Graphing Calculator Activities for Enriching Middle School Mathematics

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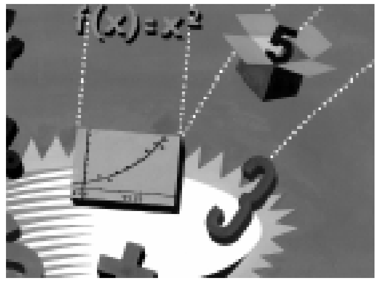

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Preface

Graphing calculators can now be purchased for less than the cost of the basic four-function calculator of twenty-five years ago. These calculators have capabilities and powers that outperform the early desktop computers that still grace the corners and storage cabinets of many classrooms. The low cost and portability of today's graphing calculators make it easier and more important than ever to incorporate them into the mathematics education of our middle school students. With the graphing, plotting, programming, table-building, list-making, and statistical features of such calculators, it is possible to introduce students to important and significant mathematics involving meaningful computation; modeling with tables, graphs, and functions; analyzing and interpreting data using statistics and statistical graphs; problem solving with simulations, and a wide variety of other topics.

As the title suggests, we wrote the activities in this book to enrich the technology components of current middle school mathematics curricula. We ask students to reason, to communicate (both in oral and written forms), to problem solve, and (of course) to use a graphing calculator in each activity. We find it difficult to classify our activities into neat mathematical categories since we wrote them with an emphasis on making mathematical connections whenever possible. One attempt at such a classification, however, gave us the following clustering:

Number Sense

Estimation Games

The Shrinking Dollar

Probability, Statistics, and Simulations

Heads Up!

Heads Up! (Continued)

Collecting Pens

Collecting Pens (Continued)

Measurement and Geometry

How Do You Measure Up?

Building a Garden Fence

Algebra and Functions

Box It Up

Box It Up (Continued)

What's My Line?

A Move in the Right Direction

We wrote our activities for use with any of the TI-80, TI-82, and TI-83 calculators with the exception of *A Move In the Right Direction,* which requires the CBL™ (Calculator-Based Laboratory™) compatibility of either the TI-82 or TI-83. Since the TI-82 is currently the most widely used of the three calculator models, we show TI-82 screen shots and keystroke instructions on the student pages. In almost all cases, these do not differ considerably, if at all, from those associated with either the TI-80 or the TI-83. The teacher notes that accompany each activity give appropriate instructions for the other calculators whenever such instructions are necessary. Each activity can be completed in one to three 50-minute class periods depending upon the time spent on whole class discussion and on investigation into the additional problems that are provided at the end of most activities. For those activities that have two parts (Activities 5 and 6, Activities 7 and 8, Activities 9 and 10) we highly advise that students work through the first activity before attempting the second.

The page design of this book should make it easy for you to use in your classes. The teacher notes for each activity are on the same page as the activity text, and the notes are printed in non-reproducible blue ink, which allows you to copy the activities for your students without reproducing the teacher notes.

We would like to thank the teachers and their students who helped us evaluate and field test these activities. We greatly appreciate the contributions of:

Adam Sterenberg and his 7th grade pre-algebra students at South Christian Grade School in Kalamazoo, Michigan.

Charlene Nooney and her 6th grade students at First Assembly Christian School in Portage, Michigan.

All of the middle school teachers in the Kalamazoo Public Schools who participated in our calculator workshop sessions: Maggie Adams, Rita Bowman, Corinne Dales, Gaynell Dixon, Kevin Dykema, Darlene Goodwin, Deborah Jackson, Jeannette James, Garry Hopkins, Diane Lang, Lorie Leak, Diane Loftus, Wes Seeley, Laura Smith, Tony Wine, and Chris Wolf.

The teachers who piloted and reviewed the activities: Adelaide Diaz, Metairie Park Country Day School, Metairie, Louisiana; Karen Cabrera, Ransom Everglades School, Miami, Florida; and Jo Ann Luhtala, Ordean Middle School, Duluth, Minnesota.

Our colleagues at Texas Instruments who assisted in the preparation of this book: Jeanie Anirudhan, Nelah McComsey, and Ron Blasz.

We hope that you and your students enjoy working through the challenges presented in this set of graphing calculator activities as much as we enjoyed the challenge of developing them for you. We encourage you and your students to write to us, sharing your further comments, ideas, and successes with learning mathematics using technology.

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— *Dwayne Channell (dwayne.channell@wmich.edu)*

Annotated Table of Contents

Number Sense

Activity 1: Estimation Games

The games in this activity are designed to help students develop number sense with the operations of multiplication and division.

Activity 2: The Shrinking Dollar

Students examine the possible long term effects of inflation. The compounding effect of inflation from year to year is one example of *exponential growth*. The activity makes use of the *replay* feature of the calculator, entering data into lists, examining scatterplots, and changing viewing windows.

Measurement and Geometry

Activity 3: How Do You Measure Up?

Students use scatterplots for investigating possible relationships between two quantitative variables. Students collect data, enter it into lists, and then view a scatterplot of the data. They create lists that are defined using other lists.

