

Because the formula for impedance is a bit cumbersome and not intuitive, design calculations and measurements are often made graphically using a Smith Chart. The Smith Chart works with normalized impedance and admittance, where normalization is made with respect to the characteristic impedance of the transmission line.

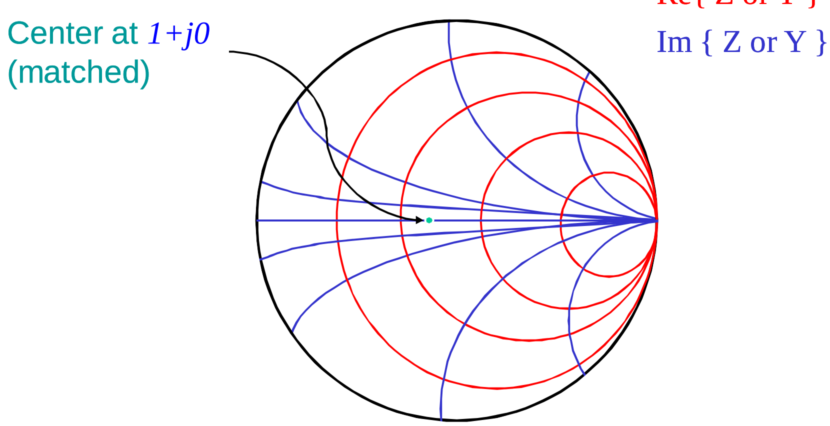
For example, the normalized impedance for a load $Z_L = 73 + j42\ \Omega$ on a $50\ \Omega$ transmission line is $Z_{LN} = 1.46 + j0.84$

By plotting the normalized load impedance on a Smith Chart, the input impedance as a function of line length can be found.

The Smith Chart also provides the value of the reflection coefficient, power delivered to load, as well as the voltage standing wave ratio (VSWR)

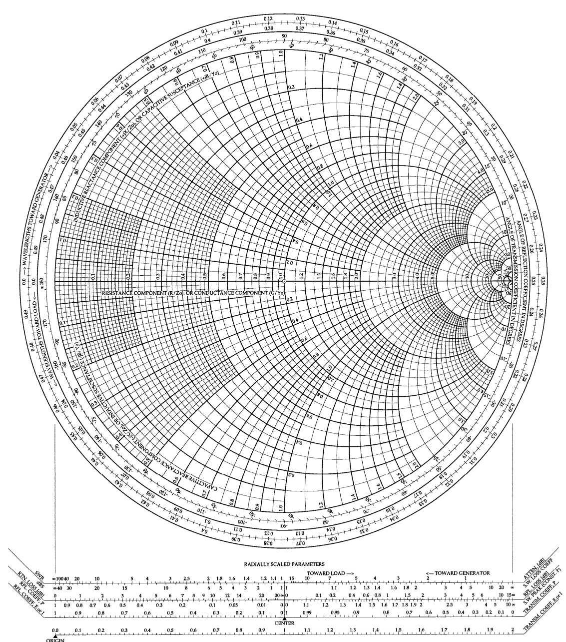
Distance measurements are given in terms of wavelengths.

The Smith Chart

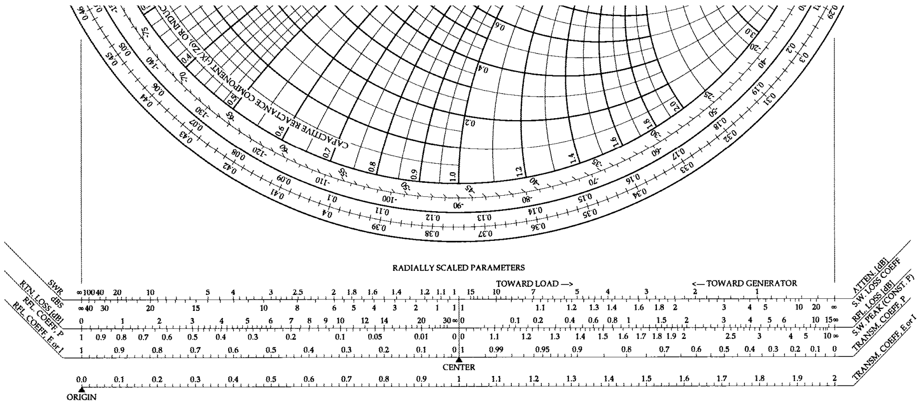


To find Z along the line for a particular Z_L, find Z_L/Z_0 on the chart and draw a circle, centered at $1+j0$ through that point. Points on that circle represent impedance on the line corresponding to distance which is read from the scale “wavelengths toward the generator”.

