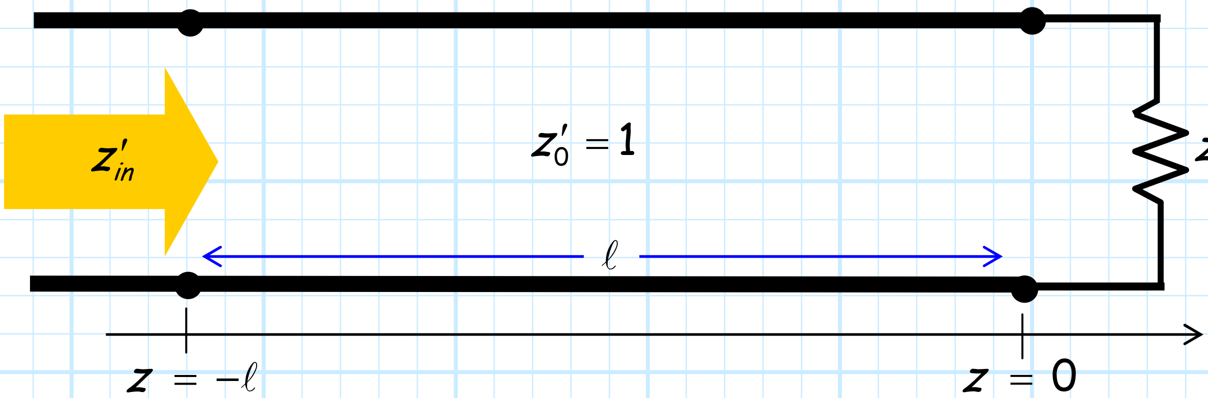
# Zin Calculations using the Smith Chart




The normalized input impedance $z'_{in}$ of a transmission line **length** $\ell$, when terminated in normalized **load** $z'_L$, **can** be determined as:

$$
\begin{aligned}
z'_{in} &= \frac{Z_{in}}{Z_0} \\
&= \frac{1}{Z_0} Z_0 \left( \frac{Z_L + j\, Z_0 \tan\beta\ell}{Z_0 + j\, Z_L \tan\beta\ell} \right) \\
&= \frac{Z_L/Z_0 + j \tan\beta\ell}{1 + j\, Z_L/Z_0 \tan\beta\ell} \\
&= \frac{z'_L + j \tan\beta\ell}{1 + j\, z'_L \tan\beta\ell}
\end{aligned}
$$

**Q:** *Evaluating this **unattractive** expression looks not the least bit pleasant. Isn't there a **less** disagreeable method to determine $z'_{in}$?*

**A:** **Yes** there is! Instead, we could determine this normalized input impedance by following these **three** steps:

1. Convert $z'_L$ to $\Gamma_L$, using the equation:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z'_L - 1}{z'_L + 1}$$

2. Convert $\Gamma_L$ to $\Gamma_{in}$, using the equation:

$$\Gamma_{in} = \Gamma_L e^{-j2\beta\ell}$$

3. Convert $\Gamma_{in}$ to $z'_{in}$, using the equation:

$$z'_{in} = \frac{Z_{in}}{Z_0} = \frac{1+\Gamma_{in}}{1-\Gamma_{in}}$$

**Q:** *But performing these **three** calculations would be even **more** difficult than the **single** step you described earlier. What short of dimwit would ever use (or recommend) this approach?*

**A:** The benefit in this last approach is that **each** of the three steps can be executed using a **Smith Chart—no** complex calculations are required!

**1. Convert $z'_L$ to $\Gamma_L$**

Find the point $z'_L$ from the impedance mappings on your Smith Chart. **Place you pencil at that point—you have now located the correct $\Gamma_L$ on your complex $\Gamma$ plane!**

For **example**, say $z'_L = 0.6 - j1.4$. We find on the Smith Chart the circle for $r$ =0.6 and the circle for $x$ =-1.4. The **intersection** of these two circles is the point on the complex $\Gamma$ plane corresponding to normalized impedance $z'_L = 0.6 - j1.4$.

This point is a **distance** of 0.685 units from the origin, and is located at **angle** of -65 degrees. Thus the value of $\Gamma_L$ is:

$$\Gamma_L = 0.685\, e^{-j65^\circ}$$

**2. Convert $\Gamma_L$ to $\Gamma_{in}$**

Since we have correctly located the point $\Gamma_L$ on the complex $\Gamma$ plane, we merely need to **rotate** that point **clockwise** around a circle ($|\Gamma| = 0.685$) by an angle $2\beta\ell$.