Activity 4: Building a Garden Fence

Students use basic concepts of perimeter and area to investigate a classic problem situation requiring maximization. Students define the length of a rectangle in terms of its width and then, using this information, define one list based on another. Students view a scatterplot of width versus area.

Probability, Statistics, and Simulations

Activity 5: Heads Up!

Students take a beginning look at randomness through an investigation, and later a simulation, of coin tossing. Data entered in lists allow students to examine both short and long term experimental probabilities and their relationship to the theoretical probability.

Activity 6: Heads Up! (Continued)

Students continue the work from *Heads Up!* using a calculator program simulation of the coin-tossing experiment. They are given a prewritten program to enter, analyze, and modify.

Activity 7: Collecting Pens

Students take a closer look at simulations. They collect and store their data using the graphing calculator, and then examine and analyze summary statistics and statistical plots (histogram and box plot) of the data.

Activity 8: Collecting Pens (Continued)

Students analyze and then later enter a simulation program using the link capabilities of their graphing calculators. Once the program is entered, they analyze the results of the simulation from the summary statistics and statistical plots.

Algebra and Functions

Activity 9: Box It Up

This classical investigation takes a numerical or tabular look at finding the maximum volume of an open box constructed by folding a rectangular sheet of material with cutout square corners. The concepts of independent and dependent variables are examined.

Activity 10: Box It Up (Continued)

Box It Up (Continued) furthers the maximum volume investigation by taking a graphical look at the problem. Students determine appropriate window settings in order to view the graph and experiment with zooming in to find the maximum value of the volume function.

Activity 11: What's My Line?

This activity focuses on strengthening student understanding of connections among graphical, tabular, and algebraic representations of simple linear functions. Students enter a simple program that allows them to determine equations for lines, in the form Y = AX +B, based on tabular and graphical information.

Activity 12: A Move in the Right Direction

Students physically provide motion data that is collected by a Calculator-Based Laboratory™ (CBL™) and then graphically displayed on their graphing calculator. Students determine if they "moved in the right direction" by comparing their graph with those provided in the activity.

Name ______________________________

Date ______________________________

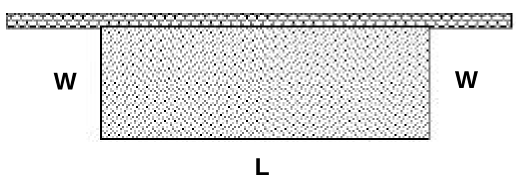
Activity 4

Building a Garden Fence

Not all problems have a single correct answer. Some have many possible solutions, and choosing the best answer may depend on what you are trying to accomplish. In this activity, you will decide how to build a garden fence to enclose the largest possible area.

The Problem

You and a friend are visiting her grandparents on their small farm. They have asked the two of you to design a small, rectangular-shaped vegetable garden along an existing wall in their backyard. They wish to surround the garden with a small fence to protect their plants from small animals.



To enclose the garden, you have 24 sections of 1 meter long rigid border fencing. In order to grow as many vegetables as possible, your task is to design the fence to enclose the maximum possible area. How many sections of fencing should you use along the width and the length of the garden?

There are many rectangular shapes that can be formed using the 24 fencing sections and, before the digging begins, you should do some calculations.

Go to the **Questions** section and answer #1, #2, and #3.

Students will use basic concepts of perimeter and area to investigate a classic problem situation requiring maximization. Depending upon grade level and length of class discussions, this activity may take up to two 50-minute class periods to complete.

Some students may find it helpful to build several sample garden designs with small square tiles. It is important that students realize that only three sides of the rectangle require fencing.

As a challenging variation on this problem, the sections of fencing could be set at a length of 0.75 meters.

Using the Calculator

A graphing calculator with list capabilities can be used to investigate this situation more thoroughly and quickly. Since Table 4.1, **Possible Dimensions for Garden Fence**, has three columns, you will use three lists in your analysis.

Calculating the Results

1. To clear the first three lists in the calculator, press [STAT] **4:ClrList** [ENTER], and then press [2nd] [L1] [,] [2nd] [L2] [,] [2nd] [L3] [ENTER].

You will use **L1** to store the possible widths and then calculate values for the corresponding lengths and areas. Once you calculate the values, you will store the lengths in **L2** and the areas in **L3**.

*Students may experiment with 24 – 2*L1 and 24 – 2L1 to see that the calculator will multiply when variables and coefficients are next to each other (juxtaposed).*

2. Press [STAT] **1:Edit** and press [ENTER].

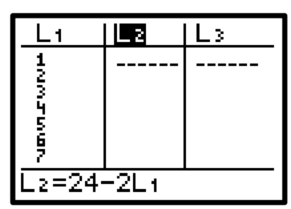
3. Enter the whole numbers from 1 to 11 (the largest possible width) into **L1** on your calculator by typing each number and pressing [ENTER] until all widths have been entered. (See the example below.)

