AN INTEGRATED MODEL FOR PREVENTIVE MAINTENANCE AND SPARE PART INVENTORY PLANNING

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ABSTRACT

AN INTEGRATED MODEL FOR PREVENTIVE MAINTENANCE AND SPARE PART INVENTORY PLANNING

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The machine in any production environment is subject to failure. Although the frequency of failures can be managed through preventive maintenance activities, it is impossible to get out of failures entirely. Firms need to carry spare parts inventory to cope with failure and ensure smooth operations through preventive maintenance. In other words, preventive maintenance and uncertain failures can be considered as the major reasons of spare part inventory. Therefore, planning preventive maintenance activities and managing spare part inventory should be handled together. In this study, we present a Dynamic Programming formulation of the joint problem of the preventive maintenance and spare parts inventory planning. The aim is to minimize the total expected cost over a finite planning horizon. Since the Dynamic Programming formulation is computationally intractable for long planning horizons and a system with a large number of machines, three heuristic approaches are proposed: (i) Myopic approach, (ii) Stationary policy and (iii) Steady-State approximation. Through computational analyses, effects of problem parameters on the performances of the proposed heuristics are investigated.

Keywords: Preventive Maintenance, Preventive Replacements, Spare Part Inventory, Joint Optimization

ÖZ

KORUYUCU BAKIM VE YEDEK PARCA ENVANTERİ İÇİN TÜMLEŞİK BİR MODEL

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Üretimde kullanılan makineler bozulabilirler. Önleyici bakımlar ile bozulma sıklıkları yönetilebilmesine rağmen, bozulmaları tamamen önlemek olanaksızdır.Firmalar bozulmalarda ve önleyici bakımlarda kullanmak için yedek parça tutmaya ihtiyaç duyarlar. Başka bir deyişle, önleyici bakım ve beklenmedik bozulmalar yedek parça envanterinin temel nedenleridir. Bu yüzden önleyici bakım ve yedek parça envanteri beraber ele alınmalıdır. Bu tez çalışmasında, önleyici bakım ve yedek parça envanteri ortak problemi bir Dinamik programlama ile modellenmiştir. Amaç belirli dönemde beklenen maliyeti en aza indirgemektir. Çok makineli üretim ortamları ve uzun periyotlu maliyet hesapları için, Dinamik programlama modelinin hesaplanması zor olduğundan, 3 sezgisel yaklaşım önerilmiştir: (i) Miyop yaklaşım, (ii) Sabit politika ve (iii) Kararlı durum yaklaşımı. Problem değişkenlerinin önerilen yaklaşımlar üzerindeki etkileri araştırılmıştır.

Anahtar Kelimeler: Koruyucu Bakım, Koruyucu Değişim, Yedek Parça Envanteri, Ortak Optimizasyon

To my family

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CHAPTER 1

INTRODUCTION

Maintenance is the set of activities to keep a component or machine in working condition. With maintenance activities, the life time of a machine increases. There are two main ways of performing maintenance: preventive and corrective. Preventive maintenance is the planned activities before failures occur, whereas corrective maintenance is performed after failures.

Maintenance activities, preventive or corrective, require usage of spare parts. During maintenance periods, spare parts must be available to use. More importantly, when a failure occurs they should be accessed immediately. Shortages of spare parts and idleness of the machine due to breakdown bears huge losses. In addition, spare parts may have order lead times. Because of these, spare parts must be held in inventory. However, they are usually too expensive to keep in inventory in large quantities. Although the frequency of failures can be managed through preventive maintenance activities, it is impossible or impractical to get out of failures entirely. Thus, maintenance planning and spare part inventory planning are interrelated activities and they must be considered together and optimized jointly.

In this study, we consider a manufacturing system that contains multiple identical and independently operating machines. Each machine has a non-repairable critical part. When a critical part fails, corresponding machine stops operating until the part is replaced. Failure rate of a critical part increases with its age. In order to replace the critical parts during preventive and corrective replacement activities, spare parts must be available in inventory. In this problem environment, we focus on joint optimization of preventive replacements and spare part inventory planning. Both preventive replacements and inventory decisions are made at discrete points in time. The total expected cost is minimized over a finite planning horizon. The main decisions to make are whether to replace each critical part preventively or not and how much to order. The total expected cost contains both replacement and inventory related costs.

We propose a Dynamic Programming formulation to find optimal replacement and stocking decisions. However, it is hard to obtain the optimal solution for a long planning horizon and for systems with large number of operating units within a reasonable time. Hence, we propose three heuristic approaches. These are (i) Myopic approach, (ii) Stationary policy, (iii) Steady-State approximation. Myopic approach ignores the impacts of the current decisions on the future events; it focuses only on minimizing the total expected cost of the immediate period. Under Stationary policy, we restrict our attention to age-based replacement and base stock policy. We define two policy parameters: On hand inventory at the beginning of a period after preventive replacements as base stock parameter and the age limit to be replaced as age-based replacement parameter. The third heuristic approach is based on approximating the finite horizon problem by an infinite horizon for a single machine. In this heuristic, the solutions of Markov Decision Process (MDP) model for the single machine case is extended to multi-machine case ignoring the inventory pooling opportunity.

In the literature, the studies generally consider a given policy then try to find best policy parameters. We do not impose any pre-determined preventive maintenance or inventory control policy as in the literature. The main objective of this study is to assess the performances of the proposed heuristic approaches under different problem parameters with respect to the solution quality.

The rest of the study is organized as follows. In Chapter 2, we present general information on maintenance policies and the studies proposing joint approaches for maintenance and spare parts inventory planning in the literature. In Chapter 3, problem definition and model assumptions are provided. Furthermore, the Dynamic Programming formulation and the heuristic approaches are introduced. In Chapter 4, we carry out an extensive computational analysis and present our findings. Finally, we conclude in Chapter 5 by summarizing the findings and offering further research directions.

CHAPTER 2

LITERATURE REVIEW

Preventive maintenance can be considered as a precaution against failure risk which increases with the age of machine. Maintenance policies and failures create demand for spare part inventories and availability of spare part inventory affects the feasibility of maintenance policies. Spare parts may be costly and there is a risk of deterioration or obsolescence during the waiting time in inventory. In addition, shortages of spare parts may cause extra cost due to loss of production. Hence, spare part inventories should be studied in detail. Kennedy et al. [15] and Rego et al. [18] provide two reviews of the studies on spare parts inventory planning. Kennedy et al. [15] categorize the studies into six: management issues, age-based replacement, multi-echelon problems, obsolescence, repairable items and special applications. Rego et al. [18] make a different classification. They investigate the studies under the categories of demand forecasting, classification of spare parts, decision to stock or not to stock, initial orders, inventory control models and obsolescence and final orders. These reviews investigate the studies in terms of only spare part inventories. However, preventive maintenance and spare part inventories should be handled together since they are interrelated.

In this study, we focus on the joint problem of preventive maintenance and spare part inventory planning. Thus, the studies related to pure maintenance or pure spare part inventory planning are not considered. In the rest of the chapter, we give general information on maintenance policies in Section 2.1. Then we present papers that propose joint approaches for maintenance and spare parts inventory in detail in Section 2.2.

2.1 General Information on Maintenance Policies

Maintenance is the set of activities to keep a component or equipment in proper condition. For a component, being in proper condition means performing its function appropriately [6]. Maintenance is made either planned (preventively) or un-planned (correctively that is after the failures). The advantages of preventive maintenance compared to corrective maintenance are summarized as follows:

i. Less cost and time [26].

ii. Reduced failure risk [9].

iii. Reduced break-down cost (since preventive maintenance is usually scheduled on idle time) [19].

iv. Increased system life-time [9].

The studies in the literature consider both preventive and corrective maintenances. Preventive maintenance is planned ahead whereas corrective maintenance is performed when the machine or unit fails. In the literature, the units that are subject to maintenance are two types: non-repairable and repairable. For non-repairable units, maintenance is equivalent to the replacement of the unit. For repairable items, maintenance can be made in two ways; either replacement or repair.

Maintenance and replacement problems are discussed in many studies in the literature. Wang [21] provides a review of studies on maintenance policies. He classifies the studies into two parts: one unit systems and multi unit systems. For one unit systems, policies are categorized as age-dependent preventive maintenance, periodic preventive maintenance, failure rate limit, sequential preventive maintenance, repair cost limit and repair number counting and reference time. For multi unit systems, he categorizes the policies as group maintenance and opportunistic maintenance. Garg and Deshmukh [8] provide another review classifying the literature into six areas: maintenance optimization models, maintenance techniques, maintenance scheduling, maintenance performance measurement, maintenance information systems and maintenance policies.

In the maintenance planning literature, the following policies are the most common ones:

- Age-based maintenance: Maintenance activities are made upon failure or at age T whichever occurs first. T can be a constant or a dynamic variable. [5]
- Block maintenance: The unit is replaced at constant time periods independent of the history of the unit. [5]
- Periodic replacement with minimal repair at failures: This policy is proposed by Barlow and Hunter [4]. Minimal repair is performed at failures whereas replacements take place at periodic replacement intervals. It is assumed that after the repair at failures, the failure rate remains the same during periodic replacement intervals.
- Sequential maintenance over a finite time span: The maintenance interval is not constant. It is changed due to the remaining life time in a finite time span.
- Group maintenance: When components on the system have interactions with each other, all components are replaced or repaired together [17].
- Condition-based maintenance: Components are monitored continuously and maintenance decisions are made with respect to the state of the components. It is also called predictive maintenance.
- Corrective maintenance: Maintenance activities are performed upon failures.

2.2 Joint Preventive Maintenance and Spare Parts Inventory Planning

Acharya et al. [1] study joint optimization of preventive replacement and spare part ordering for a system composed of independent and identical equipment pieces. The equipment pieces have increasing failure rates with time. They consider block replacement as the preventive maintenance policy and a base-stock inventory control policy. Both inventory and replacement related costs, are considered, that is, inventory holding, fixed ordering, backorder, failure replacement and preventive replacement costs are taken into account. Replaced pieces are assumed to be as good as new. Order lead time is neglected. They find optimal preventive replacement interval, base stock level and ordering interval (the interval between two successive orders) in order to minimize total cost. They investigate two situations on replacement and ordering intervals; (i) single-period model where replacement and ordering intervals are equal and (ii) multi-period model where ordering interval is a multiple of preventive replacement interval. They use an enumeration algorithm to find the joint optimal solution. They use the following time to failure probability function proposed by Yamada and Osaki [25] for numerical examples:

$$F(t) = 1 - (1 + 4t)e^{-4t}$$

where t is the time. In the numerical examples, it is shown that optimal replacement interval in the multi-period model is much lower than optimal replacement interval in the single period model. In addition, joint optimal replacement and spare ordering interval is larger than optimal value of block replacement interval when only replacement related cost is considered.

Kabir and Al-Olayan [11] consider a system which has a single operating unit and multi spare parts in inventory. They introduce a policy for stocking and age based preventive replacement with a form of (t_1, s, S). In this policy, preventive replacement is made at a predetermined age t_1 and continuous review (s, S) type of inventory policy, where S is the maximum allowable stock level and s is the reorder point, is used. Preventive replacement is performed when the age of the equipment becomes t_1 if a spare part is available, otherwise it is made as soon as the spare part becomes available. Similarly the failure replacement is made immediately if a spare is available. Otherwise the failed unit waits until the spare part becomes available. An order for (S - s) spare units are placed when the inventory level falls to the level of s. The time between two successive replacements is defined as a cycle. There is no inventory and no outstanding order is placed at time zero. If the operating unit fails at first cycle, emergency order is placed. Emergency order is placed only at first cycle. Weibull distribution is used in modeling both the order lead time and the lifetime of the unit. Both regular and emergency orders have the same lead time parameters and costs. The objective is to find the optimal policy parameters (t_1, s, S) by minimizing expected total cost. Expected total cost is composed of replacement and inventory related costs. Replacement related costs are failure replacement cost and preventive replacement cost.

Inventory related costs are emergency and regular ordering costs, inventory holding cost and shortage cost. A simulation model is developed. The simulation model runs for all predetermined t_1, S and $s \in [0, S-1]$ values. For all alternative policies, multiple simulation runs are performed. For all t_1 values, minimum cost combination of (t_1, s, S) is determined. They compare results with the age replacement policy introduced by Barlow and Prochan [5]. Barlow-Prochan age policy is supported by the optimal (s, S) inventory policy which is of jointly optimal policy to compare the expected total cost. They argue that the jointly optimal policy gives less cost than the Barlow-Prochan age replacement policy. The maximum saving compared to the Barlow-Prochan policy is 75.25%. In addition, for a large order lead time, the jointly optimal policy gives more cost savings over Barlow-Prochan policy. In their following work [12], they extend this work to include multiple identical operating units. They again compare the results of their policy with the Barlow-Prochan age replacement policy supported by the optimal (s, S) inventory policy. Their results show that the Barlow-Prochan policy has higher cost values than their policy. They claim that the combination of Barlow-Prochan policy and the optimal (s, S) inventory policy does not give global optimality with reference to their simulation results.

Kabir and Farrash [13] study the same environment as studied by Kabir and Al-Olayan [12] except that emergency ordering is not allowed. Only the regular orders are placed: if the inventory level drops to the level s, an order is placed to raise inventory level to the maximum allowable stock, S. The results are compared to the combination of Barlow-Prochan age replacement policy and the optimal (s, S) inventory policy. Interpretations of the results are the same as the studies of Kabir and Al-Olayan [11-12].

Kabir and Farrash [14] study the joint optimization of age replacement and spare ordering for an operating system with multiple identical units. They introduce fixed interval ordering for the joint optimization of age-based replacement policy and spare part ordering. They consider a periodic review policy for inventory control and age-based replacement policy for maintenance. They consider a policy with a form of (t_0, q, t_1) where t_0 is the inventory review interval, q is the order lot size and t_1 is the preventive replacement age. They develop a simulation model in order to find minimum expected total cost per unit time. The cost components are costs for failure replacement, preventive replacement, ordering, inventory holding and shortage costs. Distributions for lifetimes of the units and order lead times are assumed to follow Weibull distribution. In the computational study, different instances that differ in the number of operating units, shape and scale parameters of unit lifetime and order lead time distributions and the cost parameters are considered. Results are compared to the stocking policy proposed by Kabir and Al-Olayan [12]. Proposed policy performs 5% to 28% worse when the system consists of 3 operating units and the shape parameter of failure distribution is low. With 5 operating units, fixed interval ordering shows savings between [-6.48%, 9%] with low values of shape parameter of failure distribution. However, with the increase in shape parameter of failure distribution, the fixed interval ordering policy shows savings up to than 5% for some parameter sets even for the 3-unit situation. With 5 operating units, if shape parameter of failure distribution is high, fixed interval ordering policy gives significant savings at least 16.66%. They conclude that the fixed interval ordering policy performs better when the system consists of large numbers of operating units and the shape parameter of failure distribution is high.

Armstrong and Atkins [2] study age-based replacement and ordering decisions for a system with a single operating unit and a single spare part. The order is placed only at scheduled times and there is no emergency order option. A spare part cannot be ordered if one is already in stock or on order. The order lead time is constant. The failure rate is assumed to be increasing with time. Replacement time is neglected. Replacements are assumed to be perfect; that is, the operating unit is as good as new after replacements. The objective is to minimize the expected system cost per unit time. The decision variables are the scheduled replacement times and the scheduled time to order. Time between two replacements is defined as a cycle. The expected system cost consists of expected replacement, failure, shortage and inventory holding costs. Procurement cost is not taken into account. In the numerical examples, Weibull distribution is used for distribution of time to failure of the operating unit. They show that the cost function is pseudo convex. To illustrate the benefits of the joint optimization, they perform calculations on both sequential and joint optimization for 135 different parameter instances. In 16 of these instances, sequential optimization gives expected system cost greater than 10%. In 110 instances, the differences are smaller than 1%. Armstrong and Atkins [3] extend this work to the environment where the replacement cost depends on the age of the operating unit, there is an age dependent operating cost and a fixed time for replacement actions. They also add service constraints on the maximum expected waiting time and minimum expected fill rate. They characterize the corresponding cost functions incorporating emergency orders and random lead times where the emergency order is placed if the installed unit fails before the scheduled ordering times. They do not provide any numerical studies.

Elwany and Gabraeel [7] extend the study of Armstrong and Atkins [2]. They replace fixed life time distributions used in [2] with sensor driven remaining life time distributions. Sensor driven remaining life time distributions are derived by degradation states of the individual component. Replacement and inventory decisions are updated with the real time states of the component. They propose a sensor driven dynamic update model for equipment replacement and spare part inventory management. Their model calculates the optimal time to replacement and optimal time to order at each specific time with an updating remaining life time distribution. This update process continue according to some defined stopping rules in their numerical studies. They show that the sensor-driven decision policy gives no failures over nine cycles, whereas traditional decision policy gives three failures. Hence, updating remaining life time distribution provides better replacement and inventory decisions than traditional fixed life time distributions.

Sarker and Haque [19] study maintenance and spare part inventory planning for a system with multiple identical units. Block replacement and a continuous review inventory control policy are considered. Block replacement does not consider the unit's age or failure history. They use Weibull distribution for order lead time and lifetime of the units, and Gamma distribution for replacement durations. An emergency order is placed when the inventory level decreases to zero or below, otherwise a regular order is placed when the inventory level drops to the reorder level. A numerical study is carried out where the emergency order cost is taken to be three times of the regular order cost and the parameter values of Weibull distribution for the order lead times are the same for both regular and emergency orders. The objective is to minimize the expected total cost over a finite planning horizon. The cost components in the expected total cost are block replacement, failure replacement, regular and emergency order, inventory holding and shortage costs. Decision variables are the reorder level, the order-up-to level and the interval for block replacements. They develop a simulation model for joint optimization. They compute system down time and work-in-process inventory as performance measures in addition to total expected cost. In the numerical study, separately optimized (s, S) and T policies are considered as a benchmark to compare the cost effectiveness of the joint optimized policy. For the 10 test problem instances, separated optimized policy have higher cost in the range of [2.81% , 8.77%]. These results show that joint optimized policy gives less cost values than the combination of the separately optimized policy. However, when investigating the effects of the parameters, it is observed that separately optimized policy performs better for some instances. When shape parameters of the lifetime and failure replacement cost per unit are low, separately optimized policy gives less cost values.

Vaughan [20] considers a system with multiple identical and independent components. He considers condition based maintenance policy where maintenance activities are performed at every T period. Spare part inventory is reviewed continuously. Lead times for orders and replacements are neglected. Failure distribution of the components is Exponential; that is, the components have constant failure rate over time. Hence, total demand for the spare parts due to failures during any period is a Poisson random variable. Each unit is detected and replaced at preventive maintenance activities with a certain probability, so the number of the units to be replaced at preventive maintenance activities is assumed to be Binomial. Costs for ordering and inventory holding and penalty for shortages are considered. Replacement related costs are not taken into account. A stochastic dynamic programming formulation is developed to minimize the total expected cost of the system over a finite horizon. Decision variables are the re-order point and the maximum stock level. In the computational study, preventive maintenance period, T, is taken to be 100 and the length of recursive evaluation, t, is taken to be 499, that is the evaluation is made for five successive preventive maintenance intervals. In order to prevent the end of horizon effects, a cyclical stationary optimal policy is proposed. In this approach, the policy depends only on k which is the number of periods until the next preventive maintenance activity. For all values of k, the cyclical stationary optimal policy with the form of $(s(k), S(k))$

are enumerated. He reports that $(s(k), S(k))$ policy converges to a stationary (s, S) policy when k becomes large.

Giri et al. [9] consider replacement and ordering decisions together for a single unit system. The unit is replaced upon failures and the replaced unit is as good as a new one. Both regular and emergency orders are used. The order lead times are fixed. The lead time of emergency order is less than the lead time of regular order and on the contrary emergency order cost is higher than regular order cost. The unit in concern has an increasing failure rate. If the unit fails before the regular ordering time, an emergency order is placed and failed unit is replaced by an emergency order. However, if the unit fails between regular order time and replenishment of the regular order, the units are replaced when the order is replenished, an emergency order is not placed. A spare is held in inventory until unit fails or a defined time limit of waiting in inventory passes. Time between two replacements is defined as a cycle. Only inventory related costs; inventory holding, shortage, ordering costs, are taken into account. The expected total discounted cost is minimized over an infinite time horizon where the decision variables are regular ordering time and time limit of waiting in inventory for spare parts. This two dimensional problem is reduced to a simple one-dimensional one by giving zero and infinite values to the time limit of waiting in inventory for spare parts. That is, there are two situations: (i) no replacement until the original unit fails and (ii) replacement as soon as the spare part is replenished. The life time and failure rate of the unit follows discrete Weibull distribution proposed by Nakagawa and Osaki[16]. In the numerical example, when the failure rate is low, the time limit of waiting in inventory is selected as zero, otherwise, no limit for waiting in inventory is selected.

Hu et al. [10] study maintenance and inventory decisions for a system containing multiple identical operating units. They propose a joint strategy (T, s, S) where T is the preventive replacement age, s is the reorder level and S is the maximum stock level. Inventory is reviewed continuously. If no spare part is available at the preventive replacement time, the unit continues to operate until the spare part becomes available. It is assumed that the corresponding unit does not fail during this delay. Weibull distribution is used for the life-time distribution and the order lead time. A discrete event simulation model is developed. Total cost includes failure and preventive replacements, procurement, storage and inventory holding costs. In the simulation, they use the same parameters as used in the study of Kabir et al. [12] and results are compared with [12. Optimal values of (T, s, S) policy are different from [12]. Average cost reduction when compared to [12] is 3.61%.

Wang [22] considers a system with a large number of identical items. He utilizes a Condition-based preventive replacement policy where replacements are performed according to inspection results. Inventory is periodically reviewed and the order lead time is fixed. Cost components are ordering, inventory holding and replacement costs. There are four types of replacement costs; (i) preventive replacement cost with spares on stock, (ii) preventive replacement cost without spares on stock, (iii) failure cost with spares on stock and (iv) failure cost without spares on stock. If there is no stock available, an emergency order with a lead time of zero is placed. Thus no shortage is allowed. It is assumed that a random number of items are found defective and should be replaced at preventive maintenance activities. As a large number of identical operating units are considered, homogeneous Poisson Process approximation is used for the number of the units to be replaced between preventive maintenance activities. A Stochastic Dynamic Programming(SDP) model is developed to minimize the total expected cost per unit time over a finite time horizon. Decision variables are order interval, order quantity and preventive maintenance interval. The SDP algorithm finds only the optimal order quantity. For the optimal order interval and preventive maintenance interval, enumeration with a discrete step size is conducted. It is shown that while the rate of the arrival of defective items is increasing, a smaller order interval and more frequent preventive maintenance is needed.

Xu et al. [24] consider a multi-component system. The lifetimes of the components are identically distributed and independent. Group replacement policy is used where all operating units are replaced at fixed preventive maintenance intervals. If a unit fails, only that unit will be replaced. Weibull distribution is used for failure distribution. Inventory is reviewed periodically. An order is placed up to level S. A Monte-Carlo simulation model is presented for joint optimization of preventive maintenance and inventory control. The goal is to find total cost per unit time with optimal preventive replacements period T and order up to point S. The total cost consists of failure replacement, group replacement, stock out, ordering and storage costs. An iterative algorithm by using Monte Carlo simulation method is used for computations. The results of proposed policy are compared to the results of separately optimized group replacement and periodic review inventory policy. The proposed policy reduced the cost values by [3.41%, 10.47%]. They claim that the amount of cost saving might depend on maintenance and replacement costs and spare support costs. In addition, when failure cost increases, saving of the proposed policy reduces.

2.3 Our Contribution to The Existing Literature

We study preventive replacements and inventory planning for a system that consists of multiple machines that are identical and independent. Each machine has a non-repairable critical part to operate. If the critical part fails, corresponding machine stops operating until the critical part is replaced. Failure probability increases with age of the critical part. We assume that the machine is as good as new after preventive replacement. The critical parts have a finite life time. End-of-life parts have to be replaced at the beginning of the periods. In order to ensure smoothness of the operations, spare parts are ordered and held in inventory. We consider a periodic review inventory model where the order lead time is assumed to be zero. The cost components that we consider are procurement, inventory holding, replacement, failure and shortage costs. Both preventive replacements and inventory decisions are made at discrete points in time. Decisions that we make in each period are order quantity and to replace or not decisions for each part.

We propose a Dynamic Programming formulation that minimizes the total expected cost over a finite planning horizon. However, it is hard to obtain the optimal solution for long planning horizons and systems with a large number of machines. Hence, we propose three heuristic approaches: (i) Myopic approach, (ii) Stationary policy, (iii) Steady state approximation.

The papers that are most relevant to our study are listed in Table 2.1. These studies consider a given policy for preventive maintenance and for spare part inventory planning and then optimize policy parameters such as preventive maintenance interval or age for preventive replacement. We do not impose any pre-determined preventive maintenance and inventory control policy as in the literature. As stated before, critical parts at any age can be replaced preventively. This is our major contribution to the literature. Furthermore, we propose three heuristic approaches that are easy to implement.