When we **stop**, we are located at the point on the complex $\Gamma$ plane where $\Gamma = \Gamma_{in}$!

For **example**, if the length of the transmission line terminated in $z'_L = 0.6 - j1.4$ is $\ell = 0.307\lambda$, we should rotate around the Smith Chart a total of $2\beta\ell = 1.228\pi$ radians, or $221^\circ$. We are now at the point on the complex $\Gamma$ plane:

$$\Gamma = 0.685\, e^{+j74^\circ}$$

**This** is the value of $\Gamma_{in}$!

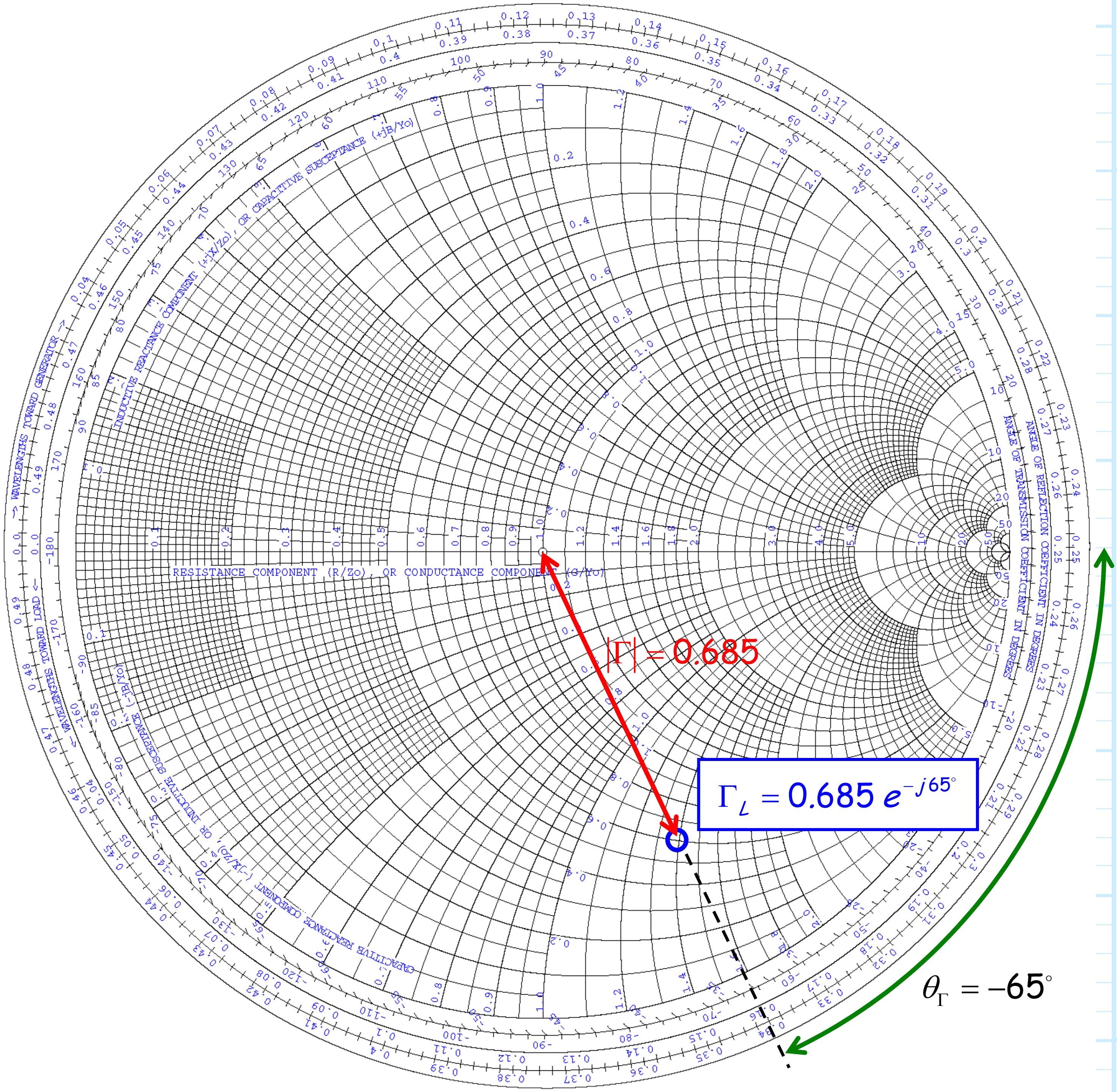
**3. Convert $\Gamma_{in}$ to $z'_{in}$**