You should have noticed in your earlier computations that the number of fencing pieces remaining for the length of the garden can be found by subtracting twice the number used for a width from the 24 fencing pieces available. We want to store in **L2** the lengths that correspond to the widths in **L1**.

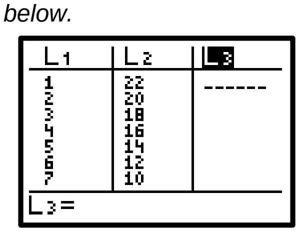
4. Press [▶] to move to the second list. Press [▲] to move to the top so that **L2** is highlighted.

Algebraically, the relationship between width and length can be expressed as L = 24 – 2W.

5. Enter [2] [4] [–] [2] [2nd] [L1] as the definition for **L2**. Your display should look like the example at the right.



Once students have pressed [ENTER], their second list should look like the display shown below.



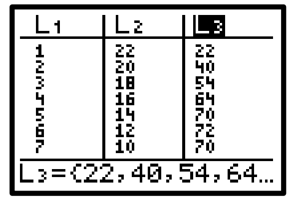
6. Press [ENTER] and the column of possible lengths should appear in **L2**.

✎ Answer #4 in the **Questions** section. This will help you find an expression to enter for **L3**.

7. Press [▶] to move to the third list and then press [▲] to move to the top so that **L3** is highlighted.

8. Press [2nd] [L1] [×] [2nd] [L2] [ENTER] to enter the expression for **L3** you found in question 4.

Student calculators should now look something like the following display:



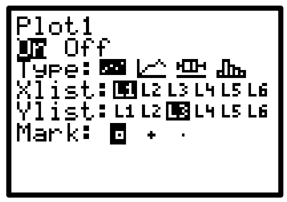
(Note: if you are using TI-80 calculators, only two lists will show on the screen.)

✐ Go to the **Questions** section and answer #5 and #6.

Displaying a Graph

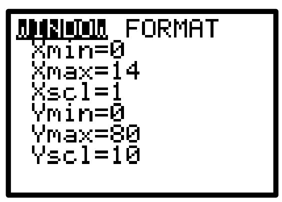
A scatterplot is often used to present a visual display of the relationship between two sets of paired data like the width and area measurements. Your calculator can produce a scatterplot display of the numbers in its lists.

1. Press [2nd] [STAT PLOT] [ENTER]. Edit the window so that yours looks like the one at the right. To make a selection, press [▸] to move the blinking cursor on top of the desired location, and then press [ENTER].



Before making the scatterplot settings, make certain that all other statistical plots and functions in the **Y=** *menu are cleared or turned off. To do so, press* [2nd] [STAT PLOT] 4 [ENTER] *and* [Y=] *followed by* [CLEAR] *on each line containing a function.*

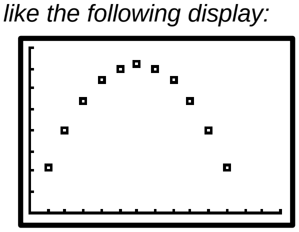
2. Press [WINDOW] and edit the numbers until your window looks like the one shown at the right.



The **Stat Plot** *windows on the TI-80 and TI-83 differ slightly from the one shown for the TI-82; however, similar selections can be made.*

3. Press [GRAPH] to view a scatterplot of the garden areas in relation to the garden widths.

Student scatterplots should look something like the following display:



4. You can view the coordinates of each plotted point by pressing [TRACE] followed by the left and right blue arrow keys.

✐ Go to the **Questions** section and answer #7, #8, and #9.

Questions

*You would have 24 – 2*3 or 18 sections for the length. The area would be 3*18 or 54 square meters.*

1. If you were to use three sections of fencing along each width of the garden, how many sections would remain to form the length? ________
What would be the area of this garden? ________
Copy these values into Table 4.1, and then enter three more possible garden sizes into the table. Try to guess the width and length of the garden with the largest possible area. Compare your results with others in your class.

There are actually 11 possible table entries with the width ranging from 1 m to 11 m in length. It is not important that students calculate all possible values since they will use a calculator to do this once they understand the need to investigate all possible combinations of width and length to determine the maximum area.

Table 4.1.

Possible Dimensions of Garden Fence

Width (m)	Length (m)	Area (m^2)
3		

You want to have the students determine the relationship L = 24 – 2W since it will be used later in developing lists.

2. If you know what the width is, how can you find the length? Write an equation that shows this relationship between width and length.

____

11 pieces along one width would use 22 of the 24 pieces leaving 2 pieces for the length. If 12 pieces were used, there would be none left for the length.

3. The smallest number of fencing pieces you can use along the garden width is one. What is the *largest* number of pieces that you can use along the width of the garden? Explain how you know this.

____

____

*Return to page 34, **Using the Calculator**.*

4. How can the values for **L3** (the areas) be determined from **L1** and **L2**? Remember that **L1** stores the possible widths and **L2** stores possible lengths.

____

Return to page 34, step 7.