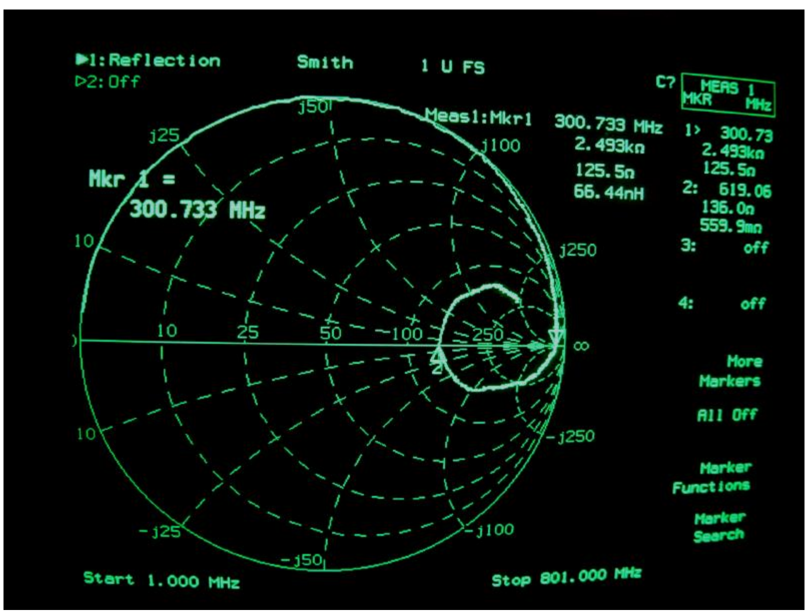
Blank Smith Chart



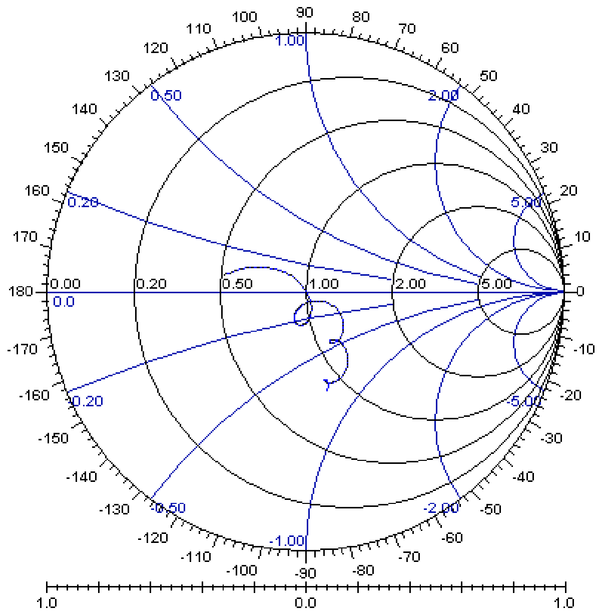
Bottom Scale of Smith Chart



Network Analyzer Smith Chart Display



Another Smith Chart Type Display



Smith Chart Example

A half-wave dipole antenna ($Z = 73 + j42\ \Omega$) is connected to a $50\ \Omega$ transmission line. How long must that line be before the real part of the input impedance is $50\ \Omega$?

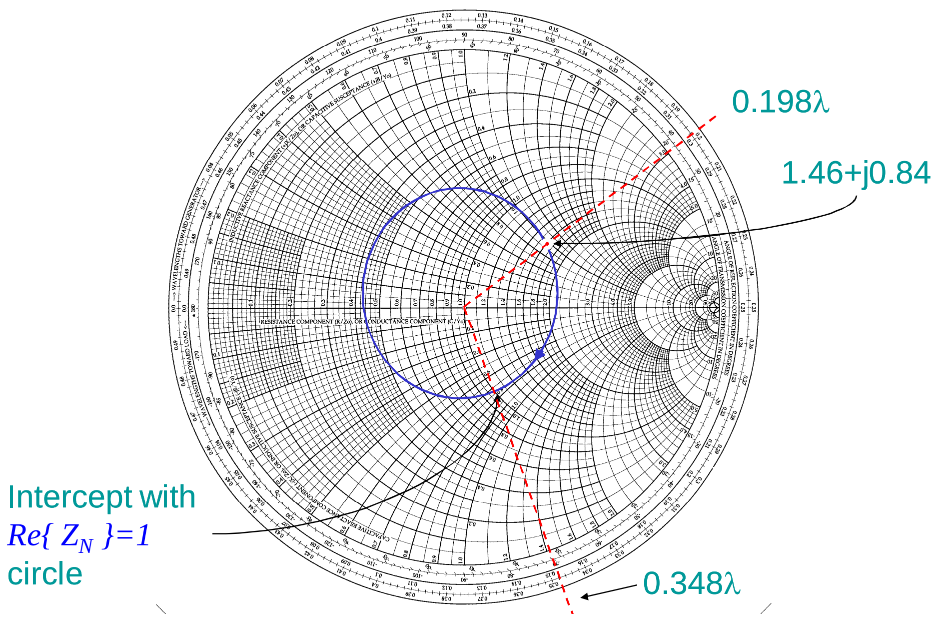
$$Re\{\ Z_{in}\ \} = 50 \quad \Rightarrow \quad \underbrace{\overset{Z_0 = 50\ \Omega}{=\!=\!=\!=\!=}}_{\ell} \quad Z_L = 73 + j42$$

Step 1: plot the normalized impedance ($1.46 + j0.84$) on the Chart

Step 2: Draw a circle through that point, with the center of the circle at $1 + j0$

Step 3: Move along the circle you drew, towards the generator, until you intercept the $Re\{\ Z_N\ \}=1$ circle is intercepted. The distance moved on the circle to get to that intercept, read from the "wavelengths toward generator" scale, represents the length of the transmission line ℓ.

Smith Chart Example



Smith Chart Example

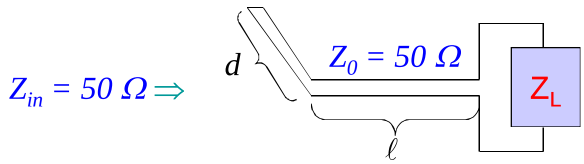
The distance moved on the scale is $0.348\lambda - 0.198\lambda = 0.15\ \lambda$. This represents the length of the transmission line, where λ is the wavelength **in the transmission line**.

The normalized input impedance for that transmission line is read from the Smith Chart to be $1 - j0.75$. This is read from the point where the circle you drew intersects the $Re\{\ Z_N\ \} = 1$ circle. The actual input impedance to the terminated line is $(1 - j0.75)50 = 50 - j37.5\ \Omega = Z_{IN}$

What we will be doing later is to add a reactive component that will cancel the reactive component of the input impedance, resulting in an input impedance equal to Z_0 (a perfect match). We will do this using “single-stub” matching.

Single-Stub Matching

As shown previously, the input impedance (admittance) of a shorted transmission line (a "stub") is purely reactive. By placing a stub in parallel with another transmission line, the reactive component can be cancelled, leaving a pure-real input impedance. This can be used to achieve a perfect match.



To do this, we need to choose ℓ so that the real part of the input admittance is equal to the characteristic admittance, and then choose d so that the reactive components cancel. This can all be done on the smith chart.

Single-Stub Matching Example

Find ℓ and d that will match our half-wave dipole antenna ($Z_L=73+j42\ \Omega$) to a $50\ \Omega$ transmission line.

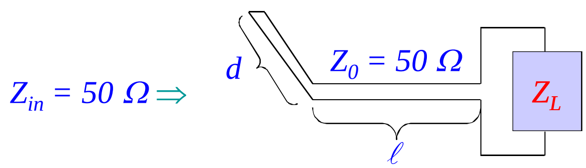
Note: Since we are going to add the stub in parallel with the transmission line, it will be easier to work with admittance rather than impedance.

Step 1: Plot the normalized load impedance ($1.46+j0.84$), and draw a circle through that point, centered at $1 + j0$. Get the normalized load admittance by drawing a line from Z_{LN} through $1+j0$ until you intersect the circle you drew on the other side. From the chart, you get a normalized load admittance of $0.52-j0.3$

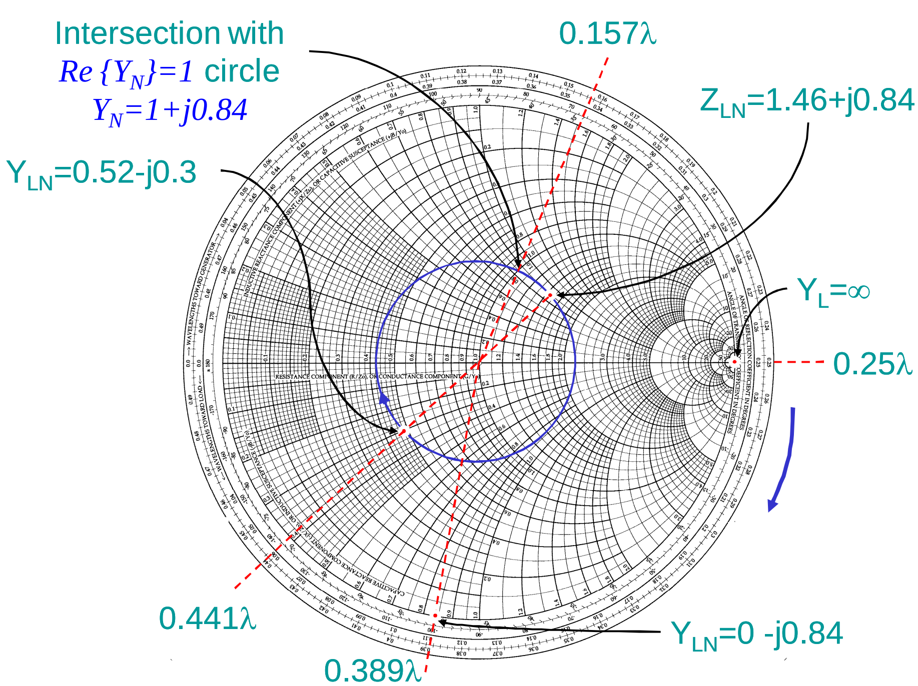
Step 2: Move towards the generator (clockwise) on the circle you drew until you intersect the $Re\{\ Y_N\ \}=1$ circle. The distance you moved to get to that intersection corresponds to the distance ℓ ($\ell = 0.5\lambda-(0.441\lambda-0.157\lambda) = 0.216\ \lambda$). From the Smith Chart, the normalized input admittance at this point is $1+j0.84$.

Single-Stub Matching Example

Step 3: In this step, we are looking for the length, d, of a shorted stub that will have an input admittance of $0\text{-}j0.84$. The load admittance of a short is infinity ($Y_L=\infty$), so that is where we will begin on the Smith Chart. We will move towards the generator from the point $Y_L=\infty$ until we intercept the $Im\{\ Y_N= -0.84\}$ line. At this point, the input admittance to the shorted stub is $0\text{-}j0.84$. The distance traveled to get to that point ($0.389\lambda\text{-}0.25\lambda=0.139\lambda$) is d.



Using these values of ℓ and d will result in a perfect match at the frequency for which it was designed. That match will degrade as the frequency varies.



Design Parameters for Example

ℓ = 0.5λ - (0.441λ - 0.157λ)= 0.216λ
d =0.389λ - 0.25λ = .139λ

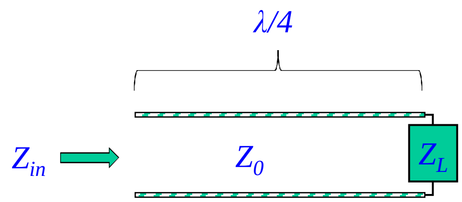
Notes:

- going around the Smith Chart once corresponds to moving a distance of 0.5 wavelengths on the transmission line
- the system will be matched at a single frequency

Quarter-Wave Matching Transformer

Used to convert any real load impedance (Z_L) to a desired real input impedance (Z_{in}).

Note: we can make our load real by placing a reactance-cancelling component in parallel with it.



Because $\lambda/4$ represents one-half rotation around the Smith Chart, the normalized input impedance is equal to the normalized admittance of Z_L:

$$\frac{Z_{in}}{Z_0} = Y_{L_N} = \frac{1}{\frac{Z_L}{Z_0}}$$

So: $Z_{in} = Z_0\left(\frac{1}{Z_L/Z_0}\right) = \frac{Z_0^2}{Z_L}$

Quarter-Wave Matching Transformer (2)

We can use this equation to find the characteristic impedance (Z_0) of the quarter-wave length on T-line:

$$Z_0 = \sqrt{Z_{in} Z_L}$$

For example: find the characteristic impedance of a quarter-wavelength section of T-line that would match our half-wave dipole antenna to a 50 Ω T-line (assume that the j42 Ω reactive component has been cancelled):

$$Z_0 = \sqrt{Z_{in} Z_L} = \sqrt{(50)(73)} = 60.4\Omega$$

Comment about stub and quarter-wave matching techniques:
Since all of the T-line dimensions are in wavelengths, the match will exist only over a narrow range of frequencies.
Different approaches need to be used to achieve broadband matching.