Table 2.1: Classification of Reviewed Studies

		System Characteristics		Maintenance Char.		Inventory Control Characteristics						
No	Study	Number of Units	Failure Rate	Policy	Maintenance Lead Time	Review Type	Num. of Units Allowed in Inv.	Procurement Cost	Shortage Cost	Emergency Order	Order Lead Times	Modeling/ Solution Approach
1	Acharya et al. [1]	Multi	Inc. with time	Block	No	Periodic	Multi	Yes	Yes	No	No	Analytic
2	Kabir et al. [11]	Single	Inc. with time	Age Based	No	Continuous	Multi	Yes	Yes	Yes	Random	Simulation
3	Kabir et al. [12]	Multi	Inc. with time	Age Based	No	Continuous	Multi	Yes	Yes	Yes	Random	Simulation
4	Kabir et al. [13]	Multi	Inc. with time	Age Based	No	Continuous	Multi	Yes	Yes	No	Random	Simulation
5	Kabir et al. [14]	Multi	Inc. with time	Age Based	No	Periodic	Multi	Yes	Yes	No	Random	Simulation
6	Armstrong et al. [2]	Single	Inc. with time	Age Based	No	Periodic	Single	No	Yes	No	Constant	Analytic
7	Armstrong et al. [3]	Single	Inc. with time	Age Based	Constant	Periodic	Single	No	Yes	Yes	Random	-
8	Elwany el al [7]	Single	Sensor-driven	Condition Based	No	Periodic	Single	No	Yes	No	Constant	Analytic
9	Sarker et al. [19]	Multi	Inc. with time	Block	Random	Continuous	Multi	Yes	Yes	Yes	Random	Simulation
10	Vaughan [20]	Multi	Constant	Condition Based	No	Continuous	Multi	Yes	Yes	No	No	Analytic (SDP)
11	Giri et al [9]	Single	Inc. with time	Corrective	No	Periodic	Single	Yes	Yes	Yes	Constant	Analytic
12	Hu et al. [10]	Multi	Inc. with time	Age Based	No	Continuous	Multi	Yes	Yes	No	Random	Simulation
13	Wang [22]	Multi	Constant	Condition Based	No	Periodic	Multi	Yes	No	Yes	Constant	Analytic (SDP)
14	Xu et al. [24]	Multi	Inc. with time	Group	No	Periodic	Multi	Yes	Yes	No	No	Simulation
15	Our Study	Multi	Inc. with time	-	No	Periodic	Multi	Yes	Yes	No	No	Analytic (DP)

CHAPTER 3

MODELS

In this chapter, we provide the mathematical models that we propose for the integrated maintenance and spare parts inventory planning. Section 3.1 provides the problem definition and assumptions. In Section 3.2, Dynamic Programming formulation is given. Section 3.3 covers three heuristic approaches that we propose: Myopic approach, Stationary policy and Steady-State approximation.

3.1 Problem Definition

We consider replacement and spare parts inventory planning in a manufacturing environment that consists of multiple machines. These machines are identical and they independently operate. Each includes a critical part. If the critical part fails, the corresponding machine does not function. The critical parts are non-repairable and they must be replaced when they fail. The failure probability increases with the age of the critical part. In order to ensure smoothness of the operations, spare parts are ordered and held in inventory. We consider a periodic review inventory model where the order lead time is assumed to be zero.

In such an environment, our aim is to characterize the optimal preventive replacement policy together with the optimal spare parts inventory policy that minimizes the total expected cost in a finite planning horizon. Total cost consists of five components: procurement, inventory holding, replacement, failure and shortage costs. The procurement, failure and replacement costs are incurred per unit basis whereas inventory holding and shortage costs are incurred per unit per period basis. There is no fixed order cost or quantity discounts for procurements.

Preventive replacement can be described as the replacement of a critical part in working condition to decrease its failure probability. The critical part and the corresponding machine are assumed to be as good as new after a replacement. Replacement time is negligible. At the end of each period, we need to decide which critical parts to replace at the beginning of next period. Such a replacement may be profitable as the expected cost from system downtime due to failures decreases when a new part is installed. When a critical part fails, it is replaced if a spare part is available. Otherwise, the corresponding machine has to wait until a spare part becomes available. The failures are detected immediately. If a critical part fails during a period and it is replaced, we assume that it will not fail in the same period again. The spare parts are ordered only at the end of the each period and order is received at the beginning of the next period. That is, order lead time is negligible. Emergency order is not allowed. We assume that all failed parts at the end of a period are to be replaced at the beginning of the next period.

The notation that we use throughout this study is provided in Table 3.1.

Table 3.1: Used Notation

Sets	
T	Length of the planning horizon
t	Index for periods, $t = 1, 2, \ldots, T$
M	Number of machines installed
i	Index for the critical parts that placed in machines, $i = 1, 2, \ldots, M$
Parameters	
N	Maximum age (life-time) of a critical part
c_s	Unit shortage cost per period
c_r	Unit replacement cost
c_f	Unit failure cost
c_p	Unit procurement cost
c_h	Unit inventory holding cost per period
$F_{i,t}$	Random variable representing whether part i fails in period t or not, $F_{i,t} \in \{0, 1\}$. 0 means that part i does not fail during period t and 1 means that it fails
K_t	Number of failed critical parts during period t, $K_t = \sum_{i \in M} F_{i,t}$
$p(a)$	Failure probability at age a

Used Notation (cont'd)

Parameters(cont'd)	
$P_{k_t}(\vec{A}_t)$	Probability that all k_t parts fail during period t for given ages $\vec{A}_t$ where $k_t \in \{0, 1, \ldots, M\}$
State Variables	
$A_{i,t}$	Age of part i at the beginning of period t before preventive replacements, $A_{i,t} = -1, 1, 2, \ldots, N$. Age -1 means that the corresponding part is failed in the previous period and it is not replaced.
$\vec{A}_t$	Vector of $A_{i,t}$'s, $\vec{A}_t$=($A_{1,t}$, $A_{2,t}$, ... , $A_{M,t}$)
$A^{'}_{i,t}$	Age of part i at the beginning of period t after preventive replacements, it is called in-period-age of the part, $A^{'}_{i,t} = 0, 1, \ldots, N-1$
$\vec{A}^{'}_t$	Vector of $A^{'}_{i,t}$'s, $\vec{A}^{'}_t$=($A^{'}_{1,t}$, $A^{'}_{2,t}$, ... , $A^{'}_{M,t}$)
I_t	Net inventory level at the beginning of period t before preventive replacements and before order is received, $I_t = -M, \ldots, M$
I_t^+	On hand inventory level at the beginning of period t before preventive replacements, I_t^+=$\max(I_t, 0)$
I_t^-	Backorder level at the beginning of period t, I_t^-=$\max(0, -I_t)$
$I^{'+}_t$	On hand inventory level at the beginning of period t after preventive replacements
Decision Variables	
Q_t	Order quantity that is placed at end of period $t-1$ to be received at the beginning of period t, $Q_t = 0, 1, \ldots, 2M$
$R_{i,t}$	Decision variable for replacement, $R_{i,t} \in \{0, 1\}$. 0 means that part i is not replaced at the beginning of period t and 1 means that it is replaced
$\vec{R}_t$	Vector of $R_{i,t}$'s, $\vec{R}_t$=($R_{1,t}$, $R_{2,t}$, ... , $R_{M,t}$)
Φ_t	Set of all possible decisions of replacements and order quantities at the end of the period $t-1$ for the period t
$f_t(I_t, \vec{A}_t)$	Minimum total expected cost from period t to T when the initial inventory level is I_t and the initial ages of the critical parts are given by $\vec{A}_t$

In each period, the sequence of events occur as Figure 3.1.

1. At the beginning of the period, the replenishment order placed at the end of

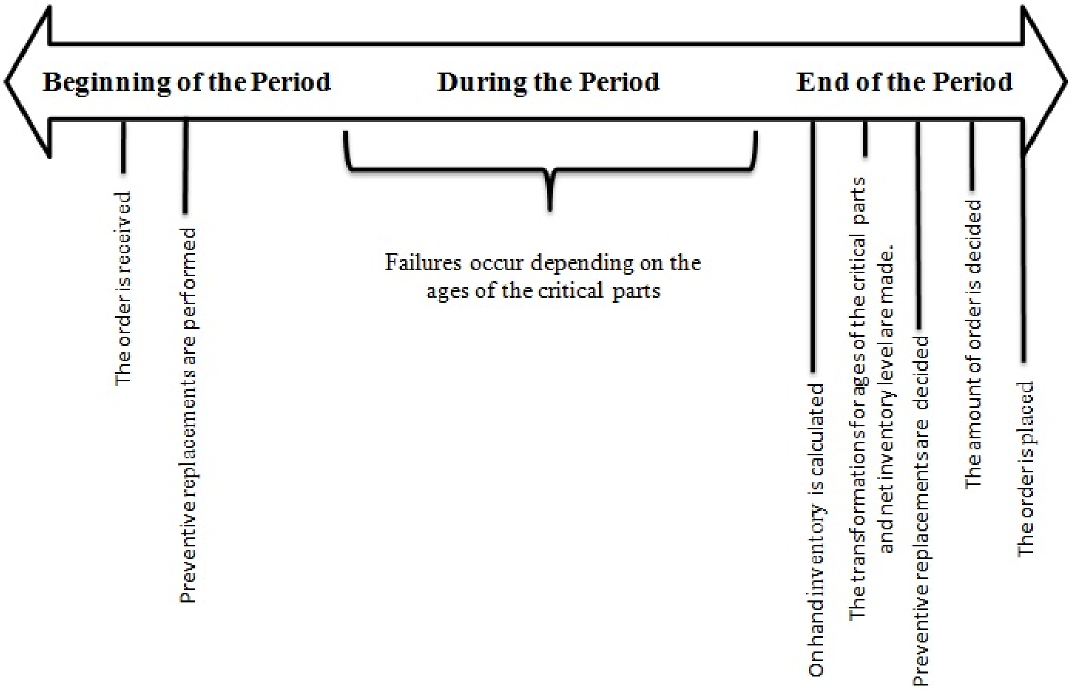

Figure 3.1: Sequence Of Events

previous period is received and preventive replacements that are determined at the end of the previous period are performed.

2. Procurement and replacement costs are incurred.

3. During the period, failures occur depending on the ages of the critical parts and corresponding failure costs are incurred.

4. If available inventory is sufficient when a critical part fails, it is replaced and a replacement cost is incurred. Otherwise, the part is scheduled to be replaced at the beginning of the next period and a shortage cost is incurred. Unit shortage cost is assumed to be greater than or equal to the sum of unit procurement cost and unit replacement cost. If a spare part is available, a failed part will definitely be replaced. That is, the system is not allowed to have shortage and positive inventory at the same time.

5. At the end of the period, on hand inventory is calculated inventory holding cost is incurred.

6. The transformations for ages of the critical parts and net inventory level are

made.

7. Preventive replacements are determined at the end of the period. The critical parts that complete their lifetimes N have to be replaced. Hence all parts at age N have to be scheduled to be replaced at the beginning of the next period.

8. The number of spare parts to order is determined and the order is placed. The order quantity includes the spare parts for preventive replacements at the beginning of the next period and on hand inventory level for the next period.

3.2 Dynamic Programming Formulation

Considering the problem environment described in Section 3.1, our objective is to minimize the total expected cost in a finite planning horizon. For this purpose, we first introduce a Dynamic Programming (DP) formulation to determine the order quantity and replacements in each period in the planning horizon. Note that this formulation does not impose any restriction on either the replacement policy or inventory policy.

Stages, decision and state variables of DP are defined as follows:

Stages: Time periods in the planning horizon, $t = 1, 2, \ldots, T$.

Decision Variables: Whether to replace or not for each critical part at the beginning of period t, $R_{i,t}$, and how much to order, Q_t, for period t.

State Variables and Transformations: The state variable is a vector including the net inventory level at the beginning of the period and the age vector of the critical parts. From one period to the next, net inventory evolves as follows:

$$I_{t+1} = I_t + Q_t - \sum_{\substack{i \in M \\ A_{i,t} \neq -1}} R_{i,t} - \sum_{i \in M} F_{i,t} \tag{3.1}$$

If on hand inventory after preventive replacements is not enough to cover the failures during the period, $\sum_{i \in M} F_{i,t} > \acute{I}_t^{+}$, there is a shortage. In this case, net inventory level at the beginning of the next period will be less than zero. Since preventive replacement decisions, $\sum_{i \in M} R_{i,t}$, already include failed critical parts, the replacement decisions of failed critical parts, $A_{i,t} = -1$, are extracted from the term $\sum_{i \in M} R_{i,t}$

when calculating net inventory level at the beginning of the next period before preventive replacements and order is received.

Other component of the state vector is the ages of the critical parts at the beginning of the period before preventive replacements, $\vec{A}_t$. Transitions from one period to the next are governed by decisions and realization of random events during that period (see Figure 3.1). If critical part i is not replaced at the beginning of period t ($R_{i,t} = 0$), and it does not fail during period ($F_{i,t} = 0$), its age increases by one. If part i fails ($F_{i,t} = 1$) during period and a spare part is available ($\acute{I}_t^+ \geq \sum_{k_t=1}^{i} F_{k_t,t}$), then, it is replaced and its age is 1 at the beginning of the next period. Otherwise, its age will be -1 at the beginning of the next period. Age transitions are summarized in Figure 3.2.

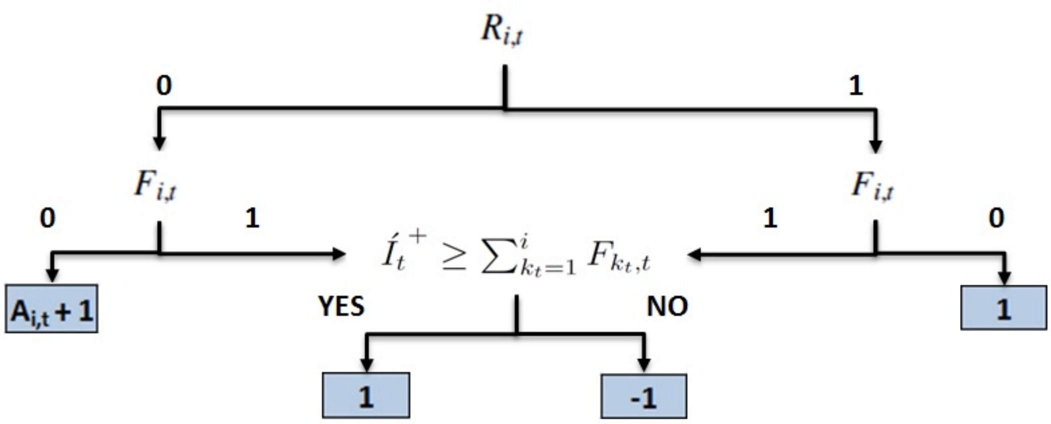

Figure 3.2: Ages Transformation of The Critical Parts

Recursive Function: Recursive function is the minimum total expected cost from period t to period T when inventory level is I_t and ages of critical parts are given by $\vec{A}_t$. The objective in DP is to find the minimum total expected cost from the first period to end of the planning horizon, $f_1(I_1, \vec{A}_1)$.

$$
f_t(I_t, \vec{A_t}) = \min_{\vec{R_t}, Q_t \in \Phi_t} \left\{ \begin{array}{l} c_r \sum_{i \in M} R_{i,t} + c_p Q_t + c_f \sum_{k_t=1}^{M} k_t P_{k_t}(\vec{\acute{A}_t}) \\ +c_r(\sum_{k_t=1}^{\acute{I_t}^+} k_t P_{k_t}(\vec{\acute{A}_t}) + \sum_{k_t=\acute{I_t}^+}^{M} \acute{I_t}^+ P_{k_t}(\vec{\acute{A}_t})) \\ +c_s \sum_{k_t=\acute{I_t}^+}^{M} (k_t - \acute{I_t}^+) P_{k_t}(\vec{\acute{A}_t}) + c_h \sum_{k_t=1}^{\acute{I_t}^+} (\acute{I_t^+} - k_t) P_{k_t}(\vec{\acute{A}_t}) \\ +E[f_{t+1}(I_{t+1}, \vec{A_{t+1}})] \end{array} \right\}
$$

$$
\forall t \in \{1, \cdots, T\} \tag{3.2}
$$

where Φ_t is defined by the following constraints:

$$
R_{i,t} \geq -\min\{A_{i,t}, 0\} \qquad \forall i \in M, t \in T \tag{3.3}
$$

$$
R_{i,t} \geq A_{i,t} - (N - 1) \qquad \forall i \in M, t \in T \tag{3.4}
$$

$$
\acute{A}_{i,t} = A_{i,t}(1 - R_{i,t}) \qquad \forall i \in M, t \in T \tag{3.5}
$$

$$
\acute{I_t}^+ = I_t + Q_t - \sum_{\substack{i \in M \\ A_{i,t} \neq -1}} R_{i,t} \qquad \forall i \in M, t \in T \tag{3.6}
$$

$$
R_{i,t} \in \{0, 1\} \qquad \forall i \in M, t \in T \tag{3.7}
$$

$$
Q_t \geq 0 \qquad \forall t \in T \tag{3.8}
$$

$$
\acute{I_t}^+ \geq 0 \qquad \forall t \in T \tag{3.9}
$$

The first term in the recursive function (3.2) calculates the cost of preventive replacements. The second term is for the cost of procurements. The third term corresponds to the expected total failure cost of the current period. Next three terms are the expected costs for replacements of failed critical parts, shortages and inventory holdings for the current period. The last term in the recursive function is for the total expected cost from period $t + 1$ to period T.

Recall that the term $P_{k_t}(\vec{\acute{A}_t})$ is the probability that exactly k out of M parts fail during period t when the ages of the critical parts after preventive replacement activities are given by $\vec{A_t}$, as described in Table 3.1. Since critical parts at different ages have non-identical probability of failure, the number of the parts that fail in a period, k_t, is the sum of nonidentical and independent Bernoulli random variables, $F_{i,t}$. We enumerate all possible values k_t's and calculate the associated probability in the numerical experiments as follows:

$$P_{k_t}(\vec{A_t}) = \sum_{\forall M_l^{k_t} \in L^{k_t}} \Big(\prod_{i \in M_l^{k_t}} p(A_{i,t}) \Big) \Big(\prod_{j \in (M - M_l^{k_t})} (1 - p(A_{i,t})) \Big)$$

where L^{k_t} is the set of all size k subset of M and $M_l^{k_t} \in L^{k_t}$.

The feasible region for the decision variables is described by constraints (3.3) to (3.9). Constraint (3.3) forces the failed units in the previous period to be replaced at the beginning of the period. Constraint (3.4) guarantees that critical parts at the end of their lifetimes are replaced. Constraint (3.5) is to find the set of in-period-ages of the critical parts after preventive replacement activities. The set of in-period-ages is required since the age of a critical part can change after preventive replacements and the failure probability of a critical part depends on its age. By Constraint (3.6), on hand inventory after preventive replacements is calculated. Since net inventory level at the beginning of the period, I_t, is less than zero if there are failed critical parts at the end of previous period and preventive replacement decisions, $\sum_{i \in M} R_{i,t}$, already include failed critical parts, the replacement decisions of failed critical parts, $A_{i,t} = -1$, are extracted from the term $\sum_{i \in M} R_{i,t}$ when calculating on hand inventory after preventive replacements.

If there are failed critical parts at the end of the planning horizon, if net inventory level at the end of the planning horizon is less than zero, they are replaced by placing a final order. If there are left-overs in inventory, they are sold at the original unit procurement cost. Therefore, the boundary condition for the recursive function can be presented as follows:

$$f_{T+1}(I_{T+1}, \vec{A_{T+1}}) = (c_r + c_p) I_{T+1}^- - c_p I_{T+1}^+ \tag{3.10}$$

3.3 Heuristic Approaches

The DP formulation presented in Section 3.2 seeks the optimal solution to the integrated maintenance and inventory planning problem without imposing any restrictions on the policies. However, it is time-consuming to apply the optimal solution for long planning horizons and for a system with a large number of machines. Run times and number of states for DP with different number of machines installed and lengths of planning horizon are in Table 3.2. Because of this computational intractability of the DP formulation, we develop three heuristic models; (i) Myopic approach, (ii) Stationary policy and (iii) Steady-State approximation, that are discussed in Section 3.3.1, 3.3.2 and 3.3.3, respectively.

Table 3.2: Run Time and Number of States for DP with Different M and T

M	T	Number of States	Run Time (Minutes)
2	5	86	0.0242
2	10	86	0.0542
2	15	86	0.0860
3	5	591	0.8234
3	10	591	1.9161
3	15	591	3.0641
4	5	3796	27.3319
4	10	3796	67.8662
4	15	3796	116.6422
5	5	23401	964.1517
5	10	23401	2486.6591
5	15	23401	4027.5571

3.3.1 Myopic Approach

Myopic approach solves a single period problem with the same terminating conditions; that is selling on hand inventory at the original unit procurement cost, procurements of required amount of spare parts for replacements of failed critical parts and replacements of these parts at the end of the period. It ignores the impacts of the current decisions on future events and expected costs, and focuses only on minimizing the total expected cost of the immediate period. Decisions depend on only the state of the current period. That is, MA gives the same solution for the same state regardless of the period within the planning horizon. Since there is no penalty cost for ordering more than needed other than holding cost, it is expected that more procurements are made under this approach. This heuristic is easy to implement for systems with a large number of machines and for long planning horizons. Actually length of the planning horizon does not affect complexity of Myopic approach formulation since it solves only one period problem.

The expected cost to minimize in this approach can be constructed by incorporating the terminating conditions in Equation (3.10) into the DP formulation in Equation (3.2):

$$\min_{\vec{R}_t, Q_t \in \Phi_t} G(\vec{R}_t, Q_t) = \left\{ \begin{aligned} & c_r \sum_{i \in M} R_{i,t} + c_p Q_t + c_f \sum_{k_t=1}^{M} k_t P_{k_t}(\vec{A}_t) \\ & + c_r \Big(\sum_{k_t=1}^{\acute{I}_t^+} k_t P_{k_t}(\vec{A}_t) + \sum_{k_t=\acute{I}_t^+}^{M} \acute{I}_t^+ P_{k_t}(\vec{A}_t) \Big) \\ & + (c_h - c_p) \sum_{k_t=1}^{\acute{I}_t^+} (\acute{I}_t^+ - k_t) P_{k_t}(\vec{A}_t) \\ & + (c_s + c_p + c_r) \sum_{k_t=\acute{I}_t^+}^{M} (k_t - \acute{I}_t^+) P_{k_t}(\vec{A}_t) \end{aligned} \right\} \tag{3.11}$$

subject to Φ_t

Note that the constraint set is the same as the one in DP formulation.

The problem described in Equation (3.11) only delivers the optimal solution to a single period. To calculate the corresponding cost of the entire planning horizon, one needs to solve (3.11) for all possible states, then the recursive function of the DP (3.2) is evaluated with the decisions of the Myopic due to state of the periods within the planning horizon.

3.3.2 Stationary Policy

In this policy, we restrict our attention to the age-based preventive maintenance and an order up-to level inventory control policy. Such a policy can be characterized by two policy parameters as follows:

$\acute{S}$	On hand inventory at the beginning of a period after preventive replacements
AL_R	The age limit for age-based replacement

Under this policy, at the end of each period, all machines with critical parts that are older than AL_R are planned to be replaced and an order to raise the inventory level after the preventive replacements to $\acute{S}$ is placed.

Note that $\acute{S}$ should be less than M since lead times are ignored and we assume that a part that is replaced correctively does not fail again in the same period. Similarly, the age limit for preventive replacements, AL_R ,should be within the interval [1,N] due to assumption that parts with an age of N should be replaced.

In order to find the optimal ($\acute{S}$, AL_R) pair, we search all possible combinations of ($\acute{S}$, AL_R) pairs within the ranges $\acute{S} \in \{0, \cdots, M\}$, $AL_R \in \{1, \cdots, N\}$. Generating all possible states and using the recursive function given (3.2), we select the best pair. Note that while using (3.2), the decisions are set such that $R_{i,t} = 1$ if $A_{i,t} \geq AL_R$ otherwise $R_{i,t} = 0$ and $Q_t = \acute{S} - I_t^+ + \sum_{i \in I} R_{i,t}$.

Under this policy, we restrict amount of on hand inventory for all periods to be the same. However it is intuitively appealing, easy to implement and gives good results for stable environments. On the other hand, it is computationally intractable like the DP formulation as for all generated policies, the recursive function of DP formulation is solved then the optimal policy is determined.

3.3.3 Steady-State Approximation – A Markov Decision Process Model

The third heuristic approach is based on approximating the finite horizon problem by an infinite horizon version. Additionally, we consider a single machine problem and extend its solution to multi-machine case ignoring inventory pooling opportunity. We expect that this approximation will give good results for long planning horizon problems since decisions are made under infinite horizon. However deciding inventory level for each machine yields overstock. A single machine problem over infinite planning horizon can be modeled as a Markov Decision Process (MDP).

We make decisions based on two state variables; (i) the inventory level at the beginning of the period, I, and (ii) the age of the critical part, A. The age of a part can be from -1 to N except 0 at the beginning of the each period and the inventory level at the beginning of the period can be -1, 0 and 1. The state space of the period has a form of (I, A).

There are two actions for all possible states: (i) whether to replace or not, R, and (ii) whether to hold a spare part for corrective replacement during the period, S. R can be 0 or 1; 0 means that the part is not replaced and 1 means that it is replaced. Again S can be 0 or 1. 0 means that an extra spare part is not held for the part and 1 means that an extra spare part for possible corrective replacements during the period is held. The courses of actions have the form (R, S). There are 4 different possible actions for a critical part; (i) $(1, 0)$, replace but not hold a spare part, (ii) $(1, 1)$, replace and hold a spare part, (iii) $(0, 0)$, not replace and not hold a spare part and (iv) $(0, 1)$, not replace but hold a spare part.

In this approach, a probability transition matrix is constructed for each possible actions. Under each possible action, the probability transition matrix is different since with the preventive replacements, the age of any critical part reduces to zero and the failure probability of the critical part depends on the age of that part. In addition, the next period age of a critical part and the inventory level of the next period depend on the availability of a spare part during the period. That is, if the critical part fails and a spare part is available, it is replaced and the age of this part is 1, the inventory level is 0 at the beginning of the next period. Else if, its age and the inventory level are -1. If the critical part does not fail and there is spare part on stock, its age increases by 1 and the inventory level is 1.

Linear programming for this MDP is constructed as follows [23]:

Sets

J		State space as a form of (I, A), $j = (-1,-1),(0,1),\cdots,(0,N),(1,1),\cdots,(1,N)$
D		Action space as a form of (R, S), $D = (1,0),(1,1),(0,0),(0,1)$

Parameters

$p(A)$		Failure probability of a critical part if its age is A
$c_j(d)$	$j \in J, d \in D$	Cost of the period which has the state variables as in state j under action d
$p_{j,k}(d)$	$j, k \in J, d \in D$	Probability that the state will be k in the next period while it is j in the current period under action d

Variables

$x_j(d)$	$j \in J, d \in D$	Steady-state probability of being in state j and taking action d

Objective function

$$\text{Minimize} \quad z = \sum_{d \in D} \sum_{j \in J} c_j(d) x_j(d) \tag{3.12}$$

$$\text{subject to}: \sum_{d \in D} x_k(d) = \sum_{e \in D} \sum_{j \in J} x_j(d) p_{j,k}(e) \qquad \forall k \in J \tag{3.13}$$

$$\sum_{d \in D} \sum_{j \in J} x_j(d) = 1 \tag{3.14}$$

$$x_j(d) \geq 0 \qquad \forall j \in J, d \in D \tag{3.15}$$

Costs of a period under any action consist of preventive replacement, procurement, expected failure, shortage and inventory holding costs. The period costs are calculated based on the state of the period and the replacement and inventory holding decisions of that action and the expected failure, shortage and inventory holding costs. The calculation of the costs, $c_j(d)$, and transition probabilities are follows:

- State (-1,-1)
  - Action (1,0): Under this action, next state is $(-1,-1)$ with probability $p(0)$ and $(0,1)$ with probability $1-p(0)$.

$$c_{(-1,-1)}(1,0) = c_p + c_r + p(0) \times (c_f + c_s)$$

  - Action (1,1): Under this action, next state is $(0,1)$ with probability $p(0)$ and $(1,1)$ with probability $1-p(0)$.

$$c_{(-1,-1)}(1,1) = 2c_p + c_r + p(0) \times (c_f + c_r) + (1-p(0)) \times c_h$$

  - Action (0,0): This action is not feasible for this state.

$$c_{(-1,-1)}(0,0) = M$$

  - Action (0,1): This action is not feasible for this state.

$$c_{(-1,-1)}(0,1) = M$$

- State (0,A)
  - Action (1,0): Under this action, next state is $(-1,-1)$ with probability $p(0)$ and $(0,1)$ with probability $1-p(0)$.

$$c_{(0,A)}(1,0) = c_p + c_r + p(0) \times (c_f + c_s), \qquad A = 1,\cdots,N$$

  - Action (1,1): Under this action, next state is $(0,1)$ with probability $p(0)$ and $(1,1)$ with probability $1-p(0)$.

$$c_{(0,A)}(1,1) = 2c_p + c_r + p(0) \times (c_f + c_r) + (1-p(0)) \times c_h, \qquad A = 1,\cdots,N$$

  - Action (0,0): Under this action, next state is $(-1,-1)$ with probability $p(A)$ and $(0,A+1)$ with probability $1-p(A)$. If $A=N$, this action is not feasible.

$$c_{(0,A)}(0,0) = p(A) \times (c_f + c_s), \qquad A = 1,\cdots,N-1$$
$$c_{(0,N)}(0,0) = M$$

– Action (0,1): Under this action, next state is $(0, 1)$ with probability $p(A)$ and $(1, A + 1)$ with probability $1 - p(A)$. If $A = N$, this action is not feasible.

$$c_{(0,A)}(0,1) = c_p + p(A) \times (c_f + c_r) + (1 - p(A)) \times c_h, \quad A = 1, \cdots, N-1$$

$$c_{(0,N)}(0,1) = M$$

- State (1,A)

  – Action (1,0): Under this action, next state is $(-1, -1)$ with probability $p(0)$ and $(0, 1)$ with probability $1 - p(0)$.

$$c_{(1,A)}(1,0) = c_r + p(0) \times (c_f + c_s), \qquad A = 1, \cdots, N$$

  – Action (1,1): Under this action, next state is $(0, 1)$ with probability $p(0)$ and $(1, 1)$ with probability $1 - p(0)$.

$$c_{(1,A)}(1,1) = c_p + c_r + p(0) \times (c_f + c_r) + (1 - p(0)) \times c_h, \quad A = 1, \cdots, N$$

  – Action (0,0): This action is not feasible for this state.

$$c_{(1,A)}(0,0) = M, \qquad A = 1, \cdots, N-1$$

  – Action (0,1): Under this action, next state is $(-1, -1)$ with probability $p(A)$ and $(0, A + 1)$ with probability $1 - p(A)$. If $A = N$, this action is not feasible.

$$c_{(1,A)}(0,1) = p(A) \times (c_f + c_r) + (1 - p(A)) \times c_h, \quad A = 1, \cdots, N-1$$

$$c_{(0,N)}(0,1) = M$$

where M is a sufficiently large number .

Constraint (3.13) is steady-state balance equation and (3.14) ensures that the total probability equals to 1. Constraint (3.15) is the non-negativity constraint for the probabilities. If MDP gives $x_j(d)$ > 0, the action d is decided to take for state j.

Under this heuristic, the decision variables are whether to replace or not and whether to hold a spare part or not for corrective replacement. The decisions depend on the age of the critical part and the inventory level of the period. Since the failure rate of a critical part is increasing with its age, it is certain that a part is not replaced at preventive replacement activities if there is an older part that is not planned to be replaced. Likewise, an extra spare part is not held for a younger part if there is an older part for which no spare is held. A noteworthy point is that if a critical part is replaced then its age will be zero. If this heuristic decides to replace and to hold an extra spare part for an age, it decides to hold a spare part for a 0-aged critical part and the age limit to hold spare part will be 0. Hence, we can represent this policy with two variables: (i) the age limit to hold spare part, AL_S, and (ii) the age limit to be replaced at preventive replacement activities, AL_R.

After finding the optimal pair of (AL_S, AL_R), the recursive function given (3.2) is evaluated to calculate the entire planning horizon cost for comparison. Note that while using (3.2), the decisions are set such that $R_{i,t} = 1$ if $A_{i,t} \geq AL_R$ otherwise $R_{i,t} = 0$ and $Q_t = \acute{I}_t^+ - I_t^+ + \sum_{i \in I} R_{i,t}$ where the $\acute{I}_t^+$ is equal to the number of the critical parts that are older then AL_S.

In this approach, on hand inventory amount is evaluated by the age limit to hold spare part. Therefore, differently from Stationary policy, on hand inventory at the beginning of a period after preventive replacements can be different under different states.

CHAPTER 4

COMPUTATIONAL STUDY

The main objective of the computational study is to assess the performances of the heuristic approaches under different problem parameters with respect to the solution quality. The main performance measure used in evaluating the performances of the heuristics is percent cost deviation from the optimal cost. For a given problem instance, percent cost deviation from the optimal, $\%\Delta$, under a certain heuristic can be expressed as:

$$\%\Delta = \frac{Z - Z^*}{Z^*} \times 100$$

where Z and Z^* are the objective function values under the heuristic and the optimal solution, respectively.

Our computational study consists of two parts. First we investigate the main effects of the parameters on $\%\Delta$ values of the heuristic approaches by performing sensitivity analysis. Then, a full factorial experiment is performed in order to investigate the parameter instances that give the best and worst performances of the heuristic approaches.

As stated in Chapter 3, the failure probability of a critical part increases with its age. We consider convex increasing failure probability for the computational study as given in Equation (4.1). With this formulation, brand new critical parts, $A_{i,t} = 0$, also have a positive failure probability.

$$p(A_{i,t}) = \frac{1}{(N+1) - A_{i,t}} \quad (4.1)$$

The algorithms to find maintenance and inventory policies are implemented in Microsoft Visual $C\sharp$ and the computational study is carried out on an Intel(R) Core(TM) i7-3537U CPU@2.00GHz processor and 8 GB of RAMS computer.

The notation for the problem parameters used in this study is provided in Table 4.1.

Table 4.1: The Problem Parameters

Parameters	Notation
Unit Shortage Cost	c_s
Unit Failure Cost	c_f
Unit Replacement Cost	c_r
Unit Procurement Cost	c_p
Unit Inventory Holding Cost	c_h
Lifetime of The Critical Parts	N
Length of The Planning Horizon	T
Number of Critical Parts Installed	M
Initial Ages of The Critical Parts	$\vec{A}_1$
Initial Inventory Level	I_1

Throughout this chapter, we will use the following abbreviation for the solution approaches.

- DP : Dynamic Programming Formulation
- MA : Myopic Approach
- SP : Stationary Policy
- SSA : Steady-State Approximation

The rest of the chapter is organized as follows: we discuss the results of the sensitivity analysis in Section 4.1. In Section 4.2, findings from full factorial experiment are discussed. Section 4.3 concludes this chapter with a general evaluation of the heuristic approaches.

4.1 Sensitivity Analysis

To perform sensitivity analysis, we first construct a base scenario assigning values for all parameters. Then, to analyze the effects of an individual parameter, we change the value of that parameter and keep the values of other parameters unvaried as in the base scenario. The values of problem parameters under the base scenario are given in Table 4.2.

Table 4.2: Parameter Values of the Base Scenario

c_s	c_f	c_r	c_p	c_h	N	T	M	$\vec{A}_1$	I_1
50	10	3	5	1	5	10	3	(2,3,4)	0

Under DP formulation, decisions are made depending on the state variables and the period. Thus, at all stages or under different states, the decisions under DP can change. That is, the optimal solution under DP cannot be represented by a well-defined policy. In MA, we solve a single period problem so it does not take the length of the planning horizon into account. The decisions under MA are made depending on the state variables. That is, the decisions can change for different state variables although they are the same for different stages. Hence, we cannot report well-defined policies under DP formulation and MA.

Under SP, we restrict ourselves to a policy with two stationary parameters: (i) amount of on hand inventory at the beginning of a period after preventive replacements, $\acute{S}$, and (ii) age limit to be replaced, AL_R. When presenting results, we represent the policy under SSA with two variables: (i) the age limit to hold spare part, AL_S, and (ii) the age limit to be replaced at preventive replacement activities, AL_R. Decisions in each period are made corresponding to the policy parameters and state variables of the periods under these two approaches. With the age limit to be replaced, which critical parts to be replaced are decided. After determining the preventive replacements, order quantity is calculated by taking into account the policy parameters $\acute{S}$ under SP and AL_S under SSA.

While presenting the results of the computational study, we report the following: (i) the objective function values of DP and the heuristic approaches, (ii) percent cost deviations of three heuristic approaches, $\%\Delta$, (iii) the best policies under SP and SSA.

4.1.1 Unit Shortage Cost per Period, c_s

Recall that if there is no on hand inventory when a critical part fails, the failed machine waits until the beginning of the next period and a shortage cost is incurred. For shortages, the required spare parts are ordered at the end of the period and the order is received at the beginning of the next period since order lead time is assumed to be zero. To analyze the effects of the unit shortage cost, its value is changed between 10 and 500 and other parameters are kept unvaried as in the base scenario. The summary of the results for the problem instances considered are provided in Table 4.3.

Table 4.3: Summary of Results for Unit Shortage Cost Analysis

	DP	Myopic		Stationary			Steady-State		
c_s	f_1	f_1	$\%\Delta$	Policy $(\acute{S}, AL_R)$	f_1	$\%\Delta$	Policy (AL_S, AL_R)	f_1	$\%\Delta$
10	181.0	184.2	1.79	2;4	182.1	0.65	0;4	190.9	5.51
20	182.6	186.4	2.09	2;4	183.5	0.48	0;4	190.9	4.58
30	183.9	188.2	2.32	2;4	184.8	0.46	0;4	190.9	3.82
40	185.2	189.4	2.25	2;4	186.1	0.48	0;4	190.9	3.11
50	186.3	190.2	2.71	2;4	187.4	1.19	0;4	190.9	2.48
70	188.1	191.2	2.63	2;4	190.0	1.98	0;4	190.9	1.50
90	189.1	191.8	1.94	3;4	190.9	1.50	0;4	190.9	0.96
110	189.6	192.0	1.54	3;4	190.9	0.96	0;4	190.9	0.73
130	189.8	192.1	1.36	3;4	190.9	0.73	0;4	190.9	0.62
150	189.9	192.2	1.27	3;4	190.9	0.62	0;4	190.9	0.57
200	189.9	192.2	1.22	3;4	190.9	0.57	0;4	190.9	0.56
250	189.9	192.2	1.21	3;4	190.9	0.56	0;4	190.9	0.56
300	189.9	192.2	1.21	3;4	190.9	0.56	0;4	190.9	0.56
400	189.9	192.2	1.21	3;4	190.9	0.56	0;4	190.9	0.56
500	189.9	192.2	1.21	3;4	190.9	0.56	0;4	190.9	0.56

Our observations with respect to an increase in c_s are as follows:

- Under SP, the amount of on hand inventory after preventive replacements, $\acute{S}$, increases whereas the age limit to be replaced, AL_R, does not change. Under SSA, inventory is held for all machines even if c_s is low and the policy does not change as c_s gets larger. That is, SSA holds more spare parts compared to other approaches since it does not consider the benefits of inventory pooling. Hence it performs worse when c_s is small and its performance improves quickly as c_s gets larger. In addition, the total expected cost of SSA does not change (Table 4.3) since there is no shortages. The policies under SP and SSA are exactly the same when $c_s \geq 90$. Under SP, the age limit to be replaced is 4 and the spare parts are held as many as the number of the installed machines. Again under SSA, the age limit to be replaced is 4 and the spare parts are held for all aged critical parts. Hence, they give same objective function value.
- When c_s is considerably high, a spare part is held for each critical part under

all approaches to decrease possible shortages. Hence the heuristics come closer to each other in terms of their $\%\Delta$ values (Figure 4.1). The total expected cost values, f_1, of the all approaches does not change after some constant values of c_s while c_s increases (Table 4.3). The reason is that by holding a spare for all machines, it makes sure that stock out does not occur so an increment in c_s does not affect the total expected cost.

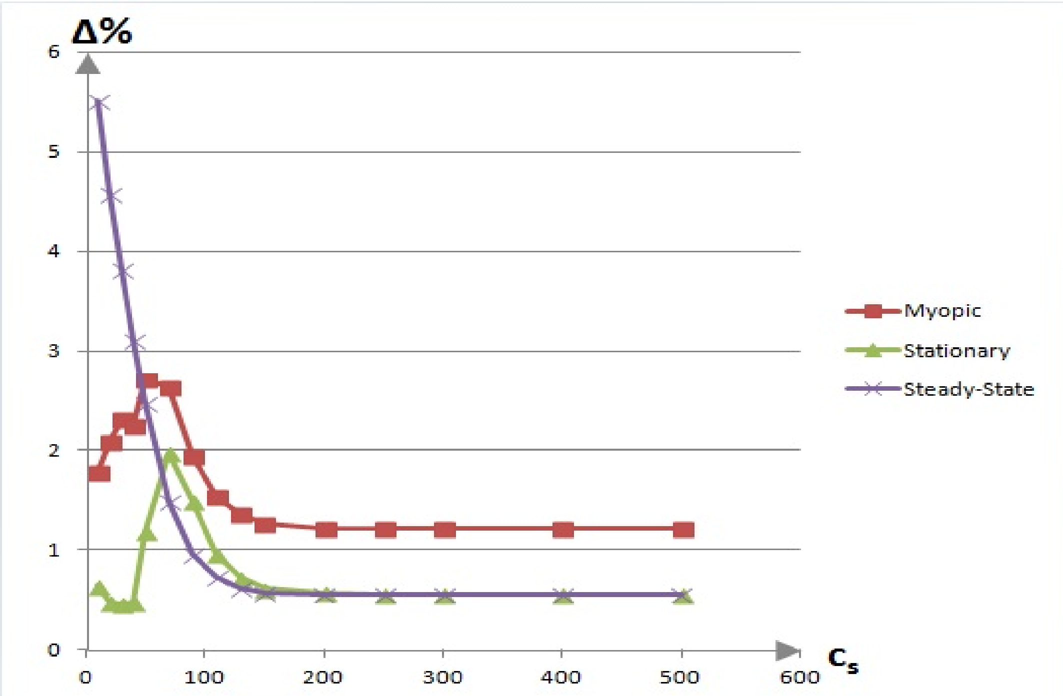

Figure 4.1: % cost deviations when c_s increases

4.1.2 Unit Failure Cost, c_f

The failure probability of a critical part increases with its age. When a critical part fails, a unit failure cost, c_f, is incurred. To analyze the effects of the unit failure cost, its value is changed between 5 and 50 and other parameters are kept unvaried as in the base scenario. The summary of the results for the problem instances considered are provided in Table 4.4.

Table 4.4: Summary of Results for Unit Failure Cost Analysis

	DP	Myopic		Stationary			Steady-State		
c_f	f_1	f_1	$\%\Delta$	Policy $(\acute{S}, AL_R)$	f_1	$\%\Delta$	Policy (AL_S, AL_R)	f_1	$\%\Delta$
5	147.5	147.6	0.06	3;5	149.5	1.38	0;5	149.5	1.38
10	186.3	190.2	2.08	2;4	187.4	0.57	0;4	190.9	2.48
15	222.2	232.8	4.78	2;4	222.9	0.33	0;4	226.5	1.95
20	256.7	258.3	0.63	2;4	258.5	0.70	0;4	262.1	2.11
30	321.0	329.4	2.64	2;3	322.5	0.48	0;3	328.2	2.25
40	383.8	400.6	4.37	2;3	384.7	0.25	0;3	390.5	1.74
50	445.3	447.0	0.37	2;3	447.0	0.37	0;3	452.7	1.66

Our observations with respect to an increase in c_f are as follows:

- With preventive replacements, age of the critical part installed on an machine is reset, as a result the failure probabilities decrease. Hence, in order to decrease the expected failures, the age limit to be replaced, AL_R, decreases under both SP and SSA (Table 4.4) as c_f increases.
- For all c_f values considered, SP performs better than the Steady-State approximation (Figure 4.2). When only $c_f = 5$ and $c_f = 20$, MA performs better than others. Although both SP and SSA restrict the decisions with two policy parameters, the policies under SSA is decided based on no inventory pooling and infinite horizon as opposed to the SP. Hence, we expect SP to perform better than SSA under most problem instances.

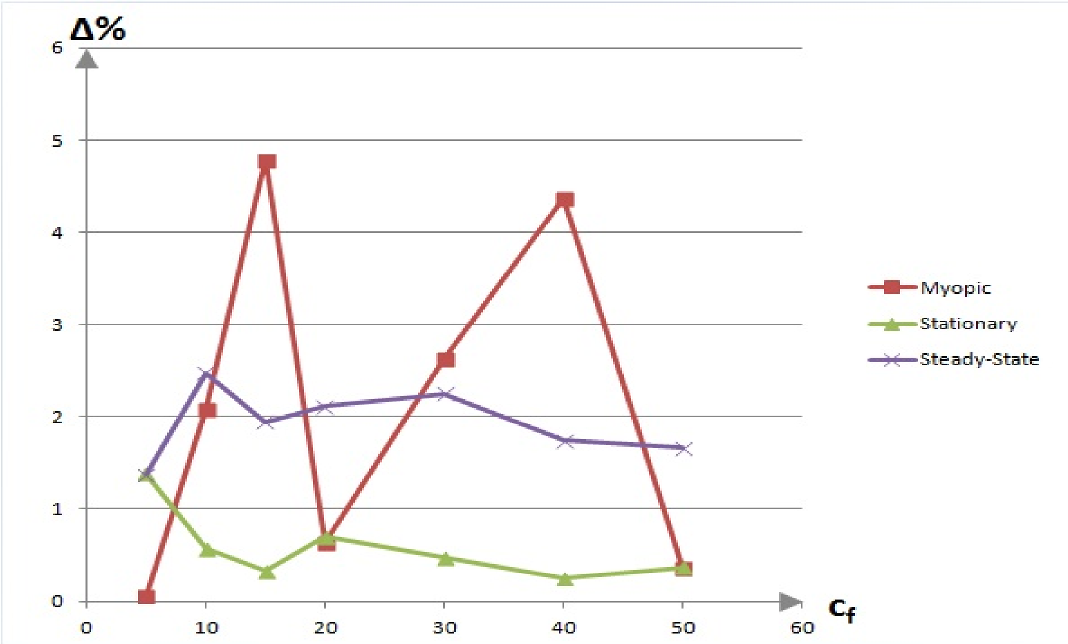

Figure 4.2: % cost deviations when c_f increases

4.1.3 Unit Replacement Cost, c_r

When a critical part is replaced preventively or correctively, a unit replacement cost is incurred. If you reduce the number of preventive replacements, the failure probability increases, i.e, need for corrective replacement increases. Hence, there is a trade off between preventive replacements and corrective replacements. To analyze the effects of the unit replacement cost, its value is changed between 0.5 and 10 and other parameters are kept unvaried as in the base scenario. The summary of the results for the problem instances considered are provided in Table 4.5.

Table 4.5: Summary of Results for Unit Replacement Cost Analysis

	DP	Myopic		Stationary			Steady-State		
c_r	f_1	f_1	$\%\Delta$	Policy $(\acute{S}, AL_R)$	f_1	$\%\Delta$	Policy (AL_S, AL_R)	f_1	$\%\Delta$
0.5	156.6	163.5	4.44	2;4	157.2	0.37	0;4	160.7	2.61
1	162.7	168.9	3.78	2;4	163.2	0.30	0;4	166.7	2.46
1.5	168.6	174.2	3.30	2;4	169.3	0.37	0;4	172.8	2.46
2	174.5	179.5	2.87	2;4	175.3	0.44	0;4	178.8	2.46
3	186.3	190.2	2.08	2;4	187.4	0.57	0;4	190.9	2.48
4	198.1	200.9	1.39	2;4	199.5	0.69	0;4	203.0	2.49
5	209.8	211.5	0.80	2;4	211.6	0.82	0;5	213.5	1.75
10	264.7	264.9	0.08	3;5	266.9	0.84	0;5	266.9	0.84

Our observations with respect to an increase in c_r are as follows:

- Recall that MA solves a single period problem ignoring the impacts of the current decisions on the future events and the expected costs of remaining periods. We observe that instead of performing preventive replacement and reducing the age of a critical part, under this policy, inventory is held for corrective replacement. To show in more detail, for different initial state variables, ie, the ages of the critical parts, we give the initial order quantity and preventive replacement decisions of DP and MA in Table 4.6. All other parameters are set as in the base scenario. Recall that the maximum lifetime of a critical part is 5 and a critical part must be replaced when it reaches to the end of its lifetime. Hence, the parts at the age of 5 under all approaches must be replaced at preventive replacement activities. As seen, under MA, no other preventive replacement is made while under the optimal policy there are preventive replacements for the critical parts that are of age 4. The performance of MA improves as c_r increases. When c_r is considerably high, its performance converges to that of the optimal policy (Figure 4.3).

Table 4.6: Initial Decisions of DP and MA Under Different Parameter Instances

	DP		Myopic	
$\vec{A_{i,1}}$	Q_1	$\vec{R_1}$	Q_1	$\vec{R_1}$
1;1;1	2	0;0;0	2	0;0;0
1;2;3	2	0;0;0	2	0;0;0
1;3;3	3	0;0;0	3	0;0;0
1;3;4	3	0;0;1	3	0;0;0
1;4;4	4	0;1;1	3	0;0;0
2;2;2	2	0;0;0	2	0;0;0
2;3;4	3	0;0;1	3	0;0;0
2;4;5	4	0;1;1	4	0;0;1
3;3;3	3	0;0;0	3	0;0;0
3;4;5	4	0;1;1	4	0;0;1
4;4;4	5	1;1;1	3	0;0;0
4;4;5	5	1;1;1	4	0;0;1

- The age limit to be replaced increases under both SP and SSA while c_r increases (Table 4.5). Under SP, on hand inventory after preventive replacements increases when c_r is considerably high whereas under SSA, a spare is kept for failures for all ages under all c_r values considered.

- SP performs better when c_r is low in contrast to the other heuristics (Figure 4.3). In addition, the policies under SP and SSA become similar when c_r is considerably high, they are exactly the same when $c_r = 10$. In this case, under SP, no preventive replacement is made and the spare parts are held as many as the number of installed machines. Similarly, under SSA, no preventive replacement is made for any age and spare parts are held for all aged critical parts. Hence, SP and SSA are identical when $c_r = 10$ in terms of both the objective function value and the decisions.

- Under SSA, the least number of preventive replacements are made when $c_r \geq 5$ (Table 4.5). Thus, the performance of SSA improves while c_r increases after that point.

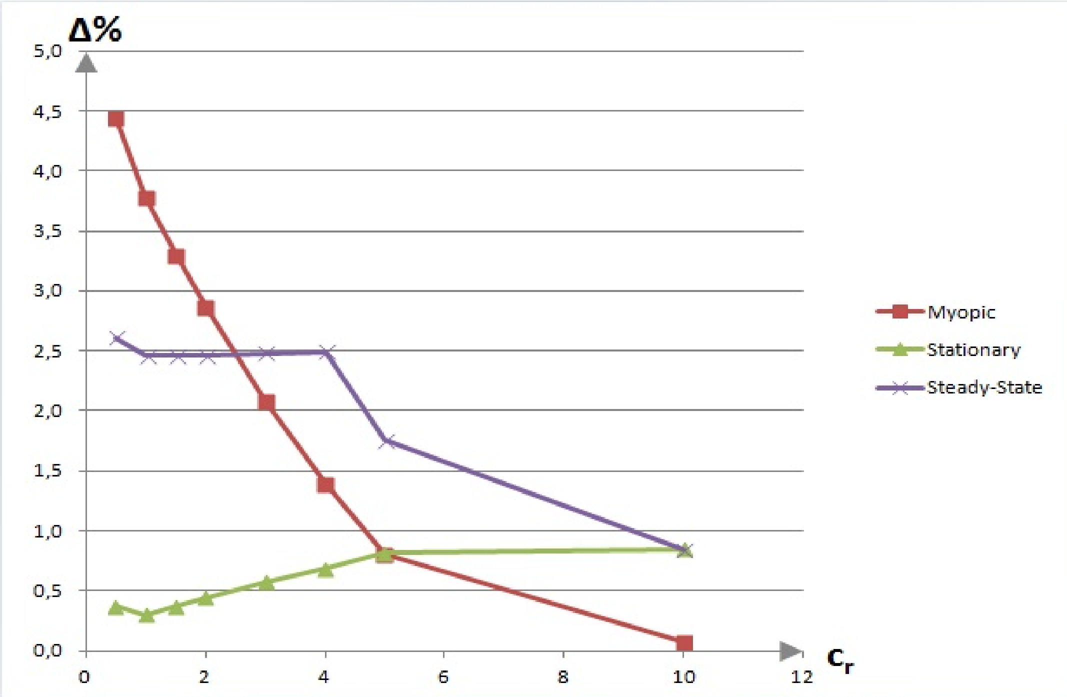

Figure 4.3: % cost deviations when c_r increases

4.1.4 Unit Procurement Cost, c_p

To analyze the effects of the unit procurement cost, its value is changed between 3 and 20 and other parameters are kept unvaried as in the base scenario. The summary of the results for the problem instances considered are provided in Table 4.7.

Table 4.7: Summary of Results for Unit Procurement Cost Analysis

	DP	Myopic		Stationary			Steady-State		
c_p	f_1	f_1	$\%\Delta$	Policy $(\acute{S}, AL_R)$	f_1	$\%\Delta$	Policy (AL_S, AL_R)	f_1	$\%\Delta$
3	162.7	168.9	3.78	2;4	163.2	0.30	0;4	166.7	2.46
5	186.3	190.2	2.08	2;4	187.4	0.57	0;4	190.9	2.48
7	209.8	211.5	0.80	2;4	211.6	0.82	0;5	213.5	1.75
10	243.1	243.5	0.19	3;5	245.5	1.02	0;5	245.5	1.02
15	296.8	296.9	0.01	3;5	298.9	0.70	0;5	298.9	0.70
20	350.1	350.2	0.01	3;5	352.3	0.61	0;5	352.3	0.61

Our observations with respect to an increase in c_p are as follows:

- While constructing MA, we treat the single period as the last period of the planning horizon. Since the left-overs are salvaged at the original unit procurement cost, the decisions under MA are not affected by c_p. In Table 4.8, we report different cost items in the total expected cost. As c_p increases, only total expected procurement cost increases, while the other cost items do not change.

Table 4.8: Total Expected Cost Items in Total Expected Cost of Myopic Approach for Unit Procurement Cost Analysis

	Myopic Approach					
c_p	Total Expected Inv. Holding Cost	Total Expected Procurement Cost	Total Expected Preventive Replacement Cost	Total Expected Failure Cost	Total Expected Failure Replacement Cost	Total Expected Shortage Cost
3	16.6	36.9	6.6	85.3	25.4	3.0
5	16.6	61.5	6.6	85.3	25.4	3.0
7	16.6	86.2	6.6	85.3	25.4	3.0
10	16.6	123.1	6.6	85.3	25.4	3.0
15	16.6	184.6	6.6	85.3	25.4	3.0
20	16.6	246.2	6.6	85.3	25.4	3.0

- As MA disregards the procurement cost, it fails to take advantage of low procurement costs. That is, its performance gets worse as c_p gets lower. As c_p gets larger, preventive replacement becomes more expensive and it becomes better to hold inventory for corrective replacement. Hence, the effects of disregarding the procurement cost diminish. When c_p is considerably high, MA gives almost the optimal solution. Such an approach becomes more favorable when c_p increases.

- As c_p gets larger, both the age limit to be replaced and on hand inventory after preventive replacements increases under SP. Since no preventive replacement is performed when c_p is high, the failure probabilities are high. SP prevents shortages by holding more spare parts. That is, as c_p gets large, inventory is held for corrective replacement instead of performing preventive replacement. Even though its performance gets worse under moderate c_p values in general, SP performs very close to the optimal solution for all values of c_p considered.

- As c_p gets larger, the age limit to be replaced increases under SSA like SP. However age limit to hold an extra spare part does not change when c_p gets higher since the policy under SSA holds already a spare part for all aged critical parts. Hence, the policies under SP and SSA becomes identical and the performance of SSA improves as c_p gets larger (Figure 4.4).

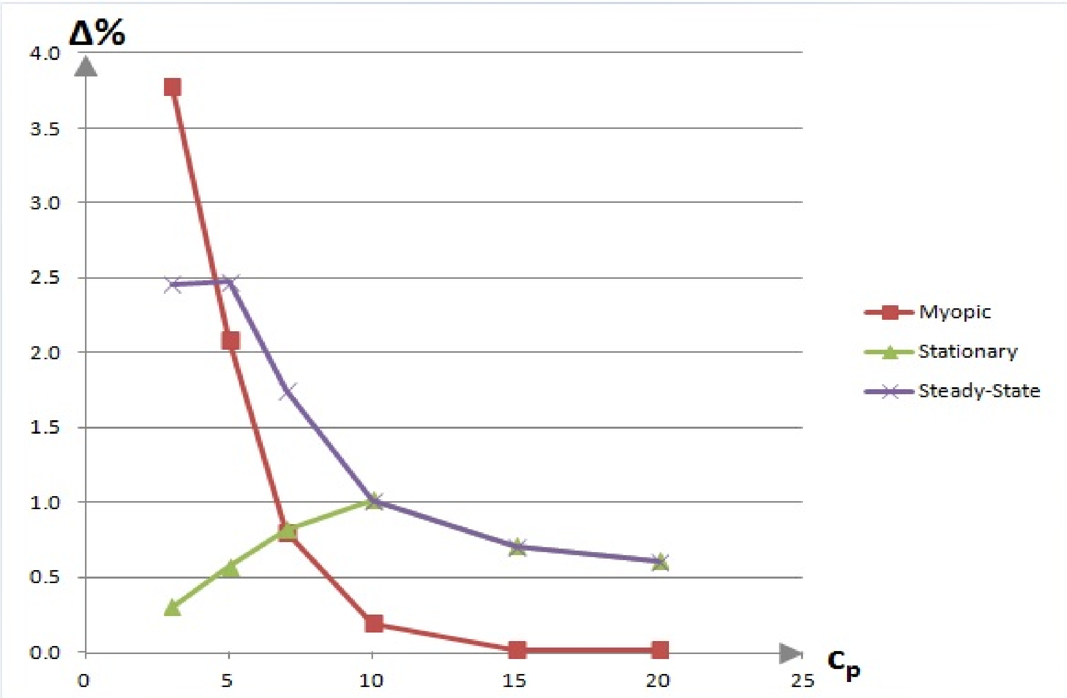

Figure 4.4: % cost deviations when c_p increases

Above analysis is conducted with a constant inventory holding cost. Next, we keep the holding cost rate constant (20% as in the base case) while c_p increases. The summary of the results for the problem instances considered are provided in Table 4.9.

Table 4.9: Summary of Results for Unit Procurement Cost Analysis When Inventory Holding Rate is 20%

		DP	Myopic		Stationary			Steady-State		
c_p	c_h	f_1	f_1	%Δ	Policy ($\acute{S}, AL_R$)	f_1	%Δ	Policy (AL_S, AL_R)	f_1	%Δ
3	0.6	156.5	161.9	3.44	3;4	157.6	0.66	0;4	157.6	0.66
5	1	186.3	190.2	2.08	2;4	187.4	0.57	0;4	190.9	2.48
7	1.4	215.3	217.6	1.05	2;4	216.8	0.68	0;5	222.1	3.16
10	2	257.1	257.7	0.22	2;5	258.1	0.38	0;5	267.0	3.84
15	3	322.9	322.9	0.01	2;5	323.1	0.05	0;5	341.8	5.87
20	4	388.0	388.0	0.00	2;5	388.0	0.01	0;5	416.7	7.41

Our observations with respect to an increase in c_p when holding rate is 20% are as follows:

- Under these problem instances, MA has the same behavior as c_p gets larger. However, all cost items in the total expected cost change in this case (Table 4.10) since unit inventory holding cost increases as well as c_p.

Table 4.10: Total Expected Cost Items in Total Expected Cost of Myopic Approach for Unit Procurement Cost Analysis When Inventory Holding Rate is 20%

		Myopic Approach					
c_p	c_h	Total Expected Inv. Holding Cost	Total Expected Procure-ment Cost	Total Expected Preventive Replacement Cost	Total Expected Failure Cost	Total Expected Failure Replacement Cost	Total Expected Shortage Cost
3	0.6	11.6	37.8	6.5	85.3	25.5	1.0
5	1	16.6	61.5	6.6	85.3	25.4	3.0
7	1.4	20.3	84.4	6.7	85.2	25.2	5.4
10	2	26.7	119.5	6.8	85.2	25.1	7.3
15	3	35.6	177.8	7.0	85.1	24.9	10.6
20	4	47.2	236.8	7.0	85.1	24.9	10.9

- As c_p gets larger, the age limit to be replaced increases but on hand inventory after preventive replacements decreases under SP in this case. Similar to the previous analyses, SP performs very close to the optimal solution for all values of c_p considered.