5. Scroll through the values in **L3**. Are the values you computed earlier contained in this list? Describe any patterns you see in the data values contained in **L3.**

____

____

____

6. Examine the third list to find the dimensions of the rectangular garden area that has the largest possible area. Complete the following sentence to provide a solution to the original question:

A width of 6 m and a length of 12 m give the maximum area of 72 m^2.

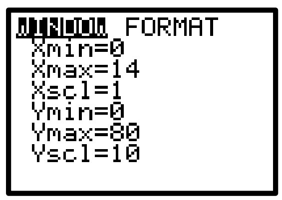
A rectangle with a width of _______ meters and a

length of _____________meters gives the largest

possible garden area of ___________ square meters.

*Return to page 35, **Displaying a Graph**.*

7. When creating the scatterplot of the areas, you entered the settings shown at right for the display window. Why do you think these values were used?



Chosen minimum and maximum values enclose the range of possible x and y values and provide a frame for the graph. Scale selections are reasonable divisions for each range of data.

____

____

____

8. You used TRACE to move through the data points in the scatterplot. Which point corresponds to the maximum area? What sets it apart from the other points on the plot?

Students should note that the coordinates of the point with the largest area appear at the highest point on the scatterplot. Comparing with their analysis of the lists, students should see the increase, followed by a decrease, of the areas as the width steadily increases. Algebra students may express this as a quadratic relationship between width and area based upon the relationship $A = W(24 – 2W)$.

____

____

____

You might find it useful to have students construct a scatterplot of widths and lengths. The discussion of the needed window dimensions and the resulting linear relationship (L = 24 – 2W) would reinforce important concepts.

9. How do any patterns that you observed in the lists show up in the scatterplot of the data?

____

____

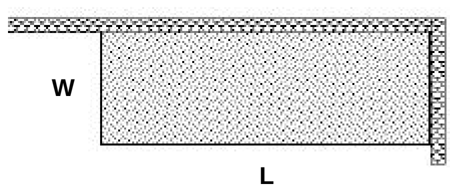
____

Problems for Additional Exploration

Use the list capabilities of your graphing calculator to investigate each of these situations. In each case, produce a scatterplot of the widths and areas. Assume you still have the 24 sections of fencing to use in forming your border.

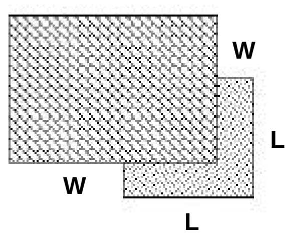
1. A friend suggests that you plant your grandparent's garden at a back corner of the yard so that the existing fence can border two of the four sides of your garden. What are the dimensions of garden with the largest possible area? Is this configuration an improvement over the original plan? Explain your reasoning.

*Widths may range from 1 to 23 meters with L = 24 – W and A = L*W. The largest possible area, 144 square meters, is formed with W = 12 meters and L = 12 meters. Since you have only two dimensions to border with the fencing, the area increases from the original plan. However, the garden is limited to placement in corners of the yard.*



2. Suppose the garden were placed at the corner of a barn so that it was positioned as shown below. What dimensions would give the largest garden area?

Widths may range from 1 to 6 meters with $L = 12 - W$ (or $L=(24-2W)/2$) and $A = 2LW - W^2$ (or some equivalent form). The largest possible area, 48 square meters, is formed when W = 4 meters and L = 8 meters.



EXPLORATIONS

Name ____________________

Date ____________________

Activity 7

Collecting Pens

In many real world problems involving probability and randomness, it is difficult, if not impossible, to determine answers through direct computation. In such cases, solutions can often be approximated using simulation strategies. In a simulation, an experiment is designed to resemble the important characteristics of the situation we are examining. Many trials of this experiment are conducted and analyzed to estimate a desired answer. When the simulations involve random outcomes they are often referred to as Monte Carlo simulations. In their simplest form, Monte Carlo simulations may involve flipping coins, rolling dice, or spinning spinners. These simulations can also be carried out using random numbers produced by a calculator or computer. The following problem is an example of a situation that can be solved using a Monte Carlo procedure.

The Problem

The Kellogg Company (the cereal makers) once placed a free felt-tipped marker in each box of *Kellogg's® Raisin Bran* they distributed. The back of the box proclaimed:

> FREE INSIDE — BRUSH MARKER ...
>
> You'll find one of the 6 washable colors in each specially marked package of *Kellogg's® Raisin Bran* cereal.
>
> Start collecting all 6 NOW!

To help you get started analyzing this problem, answer #1 through #3 in the **Questions** section.

This activity will most likely require two 50-minute class periods. If your students have not studied simulations before this time, you may want to have them first work through Activities 5 and 6 in this book.

Students have most likely participated in such collection activities themselves. It is not unusual to find some advertising scheme like this. You may want to search the grocery shelves for some cereal with a similar enticement.

If this promotion were still being conducted today, you could solve this problem by actually going out and buying boxes of cereal. This shopping trip would involve purchasing boxes of cereal and seeing which of the six markers was enclosed in each box. Your spending spree would end when you had a complete set of six pens. Since this is not practical, you need to design a simulation model that accurately reflects the essential characteristics of the shopping trip.

Discuss this problem further in your group and decide on a strategy for obtaining an approximate solution using the calculator's random numbers. If you have completed the *Heads Up!* activities in this book, you should recall that the command **iPart 6rand+1** generates random whole number values from the set {1, 2, 3, 4, 5, 6}.

✎ As you analyze the problem and design an experiment to investigate it, think about and answer #4 through #8 in the **Questions** section.

Using the Calculator

Opening a box of cereal and determining which color pen is inside is an experiment with six equally likely outcomes (Red, Tan, Yellow, Blue, Green, and Violet, for example). Entering the **iPart 6rand+1** command is also an experiment with six equally likely outcomes. The outcomes from these two experiments can be matched as shown in the following table.

Table 7.1. Pen Colors

Pen Color	Red	Tan	Yellow	Blue	Green	Violet
iPart 6rand+1 Result	1	2	3	4	5	6

As your group may have suggested, you can simulate purchasing a box of cereal by entering **iPart 6rand+1** on your calculator with the resulting number determining which color of pen is in the box. The calculator command can be repeated until each of the six possible outcomes has been obtained. The total number of times the command was entered gives an estimate of the answer for the original problem.

Use your calculator to simulate one customer purchasing cereal and collecting a set of pens.

Calculating the Results

1. Press [MATH] [▸] (highlights **NUM**) **2:iPart** 6. Press [MATH] [◂] (highlights **PRB**) **1:rand** [+] 1 [ENTER] to get the first random number.

*On the TI-83, press **3** to select **iPart**.*

2. Press [ENTER] repeatedly to get additional numbers.

✐ Keep a tally of the number of times each number comes up using tables 7.2, 7.3, and 7.4 in the **Questions** section. When all six numbers have appeared at least once, move to the next table and repeat the simulation for two additional customers.

The results from your three “customers” do not necessarily represent the expectations for the average customer. It is best to compile the results of many trials of a simulation and to use these combined data to arrive at an estimate of the desired answer.

The data might best be collected using a show of hands with one person selected to record the number of customers who were able to get all six pen colors by purchasing the given number of boxes of cereal.

✐ Use Table 7.5, **Frequency Table**, in the **Questions** section to summarize the results of all of the trials conducted in your class. Then answer #9 through #11 in the **Questions** section.

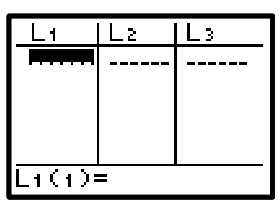
If your class schedule requires it, this might be a good point to stop the first part of the lesson.

Summary Statistics on the Calculator

You can use your calculator to get other views and summary statistics for the data. The data must be entered into the calculator list storage before statistics and graphs can be produced.

1. Press [STAT] **4:ClrList** [2nd] [L1] [,] [2nd] [L2] [ENTER] to clear the needed lists. The display should look like the one at the right.

2. Press [STAT] **1:Edit** to gain access to the calculator’s list storage. The display should look something like the one at the right.



3. Type the numbers from the first column of the table (representing the number of boxes purchased) into the first list (**L1**) on your calculator. Type a number and press [ENTER], repeating until all numbers have been entered.

4. Press [▶] to move to the second list. Type the numbers from the second column (representing number of customers) into the second list (**L2**) on your calculator. Make certain to enter the numbers in this second column on the corresponding row as found in your table.

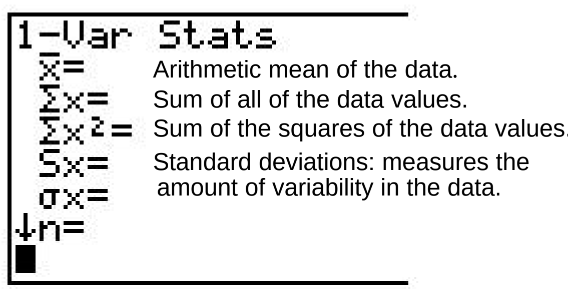
Once the paired data has been entered, you can obtain descriptive statistics on the data in Table 7.5, **Frequency Table**.

The command **1-Var Stats L1**,**L2** *informs the calculator that the data has been stored as a frequency table with the data in* **L1** *and the frequencies in* **L2**.

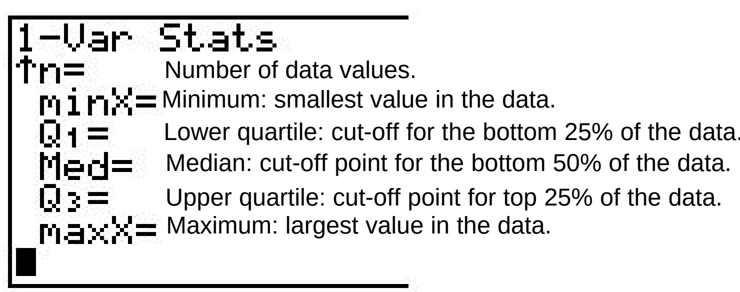
5. Press [STAT] [▶] **1:1-Var Stats** [2nd] [L1] [,] [2nd] [L2] [ENTER] to produce a list of statistics summarizing information on the number of boxes purchased by all customers represented in your class simulations.

The calculator produces a variety of statistics, many of which are not needed for our current analysis. Note the arrow pointing down ↓ in front of the last line. This is an indication that more information is available from the last set of calculations. If you press [▼] several times, you can scroll to a second screen of statistics. The following graphics give a brief explanation of the statistics presented on the two screens.

Many of these statistics are beyond the consideration of middle school students and are used in more advanced settings. In particular, the two measures of standard deviation are very difficult to explain without much more exposure to the concepts of variability and deviation scores, topics often introduced in high school level statistics classes.



The statistics shown in the second screen will be familiar to those who have studied box-and-whisker plots. If students have completed quartile analyses before, it would be appropriate to examine the frequency table and determine the "correctness" of the calculator values from the tabled data.



✎ Based upon the statistics generated by your calculator, answer #12 in the **Questions** section.

Displaying a Statistical Graph

Your calculator has statistical graphing, or plotting, features that allow the construction and display of a histogram and a box-and-whisker plot of the six-pens data. This can be done by defining the types of graphs desired and by setting appropriate window limits for viewing the graphs. You must also make certain that any function-graphing capabilities are cleared or turned off. The usual order is:

- **a.** Clear or disable any functions stored in the [Y=] menu.
- **b.** Define an appropriate viewing window.
- **c.** Define and activate the appropriate [STAT PLOT] features.

1. To clear the functions stored in the [Y=] menu, press [Y=], move the cursor over the first character of any function you find, and press the [CLEAR] key. Repeat until all functions are cleared.

2. Return to the home screen by pressing [2nd] [QUIT].

To set an appropriate viewing window for the plots associated with the data, you need to define an interval that contains all of the values in the list. Look at the set of statistics you calculated earlier. Find and record the minimum and the maximum values for the data.

These minimum and maximum values are needed in determining the limits along the x-axis.
Make certain students can explain why this is so.

Minimum value: ________ Maximum value: ________

For a histogram you also need to know the maximum *frequency* for any of the values in order to determine the “height” of the graph. This information can be found through a visual search of the second list **L2** containing the frequency counts associated with the data values.

3. Press [STAT] [ENTER] to get to the list storage. Use [▼] and [▶] to scroll through and locate the maximum value in **L2**. Record your finding below.

Similarly, this largest frequency is needed to determine the limits along the y-axis.

Students should be asked to explain its importance.

Largest frequency : ________

Knowing these three values, you are ready to define the limits of the viewing window for the statistical plots.

4. Press [WINDOW] and use [▼] to highlight the **Xmin** setting. Set **Xmin** at **5** (one less than the minimum) and press [ENTER].

5. Set **Xmax** to two more than the maximum, **Xscl** to **1**, **Ymin** to **-5**, **Ymax** to one more than the largest frequency, and **Yscl** to **1**. (These settings produce a small border around the actual graph.) Press [2nd] [QUIT] to return to the home screen.

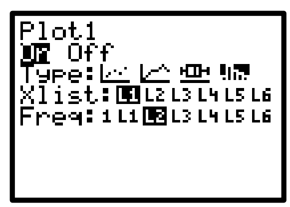
   It is important to note that the **Xscl** value serves a useful purpose in the plot of a histogram. It is used to define the width of the interval used for each bar in the graph. By defining **Xscl=1**, you are asking for a histogram that displays the frequency counts for each data value in the set. (If **Xscl** were set equal to 2, you would be asking for a frequency count on the 6's and 7's grouped together, the 8's and 9's grouped together, and so on.)

The histogram is just a graphical view of the frequency table. Students should arrive at similar conclusions after viewing both the table and the histogram.

The last step in obtaining a statistical plot is to define the type of plot desired and the data lists to be used in constructing the plot. This can be done by accessing the **Stat Plot** menu.

6. Press [2nd] [STAT PLOT] to gain access to the statistical plotting menus. Press **1:Plot1** to access the options for **Plot1**.

7. Using the blue arrow keys to highlight an option and the [ENTER] key to select that option, set the options to match the window at the right.



   You now have given the calculator all the information needed to plot the histogram.

8. To see the graph displayed, press the [GRAPH] key.

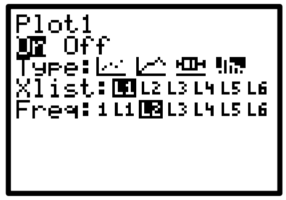
Note the distribution of the data. Do the values occur with the same frequency throughout the entire interval? If not, where do most of the values lie?

***Note**: You can use the [TRACE] key followed by the blue arrow keys to read coordinates from the histogram. To remove the coordinates from the display, press the [CLEAR] key.*

A box-and-whisker plot is also an appropriate display for the six-pens data. You have completed most of the work needed to obtain this plot. To display this plot, return to the statistical plotting menu and redefine the type of graph.