When you get finished rotating, and your pencil is located at the point $\Gamma = \Gamma_{in}$, **simply lift your pencil and determine the values *r* and *x* to which the point corresponds!**

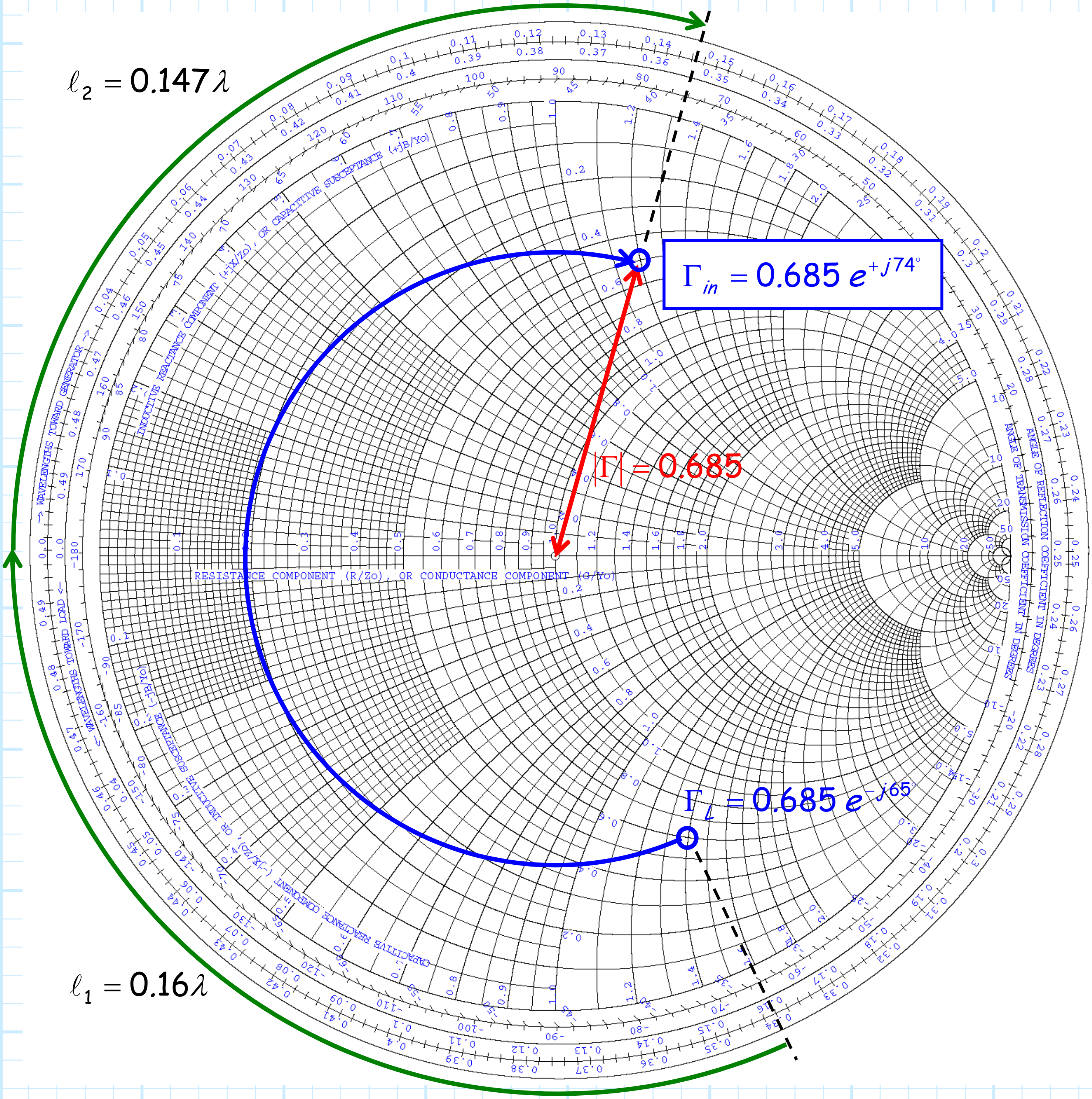
For **example**, we can determine directly from the Smith Chart that the point $\Gamma_{in} = 0.685\, e^{+j74^\circ}$ is located at the **intersection** of circles $r = 0.5$ and $x = 1.2$. In other words:

$$z'_{in} = 0.5 + j1.2$$

## Step 1



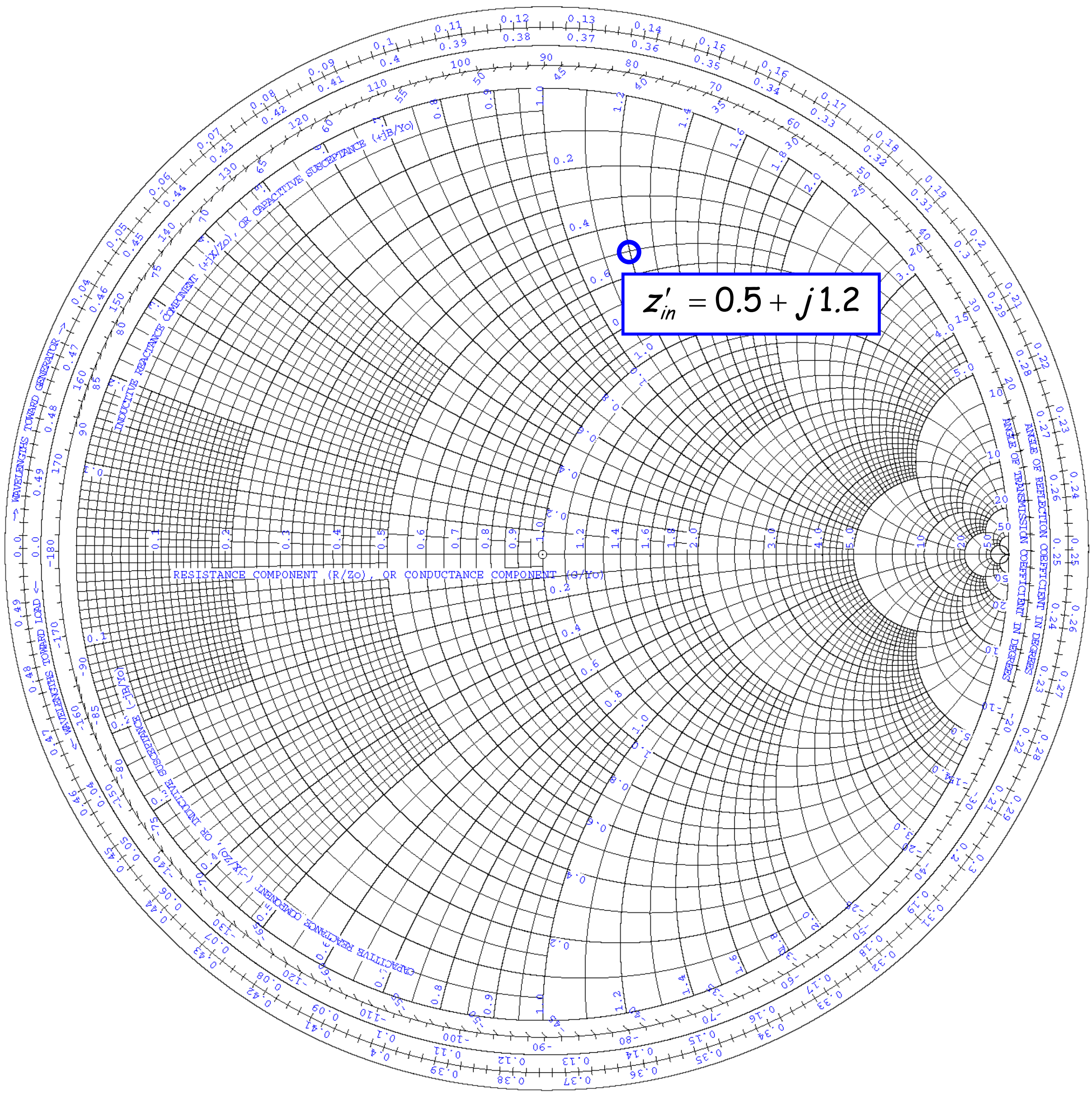
## Step 2




$$\ell = \ell_1 + \ell_2 = 0.160\lambda + 0.147\lambda = 0.307\lambda$$

$$2\beta\ell = 221^{\circ}$$

## Step 3


$z'_{in} = 0.5 + j1.2$
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD