Mortgage Pools, Pass-Throughs, and CMOs

Concepts and Buzzwords

- Fixed-Rate Mortgages
- Prepayment Risks
- Valuation of Mortgage Pools (Pass-Throughs)
- CMOs
- Interest Rate Sensitivity

- market risk, idiosyncratic risk, path-dependence, burnout, OAS, negative convexity, negative duration, tranche, PAC, TAC, Z-Bond

Readings

- Veronesi, Chapter 12
- Tuckman, Chapter 21

Basic Fixed Rate Mortgage

- With a basic fixed rate mortgage, the borrower is scheduled to make **level monthly payments** consisting of

  interest on the amount of the loan outstanding, at the pre-determined fixed mortgage rate, and

  principal payments which reduce the outstanding loan balance.

- The size of the monthly payment is set so that the original loan is paid off after a prespecified amount of time, typically 30 years.

- In other words, the fixed monthly payment makes the present value of the 30-year stream, discounted at the **mortgage rate**, equal to the principal amount of the loan.

Monthly Payment

- By convention, the quoted mortgage rate is *annualized with monthly compounding.*

- Using the annuity formula from the yield lecture, we can get a closed form expression for the monthly payment:

$$\text{prin} = \sum_{n=1}^{360} \frac{\text{pmt}}{(1 + r_m / 12)^n} = \frac{\text{pmt}}{r_m / 12} (1 - (1 + r_m / 12)^{-360})$$

$$\Rightarrow \text{pmt} = \frac{\text{prin} \times r_m}{12(1 - (1 + r_m / 12)^{-360})}$$

- Example: If the original balance is $100,000 and the mortgage rate is 7.25%, then the monthly payment is $682.18.

Amortization Schedule for 30-Year, Monthly 7.25% Mortgage

Month	Beginning Principal Balance	Monthly Payment	Monthly Interest	Scheduled Principal Repayment	Ending Principal Balance
1	100,000.00	682.18	604.17	78.01	99,922
2	99,921.99	682.18	603.70	78.48	99,844
3	99,843.51	682.18	603.22	78.96	99,765
4	99,764.55	682.18	602.74	79.43	99,685
360	678.08	682.18	4.10	678.08	0

Note that on any month, the present value of the remaining stream of payments, discounted at the fixed mortgage rate equals the remaining principal balance.

Semi-Annual Payment Formula

- We'll assume semi-annual payments so we don't have to rebuild our binomial tree.
- For a T-year fixed rate, level pay mortgage with semi-annual mortgage (coupon) rate c, the formulas become

$$\text{prin} = \sum_{n=1}^{2T} \frac{\text{pmt}}{(1 + c/2)^n} = \frac{\text{pmt}}{c/2}(1 - (1 + c/2)^{-2T})$$

$$\Rightarrow \text{pmt} = \frac{\text{prin} \times c/2}{1 - (1 + c/2)^{-2T}}$$

- Example: For a 2-year, 5.5% mortgage with semi-annual payments and $100 principal, the semi-annual payment is $26.74.

Amortization Schedule for 2-Year, 5.5% Semi-Annual Mortgage

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.50	100.00	26.74	2.75	23.99	76.01
1.00	76.01	26.74	2.09	24.65	51.36
1.50	51.36	26.74	1.41	25.33	26.03
2.00	26.03	26.74	0.72	26.03	0.00

- We can think of this as
  - a single mortgage,
  - a pool of identical mortgages, or
  - a *pass-through* security that receives a fixed fraction of all cash flows that flow through the pool.

Benchmark 1: Mortgage Value Assuming No Prepayment

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.50	100.00	26.74	2.75	23.99	76.01
1.00	76.01	26.74	2.09	24.65	51.36
1.50	51.36	26.74	1.41	25.33	26.03
2.00	26.03	26.74	0.72	26.03	0.00

- With no prepayment, the mortgage would just be a stream of four fixed cash flows, each equal to 26.74.
- It could be valued as a package of zeroes:
  26.74*(0.973047+0.947649+0.922242+0.897166) = 100.02

Mortgagor's Prepayment Option

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.50	100.00	26.74	2.75	23.99	76.01
1.00	76.01	26.74	2.09	24.65	51.36
1.50	51.36	26.74	1.41	25.33	26.03
2.00	26.03	26.74	0.72	26.03	0.00

- The mortgagor has the option to pay off the mortgage at any time without penalty by paying the remaining principal balance.
- For example, with the mortgage above, the mortgagor can pre-pay an additional 76.01 at time 0.5 (on top of his scheduled payment of 26.74) and remove his obligation to pay the remaining three payments.
- Or the borrower could pay 51.36 at time 1 and get out of the remaining two payments, etc.

Mortgagor's Prepayment Option

- Think of paying off the mortgage as buying back the remaining stream of payments.
- Then the prepayment option is an American call option where
  - the underlying asset is the remaining stream of payments
  - the strike price is the remaining principal balance.
- Thus, the underlying asset is "wasting away" and the strike price declines over time according to the pre-determined amortization schedule. Note that the option is
  - at the money when the market yield on the remaining monthly payments is equal to the original mortgage rate,
  - in the money when the market rate is below the mortgage rate
  - out of the money when the market rate is above the mortgage rate.

Valuation

- Mortgage = (Nonprepayable) Stream of Monthly Payments - Prepayment Option.
- Valuing the mortgage boils down to valuing the prepayment option.
- If all borrowers prepaid according to a strategy that minimized the mortgage value (maximized the option value), mortgage cash flows would be a function of interest rates and could be valued by replication and no arbitrage, just as we valued callable bonds.
- Alternatively, if prepayments (option exercises) were uncorrelated with the market and independent across different loans in a given pool, a well-diversified pool would just experience the average prepayment, with little variance, by the law of large numbers, and MBSs would have nearly fixed cash flows which could be valued as a package of zeroes.

Market and Non-Market Risks

In fact, prepayments are random and subject to both market and non-market risks:

- **market**: a mortgage could prepay because rates fall (the prepayment option gets deep in the money)
- **non-market**: a mortgage could prepay even when rates are high because the mortgagor sells the property or the property is destroyed (the mortgagor may be forced to exercise the option when it is out of the money)
- **non-market**: a mortgage might not prepay even when rates fall because the mortgagor faces transaction costs
- **market**: mortgagor cannot refinance because property has lost value
- **market**: mortgagor defaults because property has lost value

Valuation...

- We can value non-market risks at their *true* expected value if they can be diversified away through pooling.

- Market risks can be hedged, and thus valued by no arbitrage (risk-neutral expected value).

- Conceptually, therefore, valuation is straightforward.

- Practically, however, we need to know the average prepayment along every interest rate path throughout the life of the mortgage to be able to value the mortgage exactly. Realistic valuation problems are very difficult.

- The examples in this lecture consider simple prepayment assumptions to illustrate some basic effects.

Benchmark 2:
## Valuation Assuming Value-Minimizing Prepayment Policy

- If the borrower prepays in a way to minimize the cost of the mortgage, then mortgage is like a callable bond.

- Each period, the borrower chooses to prepay or wait, according which action minimizes the mortgage value.

- We'll ignore the possibility of partial prepayment.

  - If partial prepayments were applied to reduce the level of each remaining payment equally, then value-minimizing would dictate prepaying all or nothing.

  - In practice, partial payments apply to the latest payments first. In an upward-sloping yield curve, this means that the options that are the least in the money, or most out of the money, must be exercised first.

Mortgage with Value-Minimizing Prepayment Policy

•At each state, the borrower can leave the loan outstanding, or else pay off the loan by paying the remaining principal balance in addition to the currently scheduled payment.

•**Class Problem**: Assume the borrower chooses the action that minimizes the mortgage value and fill in the tree of decisions and values.

At each node, the mortgage value is the minimum of remaining principal and wait value.

Time 0	Time 0.5	Time 1	Time 1.5
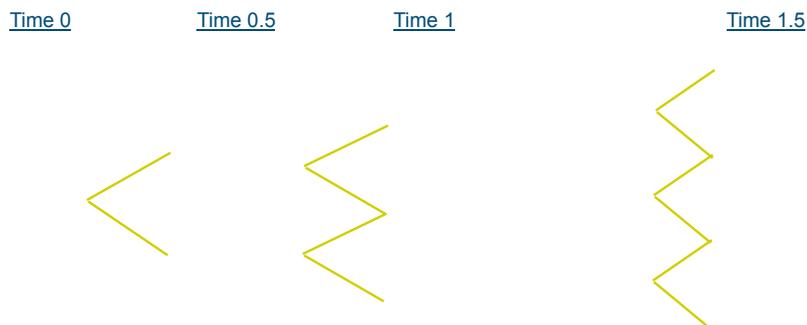
The number at each node represents the value of the remaining cash flows from the mortgage, excluding the currently *scheduled* payment.

Some Prepayment Measures and Deterministic Prepayment Scenarios Used in Practice

•SMM – Single Monthly Mortality rate: proportion of remaining pool that prepays over the month

•CPR – Conditional Prepayment Rate: annualized prepayment rate (SMMx12)

•12-Year Average Life scenario: assume no prepayment until year 12, then all at once

•FHA experience: schedule of prepayments based on data

•PSA (Public Securities Association) convention for 30-year mortgages: 0.2% CPR in month 1, 0.4% CPR in month 2, …, 6% CPR in month 30, then 6% CPR in months 31-360.

Practitioners sometimes quote prepayment scenarios as a percent of this PSA schedule.

Benchmark 3:
Mortgage Value Assuming Deterministic (Idiosyncratic) Prepayments

- Note that under the value-minimizing prepayment policy,
  - there is a 50% (risk-neutral) chance the mortgage prepays at time 0.5,
  - given that it doesn't prepay at time 0.5, there is a 25% chance the mortgage prepays at time 1.5.
- Now suppose instead, that each borrower has a 50% chance or prepaying at time 0.5, *regardless of interest rates*, and given no prepayment at time 0.5, there is a 25% chance of prepaying at time 1.5, *regardless of interest rates.*
- In a large enough pool of independent risks like these, these idiosyncratic risks average out (the law of large numbers). Therefore, we can approximate the effect of this kind of prepayment risk by assuming that, with certainty,
  - 50% of the mortgages prepay in full at time 0.5,
  - of the remaining mortgages, 25% prepay at time 1.5.
- Using this approximation, we can value the mortgage as a stream of fixed cash flows.

Mortgage Value Assuming Deterministic Prepayments

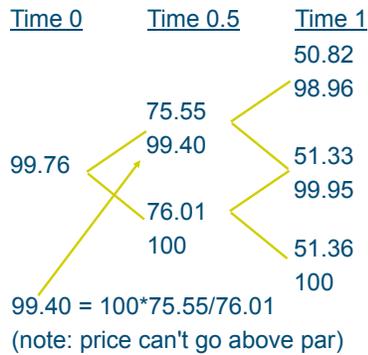| Date	Beginning Balance	Scheduled Payment	Interest	Principal	Pre-payment	Ending Balance
0.50	100.00	26.74	2.75	23.99	38.00	38.00
1.00	38.00	13.37	1.05	12.33	0.00	25.68
1.50	25.68	13.37	0.71	12.66	3.25	9.76
2.00	9.76	10.03	0.27	9.76	0.00	0.00

- **Class Problem:** What would the mortgage be worth if these prepayments occurred with certainty?

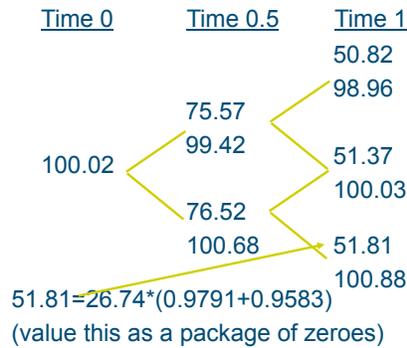## Mortgage Price as a Percent of the Remaining Principal Balance

At each node below, the first number is the total pool value, and the second is the value as a percent of remaining principal.
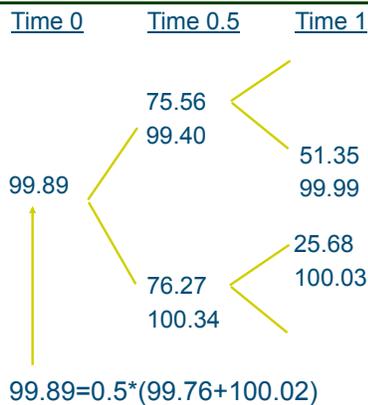
Value-Minimized Mortgage

Time 0	Time 0.5	Time 1
		50.82
		98.96
	75.55	
	99.40	
99.76		51.33
		99.95
	76.01	
	100	51.36
		100

$99.40 = 100 \cdot 75.55/76.01$

(note: price can't go above par)

Mortgage with No Prepayments

Time 0	Time 0.5	Time 1
		50.82
		98.96
	75.57	
	99.42	
100.02		51.37
		100.03
	76.52	
	100.68	51.81
		100.88

$51.81 = 26.74 \cdot (0.9791 + 0.9583)$

(value this as a package of zeroes)

Mortgage Pool with Both Types

Now consider a pool consisting half of mortgages that never prepay and half of mortgages that optimally prepay.

Time 0	Time 0.5	Time 1
	75.56	
	99.40	
		51.35
99.89		99.99
		25.68
		100.03
	76.27	
	100.34	

$99.89 = 0.5 \cdot (99.76 + 100.02)$

- In the beginning, the value of the pool is just the average of the value of the two types.
- However, over time, if the optimal prepayers do prepay, then the composition of the pool changes. The remaining pool consists of non-prepayers.
- Often, older mortgages prepay slower because the faster prepayers have dropped out. This slow down in prepayment speeds is called *burnout*.
- If prepayments depend on the level of interest rates, then the mortgage pool value is also *path-dependent*. For example, at time 1 in the middle state, the mortgage price is higher if the path was down-up than up-down, because the optimal prepayers are gone.

Valuation Approach in Practice

Rather than develop "rational" models of prepayment behavior that detail the decision-making process of mortgagors, practitioners usually take a different approach to valuation:

1) Estimate an empirical model of prepayments as a function of current and past interest rates, pool age and size, seasonality, and other variables.

   For example, prepayment rate this period = $\alpha*\exp(\beta 1*(\text{coupon minus rate}) + \beta 2*(\text{lagged rates}) + \beta 3*(\text{percent of pool outstanding}) + \beta 4*(1 \text{ if summer}) + ...$
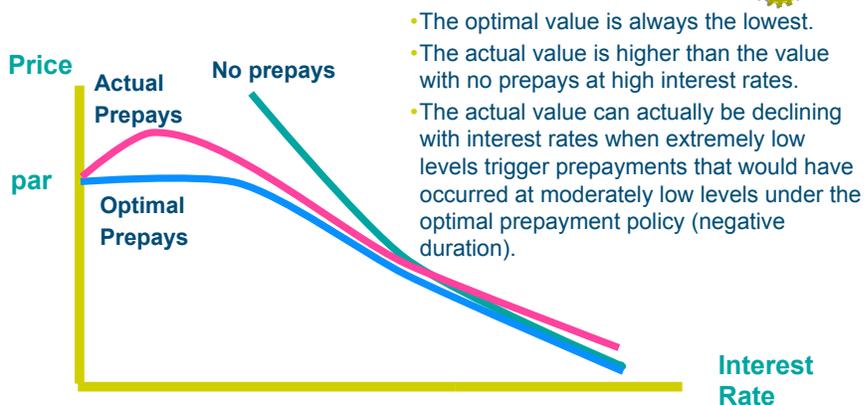
Valuation Approach in Practice…

2) Calculate the value of this empirically estimated payoff function as its risk-neutral expected discounted value as follows:

   a) Simulate mortgage cash flows along thousands of different paths in the interest rate tree using the estimated prepayment function to determine prepayments along each path.

   b) Discount the cash flows along each path back to time 0, using the short rates along the path.

   c) Average the discounted payoff value across the different paths, weighting by the risk-neutral probability of each path.

   d) The simulation is necessary because there are typically too many paths to do the calculation exactly. For example, in a 30-year monthly tree, there are 2^{360} paths.

Option-Adjusted Spread (OAS)

- The option-adjusted spread on a given mortgage-backed security implied by its market price is the spread one would need to add to each of the short rates on the interest rate tree to make the model price equal the market price.

- This gives a measure of the cheapness of the mortgage-backed security, after accounting for the presence of the borrower's prepayment option.

Price-Rate Curve for Pool

Price

Actual Prepays

No prepays

par

Optimal Prepays

- The optimal value is always the lowest.
- The actual value is higher than the value with no prepays at high interest rates.
- The actual value can actually be declining with interest rates when extremely low levels trigger prepayments that would have occurred at moderately low levels under the optimal prepayment policy (negative duration).

Interest Rate

CMOs

- The simplest way to create securities from a pool of mortgages is to create pass-throughs, each receiving a fixed share of the cash flows that pass through the pool.

- In practice, issuers of MBS also carve up the cash flows from a mortgage pool in more exotic ways.

- The resulting securities are called *collateralized mortgage obligations* (CMOs).

  - IOs and POs are examples of CMOs, also called strips.

  - Another scheme is to form *sequential tranches*.

Basic Tranche CMOs

- Each tranche is a claim to a certain amount of principal, plus interest on that principal. The tranches are ordered.

- At first, Tranche A receives all principal payments and prepayments to the pool, plus interest on that principal, until its principal is paid off.

- All other tranches receive only interest on their outstanding principal.

- After tranche A is paid off, then tranche B receives all principal payments, plus interest. Later tranches receive only interest.

- After tranche B is paid off, tranche C gets principal, etc.

Basic Tranche CMOs

- For example, consider the 2-year, 5.5% semi-annual mortgage from the previous example.
- Divide this into three tranches.
  - Tranche A gets the first $30 of principal.
  - Tranche B gets the next $30 of principal.
  - Tranche C gets the last $40 of principal.

Cash Flows to Tranches Assuming No Prepayments

Pool

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.50	100.00	26.74	2.75	23.99	76.01
1.00	76.01	26.74	2.09	24.65	51.36
1.50	51.36	26.74	1.41	25.33	26.03
2.00	26.03	26.74	0.72	26.03	0.00

Tranches

	A	30.00	B	30.00	C	40.00
Date	A Int	A Prin	B Int	B Prin	C Int	C Prin
0.50	0.83	23.99	0.83	0.00	1.10	0.00
1.00	0.17	6.01	0.83	18.64	1.10	0.00
1.50	0.00	0.00	0.31	11.36	1.10	13.97
2.00	0.00	0.00	0.00	0.00	0.72	26.03

Valuation Assuming No Prepayment

- With no prepayment, each tranche would just be a stream of four fixed cash flows, which could be valued as a package of zeroes:

- A: (0.83+23.99)*0.9730 + (0.17+6.01)*0.9476 = 30.00

- B: 0.83*0.9730 + (0.83+18.64)*0.9476 + (0.31+11.36)*0.9222 = 30.01

- C: 1.10*0.9730 + 1.10*0.9476 + (1.10+13.97)*0.9222 + (0.72+26.03)*0.8972 = 40.01

- Note that the values of the tranches sum to the value of the pool: 30.00 + 30.01 + 40.01 = 100.02

Cash Flows to Tranches Assuming 50% Prepayment at Time 0.5, and 25% Prepayment at Time 1.5

Pool

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Pre-payment	Ending Balance
0.50	100.00	26.74	2.75	23.99	38.00	38.00
1.00	38.00	13.37	1.05	12.33	0.00	25.68
1.50	25.68	13.37	0.71	12.66	3.25	9.76
2.00	9.76	10.03	0.27	9.76	0.00	0.00

Tranches

	A	30.00	B	30.00	C	40.00
Date	A Int	A Prin	B Int	B Prin	C Int	C Prin
0.50	0.83	30.00	0.83	30.00	1.10	2.00
1.00	0.00	0.00	0.00	0.00	1.05	12.33
1.50	0.00	0.00	0.00	0.00	0.71	15.92
2.00	0.00	0.00	0.00	0.00	0.27	9.76

Valuation Assuming 50% Prepayment at Time 0.5, and 25% Prepayment at Time 1.5, with Certainty

- With deterministic prepayments, each tranche would just be a stream of four fixed cash flows, which could be valued as a package of zeroes:

- A: (0.83+30.00)*0.9730 = 29.99

- B: (0.83+30.00)*0.9730 = 29.99

- C: (1.10+2.00)*0.9730 + (1.05+12.33)*0.9476 + (0.71+15.92)*0.9222 + (0.27+9.76)*0.8972 = 40.01

- Again, the values of the tranches sum to the value of the pool (allowing for rounding error):

  - 29.99 + 29.99 + 40.01 = 100.00

Tranche A
Assuming Optimal Prepayments

At each node, the CMO value is the value of the remaining cash flows, excluding the currently scheduled cash flow.

Time 0

29.99 =
(0.83+23.99+
0.5(5.99+6.01))
*0.9730

Time 0.5

5.99
=(0.17+6.01)*0.9709

6.01

- ✳ Note that the value-minimization is done by the borrower at the level of the *whole mortgage*.

- ✳ For the derivatives, *there is no further optimization*. The derivative (e.g., IO, PO, CMO) cash flows are simply derived from those of the whole mortgage.

Tranche B
Assuming Optimal Prepayments

At each node, the CMO value is the value of the remaining cash flows, excluding the currently scheduled cash flow.

Class Problem: Fill in the tree of values of Tranche B below:

Time 0	Time 0.5	Time 1

Tranche C
Assuming Optimal Prepayments

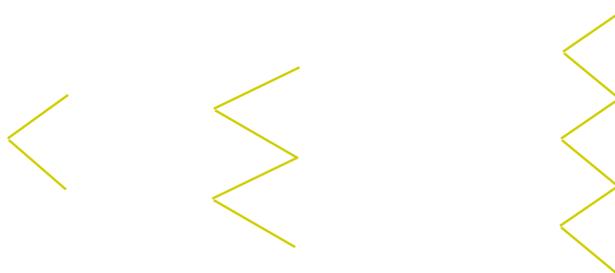At each node, the CMO value is the value of the remaining cash flows, excluding the currently scheduled cash flow.

Class Problem: Fill in the tree below:

Time 0	Time 0.5	Time 1	Time 1.5

Interest Rate Sensitivity of Mortgage and CMOs with No Prepayments

	Time 0.5	Interest Rate Deltas
	75.57	• Mortgage:
	5.99	$\cdot$ -(75.57-76.52)/(0.06004-0.04721)= 75
	29.89	• A:
Mortgage: 100.02	39.68	$\cdot$ -(5.99-6.03)/(0.06004-0.04721) = 3
A: 30.00		• B:
B: 30.01		$\cdot$ -(29.89-30.15)/(0.06004-0.04721)= 20
C: 40.01	76.52	• C:
	6.03	$\cdot$ -(39.68-40.34)/(0.06004-0.04721)= 51
	30.15	
	40.34	

40.34 = 1.10*0.9769 + (1.10+13.97)*0.9538 + (0.72+26.03)*0.9309

With no prepayments, each security is just a stream of fixed cash flows.

Interest Rate Sensitivity of Mortgage and CMOs with Optimal Prepayments

	Time 0.5	• Interest Rate Deltas
	75.55	• Mortgage:
	5.99	-(75.55-76.01)/(0.06004-0.04721)= 36
	29.89	• A:
Mortgage: 99.76	39.67	-(5.99-6.01)/(0.06004-0.04721) = 1
A: 29.99		• B:
B: 29.94	76.01	-(29.89-30.00)/(0.06004-0.04721)= 9
C: 39.83	6.01	• C:
	30.00	-(39.67-40.00)/(0.06004-0.04721)= 26
	40.00	

• The C tranche absorbs most of the reduction in both value and interest rate delta caused by optimal prepayments.
• With no prepayments, C has the biggest upside in the low interest rate states.
• Optimal prepayment, which takes away the upside, has the biggest impact on C.

Security Design

- Other CMO structures: IOs, POs, Z-Bonds, PACs, TACs, Companion tranches, floaters and inverse floaters
- Why do issuers and dealers divide mortgage pools into so many different kinds of securities?
- Different securities offer very different risk characteristics.
  - The A tranche is much less subject to prepayment risk than the Z tranche.
  - The PAC cash flows are less risky than the companion.
- Introducing new securities increases the set of payoff patterns that investors can achieve.
- The impact of financial innovation on market prices is ambiguous.
- Presumably, dealers innovate only when doing so increases their total sale revenues. This might occur if different investors have different risk preferences or different abilities to hedge risk.

Z-Bond

Another way to structure CMO tranches involves a Z Tranche or Z bond--a security that receives no principal, *or interest*, until all previous tranches are paid off.

Pool assuming no prepayments

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance
0.50	100.00	26.74	2.75	23.99	76.01
1.00	76.01	26.74	2.09	24.65	51.36
1.50	51.36	26.74	1.41	25.33	26.03
2.00	26.03	26.74	0.72	26.03	0.00

Tranches

	A	30.00	B	30.00	Z	40.00	
Date	A Int	A Prin	B Int	B Prin	Z Int	Z Prin	Z Balance
0.50	0.83	25.09	0.83	0.00	0.00	0.00	41.10
1.00	0.13	4.91	0.83	20.87	0.00	0.00	42.23
1.50	0.00	0.00	0.25	9.13	1.16	16.20	26.03
2.00	0.00	0.00	0.00	0.00	0.72	26.03	0.00

Tranches with Z Bond Assuming 50% Prepayment at Time 0.5, and 25% Prepayment at Time 1.5

Pool

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Prepayment	Ending Balance
0.50	100.00	26.74	2.75	23.99	38.00	38.00
1.00	38.00	13.37	1.05	12.33	0.00	25.68
1.50	25.68	13.37	0.71	12.66	3.25	9.76
2.00	9.76	10.03	0.27	9.76	0.00	0.00

Tranches

	A	30.00	B	30.00	Z	40.00	
Date	A Int	A Prin	B Int	B Prin	Z Int	Z Prin	Z bal
0.50	0.83	30.00	0.83	30.00	1.10	2.00	38.00
1.00	0.00	0.00	0.00	0.00	1.05	12.33	25.68
1.50	0.00	0.00	0.00	0.00	0.71	15.92	9.76
2.00	0.00	0.00	0.00	0.00	0.27	9.76	0.00

Other CMO Structures

- Planned Amortization Class (PAC)
  - PAC bonds have nonrandom payments as long as prepayment speeds stay within a specified range.
  - Support Class (or Companion class) receives residual payments.
  - e.g., 4 PAC Tranches (A-D), 4 Companion Tranches (E-H)
- Floating-Rate Tranches
  - Floating-rate tranches can be created from any fixed-rate tranche by creating a floater and inverse floater.

IOs and POs

- The most basic MBS is a pass-through, a security that receives a fixed fraction of all cash flows that flow through the pool.
- As an alternative, MBS issuers sometimes strip pass-throughs into two securities:
  - IOs -- Securities that receive interest only.
  - POs -- Securities that receive principal only.
- For the pass-through, optimal prepayments reduce value, but suboptimal prepayments can actually increase value.
- For the PO, prepayments are unambiguously good:
  - get the same cash flows earlier
- For the IO, prepayments are unambiguously bad:
  - get altogether less cash flow

IO and PO from the 2-Year 5.5% Mortgage With No Prepayments

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Ending Balance	IO	PO
0.50	100.00	26.74	2.75	23.99	76.01	2.75	23.99
1.00	76.01	26.74	2.09	24.65	51.36	2.09	24.65
1.50	51.36	26.74	1.41	25.33	26.03	1.41	25.33
2.00	26.03	26.74	0.72	26.03	0.00	0.72	26.03

- With no prepayment, each security has fixed cash flows and can be valued as a package of zeroes:
  - IO: 2.75*0.9730 + 2.09*0.9476 + 1.41*0.9222 + 0.72*0.8972 = 6.60.
  - PO: 23.99*0.9730+24.65*0.9476+25.33*0.9222+26.03*0.8972=93.42.
- The IO plus the PO reconstitute the whole mortgage, so their values must satisfy: IO value + PO value = Mortgage value.
  Indeed, 6.60 + 93.42 = 100.02.

IO and PO from the 2-Year 5.5% Mortgage With Deterministic Prepayments

Date	Beginning Balance	Scheduled Payment	Interest	Principal	Pre-payment	Ending Balance	IO	PO
0.50	100.00	26.74	2.75	23.99	38.00	38.00	2.75	62.00
1.00	38.00	13.37	1.05	12.33	0.00	25.68	1.05	12.33
1.50	25.68	13.37	0.71	12.66	3.25	9.76	0.71	15.92
2.00	9.76	10.03	0.27	9.76	0.00	0.00	0.27	9.76

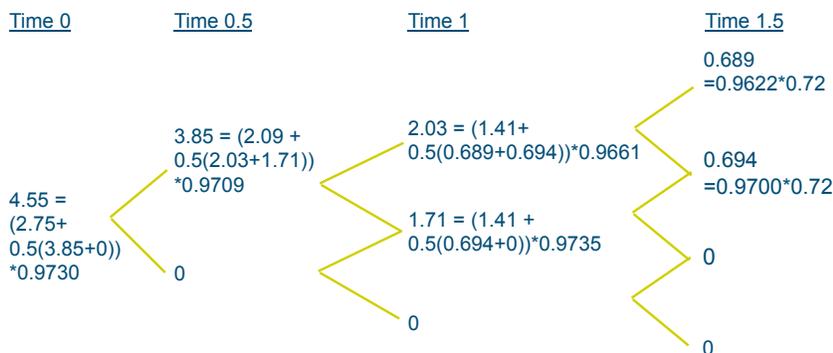- With deterministic prepayment, each security has fixed cash flows and can be valued as a package of zeroes:
  - IO: 2.75*0.9730 + 1.05*0.9476 + 0.71*0.9222 + 0.27*0.8972 = 4.56.
  - PO: 62.00*0.9730+12.33*0.9476+15.92*0.9222+9.76*0.8972=95.44.
- IO value + PO value = Mortgage value
  - 4.56 + 95.44 = 100.00

☀ The prepayments reduce the IO value and increase the PO value.

IO from Mortgage with Value-Minimizing Prepayment Policy

At each node, the IO value is the value of the remaining cash flows, excluding the current interest payment.
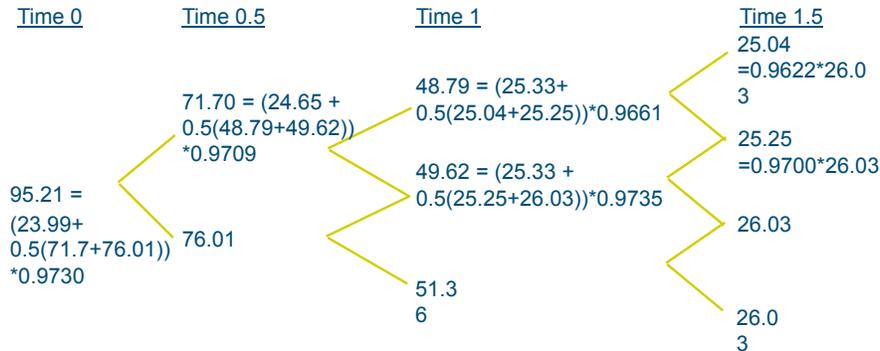
Time 0   Time 0.5   Time 1   Time 1.5

                                                        0.689
                                                        =0.9622*0.72

                  3.85 = (2.09 +   2.03 = (1.41+
                  0.5(2.03+1.71))   0.5(0.689+0.694))*0.9661   0.694
                  *0.9709   =0.9700*0.72

4.55 =
(2.75+
0.5(3.85+0))   1.71 = (1.41 +   0
*0.9730   0   0.5(0.694+0))*0.9735

                                        0   0

                                                                   0

☀ Note that the value-minimization is done by the borrower at the level of the whole mortgage. For the derivatives, *there is no further optimization*. The derivative (e.g., IO and PO) cash flows are simply derived from those of the whole mortgage.
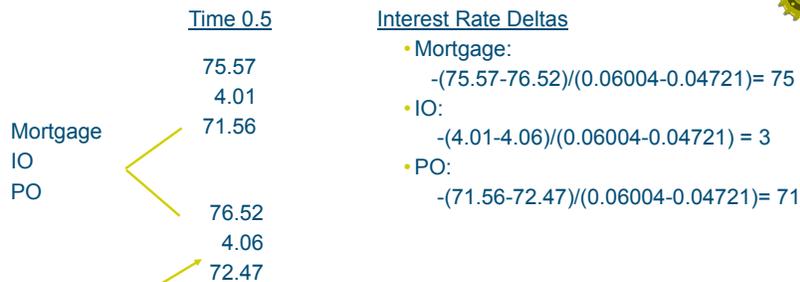
PO from Mortgage with Value-Minimizing Prepayment Policy

At each node, the PO value is the value of the remaining cash flows, excluding the current principal payment.

Time 0   Time 0.5   Time 1   Time 1.5

25.04
=0.9622*26.03

71.70 = (24.65 +   48.79 = (25.33+
0.5(48.79+49.62))   0.5(25.04+25.25))*0.9661
*0.9709

25.25
=0.9700*26.03

95.21 =
(23.99+   49.62 = (25.33 +
0.5(71.7+76.01))   76.01 0.5(25.25+26.03))*0.9735
*0.9730

26.03

51.3
6

26.0
3

Note: IO + PO = Mortgage

4.55 + 95.21 = 99.76

Interest Rate Sensitivity of Mortgage, IO, and PO with No Prepayments

Time 0.5   Interest Rate Deltas

75.57   • Mortgage:
4.01   -(75.57-76.52)/(0.06004-0.04721)= 75
71.56   • IO:
Mortgage   -(4.01-4.06)/(0.06004-0.04721) = 3
IO   • PO:
PO   -(71.56-72.47)/(0.06004-0.04721)= 71

76.52
4.06
72.47

With no prepayments, each security is just a stream of fixed cash flows.

4.06 = 2.09*0.9769 + 1.41*0.9538 + 0.72*0.9309

72.47 = 24.65*0.9769 + 25.33*0.9538 + 26.03*0.9309

Interest Rate Sensitivity of Mortgage, IO, and PO with Value-Minimizing Prepayment Policy

Time 0.5

75.55
3.85
Mortgage   71.70
IO
PO
76.01
0
76.01

Interest Rate Deltas
- Mortgage:
  -(75.55-76.01)/(0.06004-0.04721)= 36
- IO:
  -(3.85-0)/(0.06004-0.04721) = -300
- PO:
  -(71.70-76.01)/(0.06004-0.04721)= 336

With optimal prepayments
- the delta of the mortgage is lower,
- the delta of the IO is negative.

Price-Rate Curves for IO and PO