It is not unusual for a box-and-whisker plot of data for this problem to have an extremely long upper whisker (usually outliers are involved).

1. To define the box-and-whisker plot, press [2nd] [STAT PLOT] [ENTER]. Select the options shown on the window at the right.



Expect to see the lower three quartiles (the lower whisker and the box) show much less variability.

2. To display the plot, press the [GRAPH] key.

✎ Note the distribution of the data and answer #13 and #14 in the **Questions** section.

Questions

Answers can vary considerably. A popular answer is 36 (just sounds good?).

1. Assuming that equal numbers of each of the six pen colors were randomly distributed in the packaging of the cereal by the Kellogg company, about how many boxes of cereal do you think a customer would have to buy in order to collect all six of the pens?

____

The minimum is six, but only for a very lucky person. Students will be asked later to estimate the probability of actually getting all of the pens in six purchases.

2. What do you think is the minimum number of boxes a person might have to buy to get all of the pens? Explain.

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____

____

Theoretically, the maximum is 5/6 of the number of boxes distributed plus 1, but again this is very unlikely to occur. Some students may suggest 6^6 since this value is so large.

3. What do you think is the maximum number of boxes a person might have to buy before getting all of the pens? Explain.

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Return to page 62.

The pen color received in a box of opened cereal is assumed to be random.

4. What random event exists in this situation?

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There are 6 outcomes for this event (the six pen colors).

5. What are the possible outcomes for this random event?

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These outcomes are equally likely (again our assumption).

6. Are the outcomes equally likely?

____

____

7. How can the possible outcomes be modeled using the command **iPart 6rand+1**?

Each of the six outcomes can be matched to one of the 6 integers produced by the command **iPart 6rand+1***. The calculator could be used to repeatedly produce the random integers until each number has been displayed.*

8. How could the calculator be used to determine the number of boxes of cereal purchased by a single customer?

The number of times the command needed to be issued gives an estimate to the number of boxes a customer would need to purchase.

Return to page 62, ***Using the Calculator****.*

Table 7.2. Simulation for Customer 1

Pen Color	Tally
Red (1)	
Tan (2)	
Yellow (3)	
Blue (4)	
Green (5)	
Violet (6)	

Total boxes purchased by this customer ______

Table 7.3. Simulation for Customer 2

Pen Color	Tally
Red (1)	
Tan (2)	
Yellow (3)	
Blue (4)	
Green (5)	
Violet (6)	

Total boxes purchased by this customer ______

Table 7.4. Simulation for Customer 3

Pen Color	Tally
Red (1)	
Tan (2)	
Yellow (3)	
Blue (4)	
Green (5)	
Violet (6)	

Total boxes purchased by this customer ______

*Return to page 63, **Calculating the Results**.*

Table 7.5. Frequency Table:
Class Results for SIXPENS Collection Problem

Number of Boxes Purchased	Number of Customers
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

The data is being recorded in a frequency table with one column representing the actual data values and the other the frequency counts corresponding to each data value.

Answers will vary but it is expected that students will find that most customers can expect to buy between ten and twenty boxes of cereal.

The **Frequency Table** provides one way to look at the data your class has collected. Your responses to #9 through #11 should be based upon the distribution of the data in the table, not upon any further calculations.

9. About how many boxes of cereal do you believe a customer would have to buy in order to collect all six of the pens?

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10. What are the minimum and maximum numbers of boxes customers had to purchase? Were these values close to those you predicted earlier?

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11. The difference between the maximum and minimum data values in a set is called the *range*. What is the range for your class data?

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*Return to page 63, **Summary Statistics on the Calculator**.*

12. Based upon the experiments you have just conducted and the statistics produced by your calculator, how many boxes of cereal can the *average* customer expect to buy in order to collect all six pen colors? Explain your answer. For example, did you use the mean? The median? Some other summary statistic? Why? How close is your answer to your original guess?

There is no one correct way to answer this question. Accept any reasonable answer for which students are able to give support.

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Return to page 65, ***Displaying a Statistical Graph****.*

13. Look at the distribution of the data on the box-and-whisker plot. Where does most of the data lie? How does the plot convey this information?

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14. Using the [TRACE] key to see the coordinates of each point on the plot, answer the following questions:

a. A typical customer would expect to get all six pen

fewer boxes of cereal.

Use the median for this 50% figure.

A typical customer would expect to get all six pen

fewer boxes of cereal.

Use the upper quartile for this 75% figure.

b. Describe how the histogram and the box-and-whisker plot provide information about the six pens collection problem. How does the summary information provided by these graphical views differ from the tabular view used earlier?

The histogram and box plot are visual display of the data that provide at a quick glance a sense of the "average" data value (mean from the histogram and median from the box plot). The tabular view of the data provides more specific information on the complete data set; however, interpretation of the results may require a more lengthy analysis.

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Problems for Additional Exploration

Examine the frequency table. The ratio of the number of customers getting all pens in just 6 boxes to the total number of customers provides an estimate for this probability. (The estimate could end up being zero.)

