EXPLORATIONS

Chapter 12

Features Used

real( ), limit( ), NewProb, when( ), Numeric Solver

Setup

◆ 1, NewFold tline setMode("Angle", "Degree") setMode("Complex Format", "Polar")

Transmission Lines

This chapter describes how to calculate the characteristic impedance and phase velocity on transmission lines. Steady state transmission line behavior and simple matching concepts are included also. The functions **reflcoef( )**, **lineleng( )**, **zin( )**, **yin( )**, and **vswr( )** are created.

Topic 56: Characteristic Impedance

One of the most basic parameters of a transmission line is **zo**, its characteristic impedance. **zo** depends upon the geometry and the material of the transmission line. In this section, **zo** is calculated for four common transmission lines — coaxial, twin-lead, parallel plate, and microstrip. The cross-sections of these lines are shown in Figure 1.

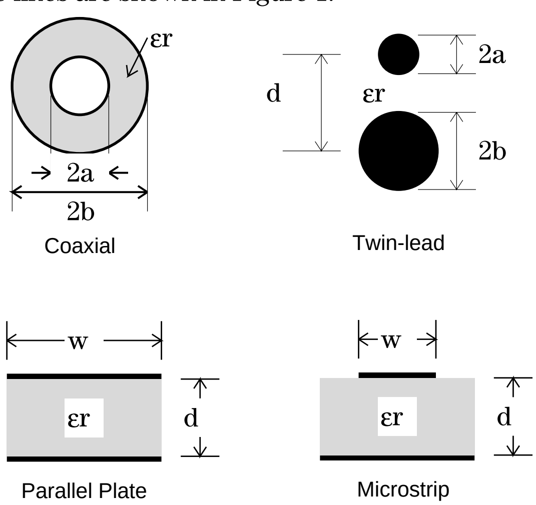


Figure 1. Transmission line cross-sections

Coaxial	$zo = \dfrac{60\ln\left(\dfrac{b}{a}\right)}{\sqrt{\varepsilon r}}$
Twin-lead	$zo = \dfrac{120\cosh^{-1}\left(\dfrac{d}{2\sqrt{ab}}\right)}{\sqrt{\varepsilon r}}$
Parallel Plate	$zo = \dfrac{120\pi d}{w\sqrt{\varepsilon r}}$
Microstrip	$F = \sqrt{\dfrac{\varepsilon r + 1}{2} + \dfrac{\varepsilon r - 1}{2\sqrt{1 + \dfrac{12d}{w}}}}$
w/d≤1	$zo = \dfrac{60\ln\left(\dfrac{w}{4d} + \dfrac{8d}{w}\right)}{F}$
w/d≥1	$zo = \dfrac{120\pi}{F\left(\dfrac{2}{3}\ln\left(1.444 + \dfrac{w}{d}\right) + 1.393 + \dfrac{w}{d}\right)}$

Table 1. Characteristic impedances

The equations shown in Table 1 are used to calculate zo of a transmission line from its geometry and material parameters. However, with the TI-89's numeric solver, any variable can be calculated when the others are known.

Coaxial and Twin-lead

1. Clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].

2. Press [APPS] **9:Numeric Solver** to display the Numeric Solver, and enter the equation for **zo** as highlighted in screen 1.

zocoax [=] **60** [×] [CATALOG] **ln(b** [÷] **a** [)] [÷] [(] [2nd] [√] [◆] [(] [alpha] **e r** [)] [)]

3. Press [ENTER] or ⊖ to display the variables in the equation.

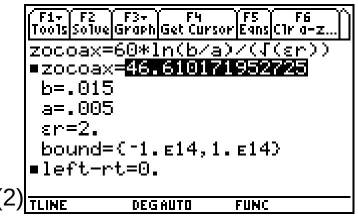
Note: *To enter ε , press* [◆] [(] [alpha] ***e***.

(1)

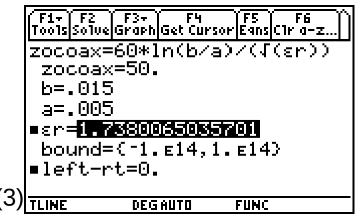
Note: *The number of digits displayed is independent of the mode settings, since it is a numeric solution.*

4. Enter values of **b**=.015, **a**=.005, and **εr**=2. Then move the cursor to **zocoax=** and press [F2] **Solve** to obtain the solution **zocoax**=46.61… Ω as shown in screen 2.

   The ▪ symbols on the left side of the screen show which variable was calculated and that the solution is exact since **left-rt=0**.

(2)


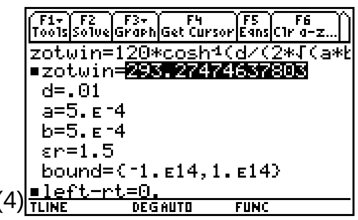
5. To calculate the value of εr which will give **zocoax**=50, enter 50 into **zocoax**, move the cursor to **εr**, and press [F2] to display the required value of 1.73… (screen 3).

(3)


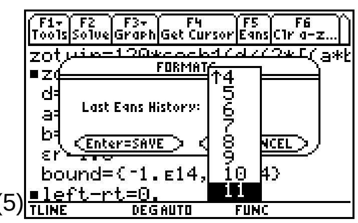
6. In a similar manner, enter the **zo** equation for the twin-lead line by moving to the top of the line.

   zotwin [=] **120** [×] [CATALOG] **cosh^{-1}(d** [÷] [(] **2** [×] [2nd] [√] **a** [×] **b** [)] [)] [)] [÷] [(] [2nd] [√] [◆] [(] [alpha] **e r** [)] [)]

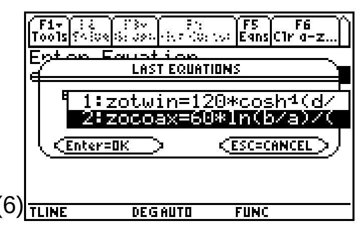
7. Set **d**=.01, **a**=**b**=.0005, and ε**r**=1.5 and solve for **zotwin**=293.27… Ω (screen 4).

(4)


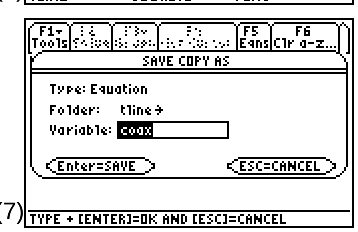
8. These two equations used can be accessed through [F5]; the number of “last” equations is eleven (11) by default and is set using [◆] [|] ⊳ (screen 5).

(5)


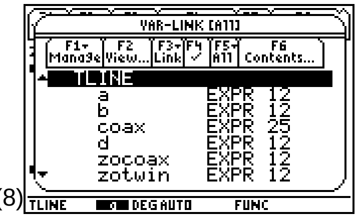
9. Press ⊳ to display both equations. Any equation in this list can be made the active equation by highlighting it and pressing [ENTER]; activate **zocoax** (screen 6).

(6)


10. Equations can be recalled to the Numeric Solver in this way as long as the number of last equations history is not exceeded. For more permanent storage, press [F1] **2:Save Copy As** ⊽. The equation is placed in the **TLINE** folder. Name it **coax** (if it is named **zocoax**, the equation name will conflict with the variable name) as shown in screen 7.

(7)


11. To verify that **coax** has been stored, press [2nd] [VAR-LINK] and scroll down to the **TLINE** folder to see **coax** and each of the variables **a**, **b**, **zocoax**, and **εr** listed (screen 8). Since the variables are listed alphabetically and Roman characters precede Greek, **εr** is at the bottom of the folder; scroll down to see it.



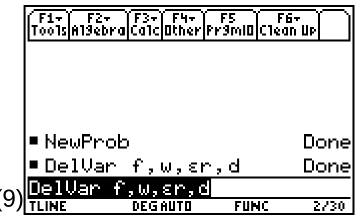
(8)

Parallel Plate and Microstrip

Although the remaining two equations can be entered into the Numeric Solver and saved in the same way, the rather complicated microstrip equation is entered and checked more easily in the Home screen.

1. Return to the Home screen, and clear the variables to be used in the microstrip equation as shown in screen 9.

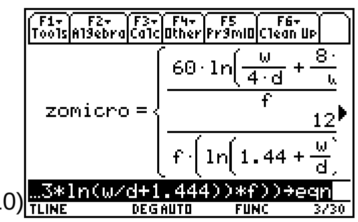
[CATALOG] **DelVar f** [,] **w** [,] [◆] [(] [alpha] **e r** [,] **d**



(9)

2. Now enter the **zo** equation (screen 10).

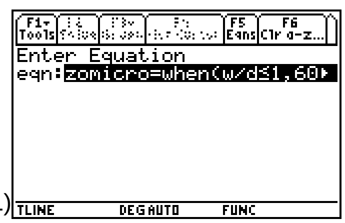
zomicro [=] [CATALOG] **when(w** [÷] **d** [2nd] [<] [=] **1** [,] **60** [×] [2nd] [LN] **8** [×] **d** [÷] **w** [+] **w** [÷] [(] **4 d** [)] [)] [÷] **f** [,] **120** [2nd] [π] [÷] [(] [(] **w** [÷] **d** [+] **1.393** [+] **2** [÷] **3** [×] [2nd] [LN] **w** [÷] **d** [+] **1.444** [)] [)] [×] **f** [)] [)] [STO▸] **eqn**



(10)

Note how the **when()** command is used to implement the two parts of the **zomicro** equation. The **→eqn** at the end stores the **zomicro** equation so that the Numeric Solver uses it.

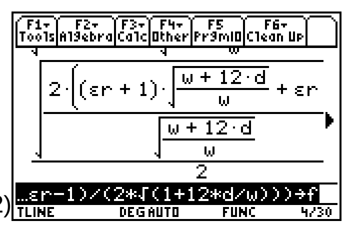
3. Press [APPS] **9:Numeric Solver** to see **zomicro** in the Numeric Solver.



(11)

4. So far **f** has not been defined. Return to the Home screen and define **f** as shown in screen 12.

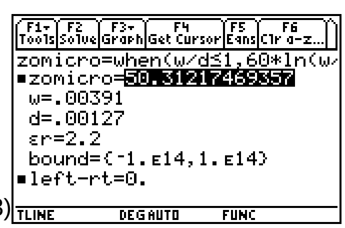
[2nd] [√] [(] [◆] [(] [alpha] **e r** [+] **1** [)] [÷] **2** [+] [(] [◆] [(] [alpha] **e r** [−] **1** [)] [÷] [(] **2** [×] [2nd] [√] **1** [+] **12** [×] **d** [÷] **w** [)] [)] [)] [STO▸] **f**



(12)

To prevent deleting **f** with **NewProb,** be sure to lock it. Alternatively, name it **ff**. See *Do This First*.

5. Display the Numeric Solver and press [ENTER]. Enter the values for a microstrip transmission line on a dielectric substrate with thickness **d**=0.00127, **εr**=2.2, and a conductor width of **w**=.00391 m. Solving for **zomicro** gives **zomicro**=50.31…Ω (screen 13).



(13)

Topic 57: Reflection Coefficient

When sinusoidal generators are used to excite a transmission line, all transient waves have decayed to zero and the line is in steady state. A common steady-state design goal is to match the source impedance to the transmission line input impedance. The input impedance of a transmission line with characteristic impedance zo and length d is given by

$$\text{zin} = \text{zo}\,\frac{1+\Gamma_L e^{-j4\pi\frac{d}{\lambda}}}{1-\Gamma_L e^{-j4\pi\frac{d}{\lambda}}}$$

for a frequency with a wavelength of λ. Since this calculation involves complex numbers, creating a function will make the calculations easier.

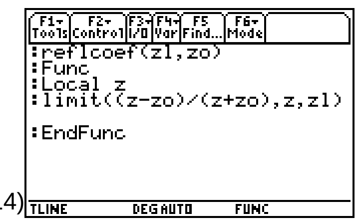
1. Clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].
2. Calculate the reflection coefficient of the load impedance as

$$\Gamma_L = \frac{\text{zl}-\text{zo}}{\text{zl}+\text{zo}}$$

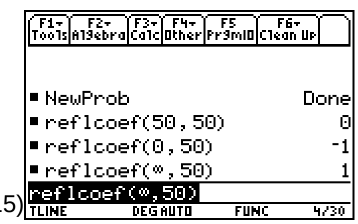
The reflection coefficient is a complex phasor with an amplitude of 1 or less.

Define the function **reflcoef** as shown in screen 14.

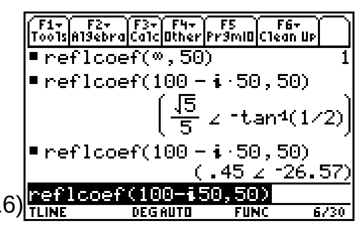
Note that **limit()** is used to handle the case of an open circuit with **zl** =∞.

(14)


3. Return to the Home screen and use **reflcoef** to calculate the reflection coefficients for real loads of **zl** =50, 0, and ∞ Ω on a line with **zo**=50 Ω (screen 15).

(15)


4. Calculate the coefficients for the complex load of 100−j50 Ω on a line with **zo**=50 Ω (screen 16).
5. Press [◆] [≈] to get the floating point value shown at the bottom of screen 16.

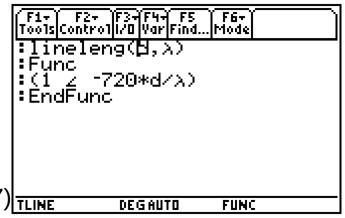
(16)


The results are 0, -1, +1, and 0.45∠-26.57°. When the load is “matched” to the line, there is no reflected signal; a short circuit reflects the incident signal with opposite polarity; and an open circuit reflects with the same polarity.

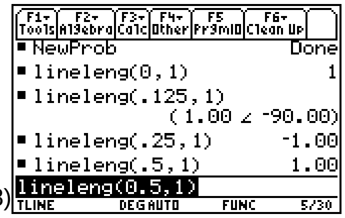
Topic 58: Phase Shift

When the load is attached to the end of a length of line, the input reflection coefficient is multiplied by exp(-j4πd/λ) which is 1∠(-720d/λ) as a phasor in degree form. This term depends only on the line length in terms of wavelength d/λ.

1. Define the function **lineleng** as shown in screen 17.
2. Return to the Home screen and clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].
3. Use **lineleng** to calculate the phase shift of a reflection coefficient for line lengths of **d**=0, 1/8, 1/4, and ½ wavelengths. Since the line length is given as a fraction of wavelength, λ=1 (screen 18).

(17)


***Note**: To enter λ, press [◆] [(] [alpha] **L**. To enter ∠, press [2nd] [∠]. To enter – , press [(-)].*

(18)


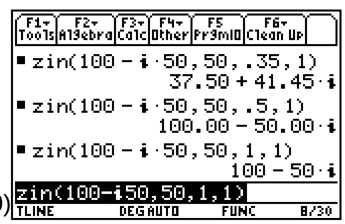
Topic 59: Input Impedance/Admittance

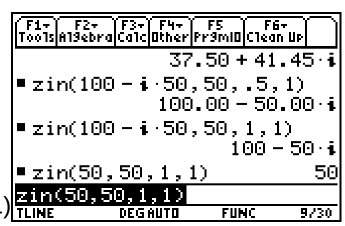
The equation for input impedance can be defined as a function.

The input impedance depends upon the line length. For lines with **d**=nλ/2, the input impedance equals the load impedance. For loads with **zl**=**zo**, the input impedance is **zo**.

1. Press [MODE] and set **Complex Format** mode to **RECTANGULAR**.
2. Define the function **zin** as shown in screen 19. **zin** uses **reflcoef** from Topic 57 and **lineleng** from Topic 58.
3. Return to the Home screen, and clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].
4. Use **zin** to calculate the input impedance of a line with **z**l =100-j50 Ω, **zo**=50 Ω, and λ=1. Use **d**=.35, **d**=.5, and **d**=1 (screen 20).
5. Calculate the input impedance for **zl** =50 Ω, **zo**=50 Ω, **d**=1, and λ=1.

(19)

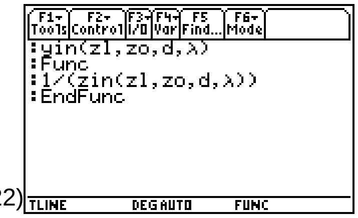

(20)


(21)


***Note**: The Complex Format mode has been switched to Rectangular so that real and imaginary results are displayed.*

6. Since connecting elements in parallel with transmission lines is common, admittance is helpful in transmission line calculations.

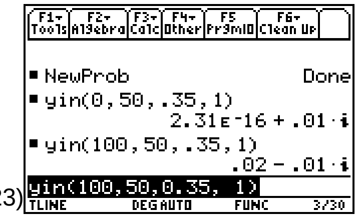
   Define the admittance calculation as a function **yin** which uses the function **zin** (screen 22).



(22)

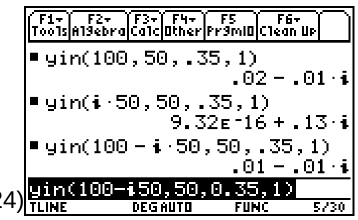
7. Return to the Home screen, and clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].

8. Use **yin** to calculate the input admittance of a **zo**=50 Ω line for **d**=0.35m and λ=1 for the real values of **zl**=0 and 100 (screen 23).



(23)

9. Calculate the input admittance for the complex values of **zl**=j50 and 100-j50. Use **zo**=50 Ω, **d**=0.35m, and λ=1m (screen 24).



(24)

Topic 60: VSWR

The reflection coefficient is difficult to measure, so an easily measured alternate parameter is used to describe mismatch, Voltage Standing Ratio (VSWR), given as

$$\text{VSWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

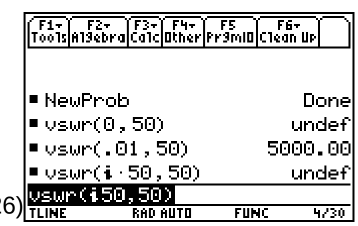
1. Define the function **vswr** (screen 25) to implement these calculations. **Vswr** uses the function **reflcoef** from Topic 57.

(25)

2. Return to the Home screen, and clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].

3. Calculate the VSWR of loads of 0, 0.01, j50 Ω, with **zo**=50 Ω (screen 26).

   The results are undefined (**undef**) for short circuits and open circuits.

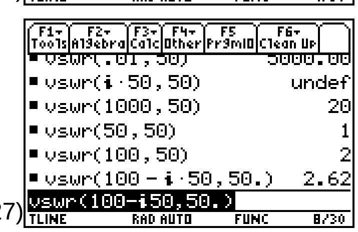


(26)

4. Calculate the VSWR of loads of 1000, 50, 100, 100-j50 Ω.

   Use a value of **50**[.] (note the decimal point) in the last entry to get the floating-point value.

   VSWR varies from 1 for a matched condition to ∞ for loads of 0, jX, or ∞ Ω.



(27)

Topic 61: Impedance Matching

A load can be matched to a transmission line, Γ_L=0, by the addition of parallel circuit elements. One method of matching a load is to insert an additional length of line between the original line and the load as shown in Figure 2. The length of this added line is chosen so that the real part of the input impedance (or admittance) equals the characteristic impedance (or admittance) of the transmission line. Then a parallel element is added to cancel the imaginary part of input admittance resulting in a matched condition. In mathematical terms the match is achieved when

real(yin(zl,zo,d, λ)) = real(gin + jbin) = 1/zo

where **zl**, **zo**, and λ are fixed and **d** varies.

The resulting value of susceptance, **jbin**, must be cancelled by a parallel element to achieve the desired match.

Calculate the parameters to match the load **zl**=100-j50 to a 50 Ω line.

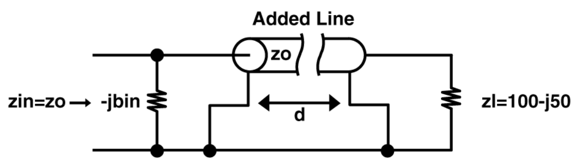
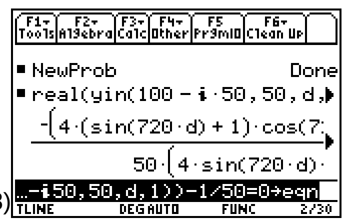


Figure 2. Matching circuit

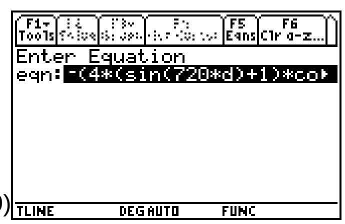
1. On the Home screen, enter the impedance matching equation as shown in screen 28.

[CATALOG] **real(yin** [(] **100** [−] [2nd] [i] **50** [,] **50** [,] **d** [,] **1** [)] [)] [−] **1** [÷] **50** [=] **0** [STO▶] **eqn**

The equation is stored in **eqn** so that the Numeric Solver can be used to find the value for **d**.

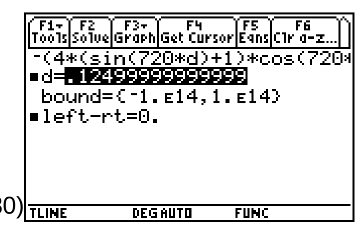
(28)


2. Press [APPS] **9: Numeric Solver**. The equation is displayed as shown in screen 29.

(29)


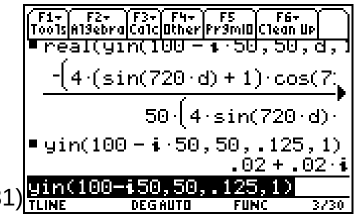
3. Press [ENTER] [F2] to solve for **d** (screen 30).

d=.125λ is one solution.

(30)


4. Therefore, **d**=.125λ is the required value for the function **yin**.

On the Home screen, calculate the parameters for z_L=100-j50Ω, **zo**=50m, **d**=.125λ, and λ=1m (screen 31).



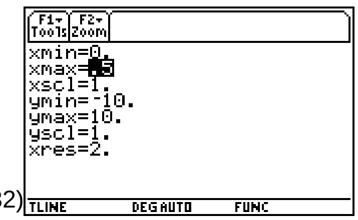
(31)

gin=.02=1/50, and the accompanying susceptance is **jbin**=j0.02. The equality of **g** and **b** is merely coincidental.

To match this load, a parallel susceptance of -j0.02 is needed. This is satisfied by an inductor since 1/2πfL=0.02 or L=1/0.04πf where the frequency must be known to determine L.

There are an infinite number of solutions, repeating every λ/2, that is, **d**=.125, .625, 1.125,… . But there are other solutions for **d** as well.

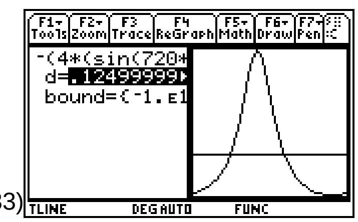
5. To see these, press [◆] [WINDOW] and set **xmin** to 0 and **xmax** to .5 as shown in screen 32.



(32)

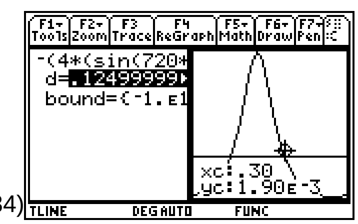
6. Press [APPS] **9:Numeric Solver** [ENTER] to redisplay the Numeric Solver.

7. Press [F3] **4:ZoomFit** to see a graph of the equation on the right of a split screen (screen 33).



(33)

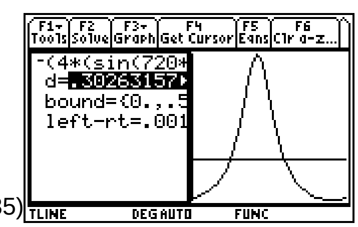
8. The Numeric Solver found the first zero; however, the second zero is also a valid solution. To find its value, press the ▶ and ▼ keys until the cursor is near the second zero (screen 34).



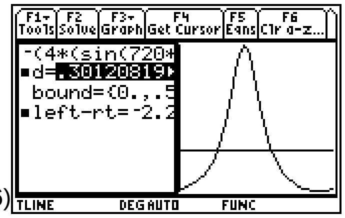
(34)

9. Press [2nd] [⊞] to switch screens, and then press [F4] **Get Cursor** (screen 35).

d now has the **x** value of the cursor.



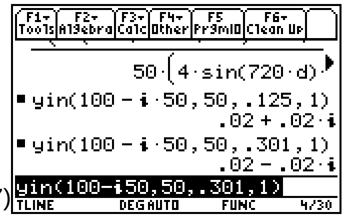
(35)



(36)

10. Press [F2] **Solve** to get the second solution (screen 36).

    The proper conductance occurs at about **d**=.301.



(37)

11. Press [HOME] [◆] **1** to display a full-sized Home screen.

12. Use the function **yin** to calculate the input admittance for **zl**=100-j50Ω, **zo**=50m, length **d**=.301λ, and λ=1m (screen 37).

    The input admittance for this length is **yin**=.020-j.020. This can be matched by using a capacitor where 2πfC=.02.

Tips and Generalizations

This chapter has again shown the power of the Numeric Solver for finding an unknown in a transcendental equation and plotting the equation versus the unknowns to see if there are multiple solutions. This chapter has also shown that the Solver remembers previous equations, which can be a great time saver.

Finding properties of transmission lines is nice; however, for the ambitious who really want to go far, Chapter 13 on antennas is the way to go.