To estimate the probability of getting all pens in 15 or fewer boxes, first determine the number of customers getting all pens with 6, 7, 8, 9, 10, 11, 12, 13, 14, or 15 purchases. Then divide this sum by the total number of customers.

Students could use **iPart 8rand+1** *and conduct the simulation again including the generation of the graphs using eight colors instead of six.*

1. Suppose that a customer buys several boxes of *Kellogg's® Raisin Bran* hoping to get all six of the pen colors. Use your class simulation results to answer each of the following.

a. If a customer buys exactly six boxes of cereal, what is the probability that he or she will get all six different pen colors?

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b. If the customer buys fifteen boxes of cereal, what is the probability that he or she will get all six pen colors?

***Note**: The customer may not have to open all fifteen boxes to get all six colors.*

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2. How would you simulate the pen collection problem if the Kellogg company decided to distribute eight different colors of pens in their cereal instead of six?

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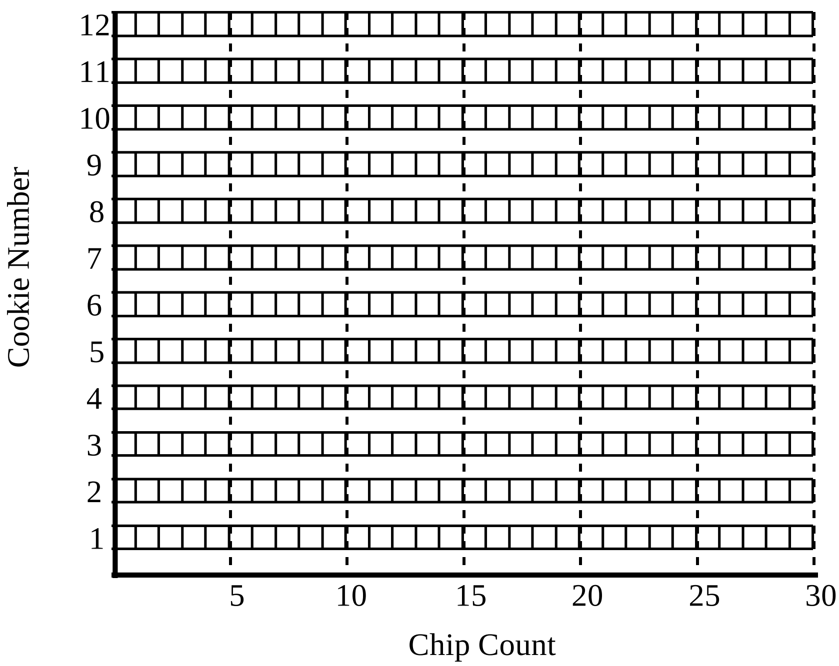
3. The Chocolate Chip Mixing problem presented below provides another example of a rather complex problem that can be solved using Monte Carlo simulation techniques..

The Cookie Bin is a small shop that bakes and sells cookies at an indoor shopping mall. The owner plans to introduce a new bite-sized chocolate chip cookie to her line of products. She would like to have each cookie contain at least five chocolate chips. To minimize the cost of ingredients, she wants to determine the least number of chips that need to be mixed into a batch of dough for a dozen cookies so as to meet the five-chip-per-cookie minimum. How many chips should she use for each batch? (Note that she will need *at least* 60 chips.)

a. Assume that each chip placed into the dough for one dozen cookies has an equally likely chance of ending up in any one of the twelve cookies made from that batch. Simulate the random placement of chocolate chips into the individual cookies in the batch. The calculator command **iPart 12rand+1** can be used to determine which of the 12 cookies is to get a given chocolate chip. Since each cookie is to contain at least five chips, repeatedly use the command and keep a frequency count of the number of chips in each cookie until the minimum number of chips needed to meet the five-chip-per-cookie requirement is reached. You can fill in the following bar graph to record your progress.

You may want to ask students to "place" the first 60 chips and then examine the distribution to see how "uneven" the chip placement is once the minimum number of chips have been mixed in.

Expect a wide variation in answers to this simulation. In sample trials, we have experienced chip counts ranging from the mid-seventies to over 160 with the median across all trials somewhere between 100 to 110.



b. How many chocolate chips were needed for the batch of cookies you "baked"?

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c. Collect the data from all batches of cookies "baked" by members of your class and enter them into a list on your calculator.

d. Construct a box-and-whisker plot of the class data.

If the baker were to mix the median number of chips into several batches of cookies, he should expect that in 50% of the batches, each cookie would contain 5 or more chips.

e. What is the median number of chocolate chips used to meet the five-chip-per-cookie minimum? How would you interpret this number (the median) for the Cookie Bin owner?

___

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Use the upper quartile.

f. What number of chips will result in each cookie having five or more chips 75% of the time?

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Answers will vary but should be supported.

g. How many chocolate chips do you think the Cookie Bin owner should put into the dough for each batch of one dozen cookies? Explain.

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