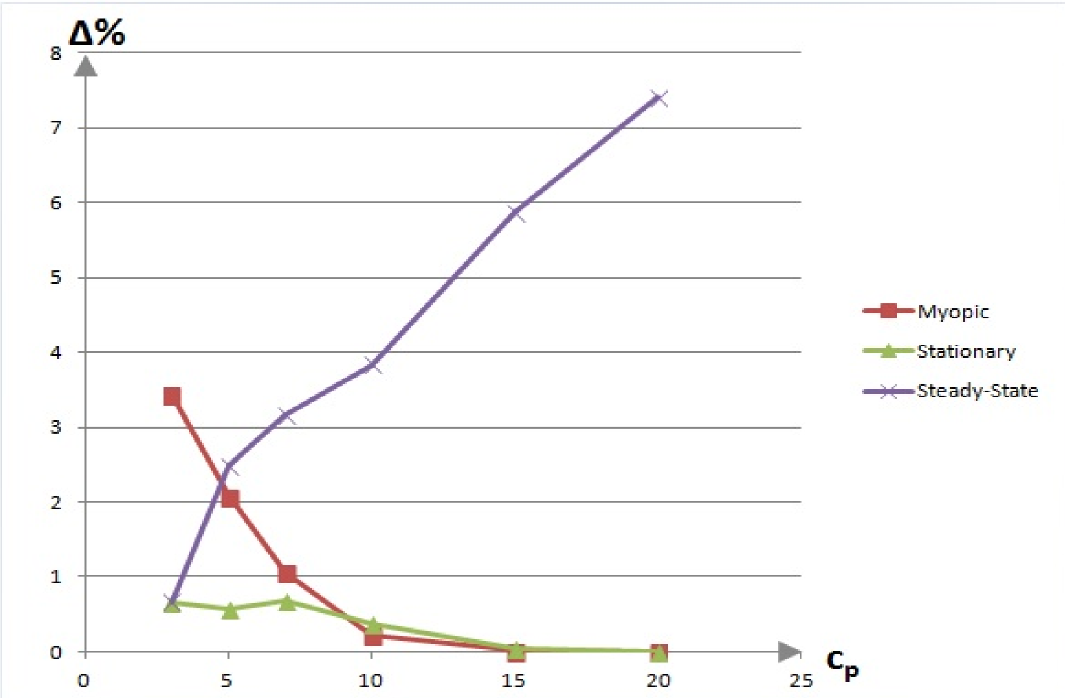

Figure 4.5: % cost deviations when c_p increases and inventory holding rate is constant and equals to 0.2

- The policies under SSA are the same as the previous case. However, the performance of SSA gets worse as c_p increases (Figure 4.4) since the policy under SSA holds inventory for all aged critical parts and c_h increases as well as c_p.

4.1.5 Unit Inventory Holding Cost per Period, c_h

For replacements after failures, required spare parts are held in inventory during the period. Inventory holding cost is incurred for leftovers. To analyze the effects of the unit inventory holding cost, its value is changed between 0.05 and 2 and other parameters are kept unvaried as in the base scenario. The summary of the results for the problem instances considered are provided in Table 4.11.

Table 4.11: Summary of Results for Unit Inventory Holding Cost Analysis

	DP	Myopic		Stationary			Steady-State		
c_h	f_1	f_1	$\%\Delta$	Policy $(\acute{S}, AL_R)$	f_1	$\%\Delta$	Policy (AL_S, AL_R)	f_1	$\%\Delta$
0.05	168.3	171.8	2.08	3;4	169.2	0.55	0;4	169.2	0.55
0.1	169.4	172.9	2.03	3;4	170.3	0.55	0;4	170.3	0.55
0.15	170.5	173.9	1.98	3;4	171.5	0.55	0;4	171.5	0.55
0.2	171.7	175.0	1.93	3;4	172.6	0.55	0;4	172.6	0.55
0.3	174.0	177.1	1.83	3;4	174.9	0.56	0;4	174.9	0.56
0.5	178.2	181.3	1.73	3;4	179.5	0.71	0;4	179.5	0.71
1	186.3	190.2	2.08	2;4	187.4	0.57	0;4	190.9	2.48
1.5	193.1	197.7	2.38	2;4	193.9	0.43	0;4	202.4	4.82
2	199.6	204.4	2.42	2;4	200.4	0.40	0;4	213.8	7.12

Our observations with respect to an increase in c_h are as follows:

- The performances of both MA and SP are robust to an increase in c_h.
- The policy under SSA does not change under all values of c_h considered. The spare parts for all aged critical parts are held so the performance of SSA gets worse (Figure 4.6) similar to the analysis of constant inventory holding cost rate (Figure 4.5). For small values of c_h, the policies under SP and SSA are identical so they give the same objective function value (Table 4.11). As c_h increases beyond 0.5, SP reacts by decreasing the spare part inventory. However, the policy under SSA does not change. As a result, SP performs better when compared to SSA.

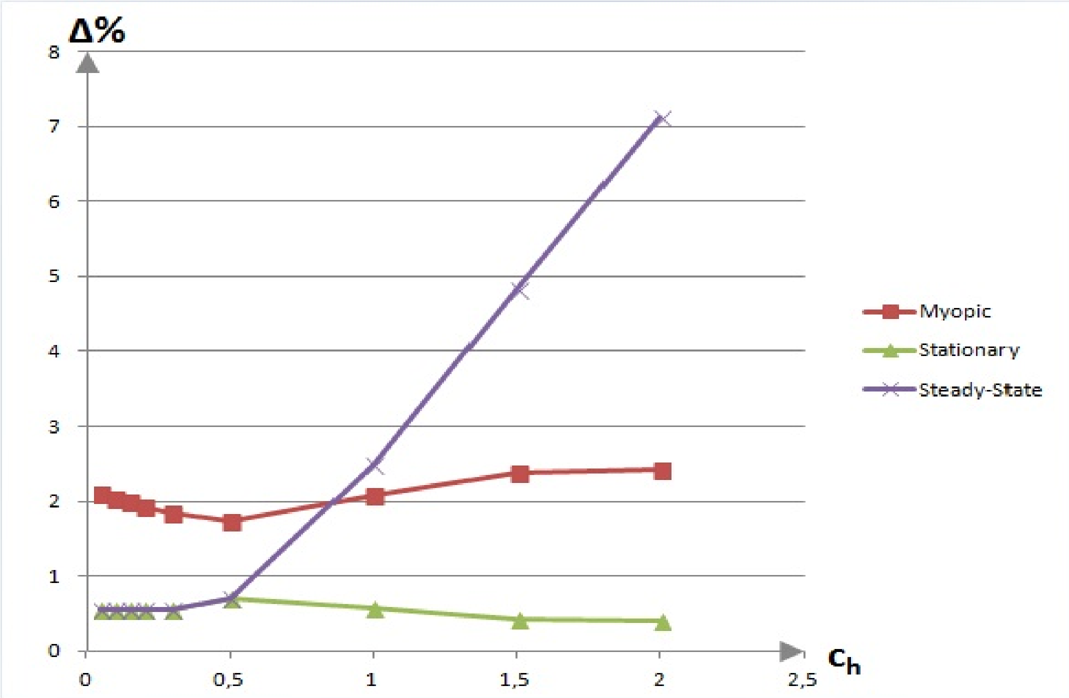

Figure 4.6: % cost deviations when c_h increases

4.1.6 Length of the Planning Horizon, T

We next analyze the impacts of the length of the planning horizon on the objective function values. To analyze the effects of the length of the planning horizon, its value is changed between 3 and 100 and other parameters are kept unvaried as in the base scenario. The summary of the results for the problem instances considered are provided in Table 4.12.

Table 4.12: Summary of Results for Length of Planning Horizon Analysis

	DP	Myopic		Stationary			Steady-State		
T	f_1	f_1	$\%\Delta$	Policy $(\acute{S}, AL_R)$	f_1	$\%\Delta$	Policy (AL_S, AL_R)	f_1	$\%\Delta$
3	59.5	62.0	4.17	2;4	60.4	1.55	0;4	61.7	3.66
5	95.0	96.8	1.87	2;4	96.3	1.43	0;4	98.1	3.30
7	131.8	135.0	2.48	2;4	132.7	0.71	0;4	135.2	2.65
10	186.3	190.2	2.08	2;4	187.4	0.57	0;4	190.9	2.48
15	277.2	283.0	2.08	2;4	278.3	0.40	0;4	283.7	2.32
20	368.2	375.7	2.03	2;4	369.5	0.33	0;4	376.5	2.25
25	459.2	468.4	1.99	2;4	460.6	0.29	0;4	469.4	2.21
30	550.3	561.1	1.96	2;4	551.7	0.26	0;4	562.2	2.18
40	732.3	746.4	1.93	2;4	733.9	0.23	0;4	747.9	2.14
50	914.3	931.8	1.91	2;4	916.2	0.20	0;4	933.7	2.12
75	1369.3	1395.2	1.89	2;4	1371.7	0.17	0;4	1397.9	2.09
100	1824.4	1858.6	1.88	2;4	1827.3	0.16	0;4	1862.2	2.08

We present our observations with respect to an increase in T as follows:

- All heuristics perform better as T gets larger.
- SP gives the best results as compared to the other heuristics and SSA gives the worst cost values (Figure 4.7).
- The policies under neither SP nor SSA changes (Table 4.12). However, these heuristics give better objective function values as T increases. SP takes the length of the planning horizon into account but SSA solves an infinite horizon problem. Thus it is expected that the policy under SSA does not change with different T values and SSA gives better results for long planning horizons.

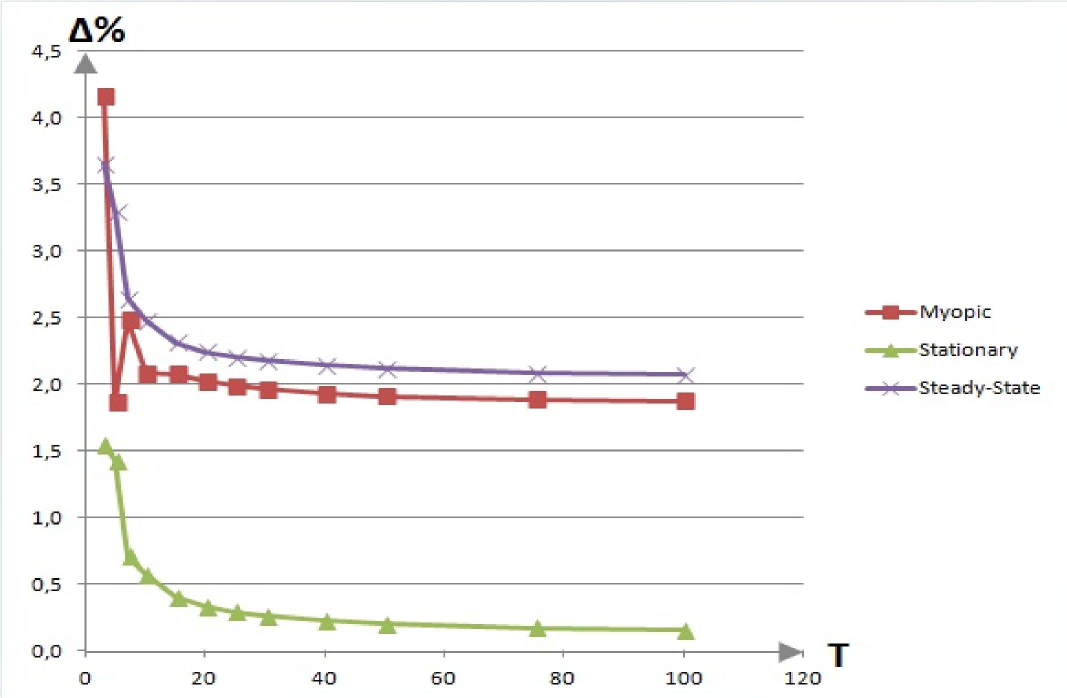

Figure 4.7: % cost deviations when T increases

4.1.7 Number of Units Installed, M

To analyze the effects of the number of the machines or equivalently the number of the critical parts installed, M, we increase the number of the machines one by one up to 4. While investigating the effects of M, the initial ages of all critical parts are taken as 3 to eliminate the effects of the initial ages of the critical parts. The summary of the results for the problem instances considered are provided in Table 4.13.

Table 4.13: Summary of Results for Number of The Critical Parts Installed Analysis

	DP	Myopic		Stationary			Steady-State		
M	f_1	f_1	$\%\Delta$	Policy $(\acute{S}, AL_R)$	f_1	$\%\Delta$	Policy (AL_S, AL_R)	f_1	$\%\Delta$
1	63.5	64.2	1.08	1;4	64.0	0.70	0;4	64.0	0.70
2	127.0	128.4	1.08	2;4	127.9	0.70	0;4	127.9	0.70
3	187.1	190.5	1.78	2;4	189.2	1.12	0;4	191.9	2.53
4	246.1	250.2	1.66	3;4	247.7	0.67	0;4	255.8	3.95

We present our observations with respect to an increase in M as follows:

- Recall that SSA considers a single machine and inventory pooling is not considered. This disadvantage from no inventory pooling increases with the number of the critical parts (Figure 4.8) so the performance of SSA gets worse while M increases.

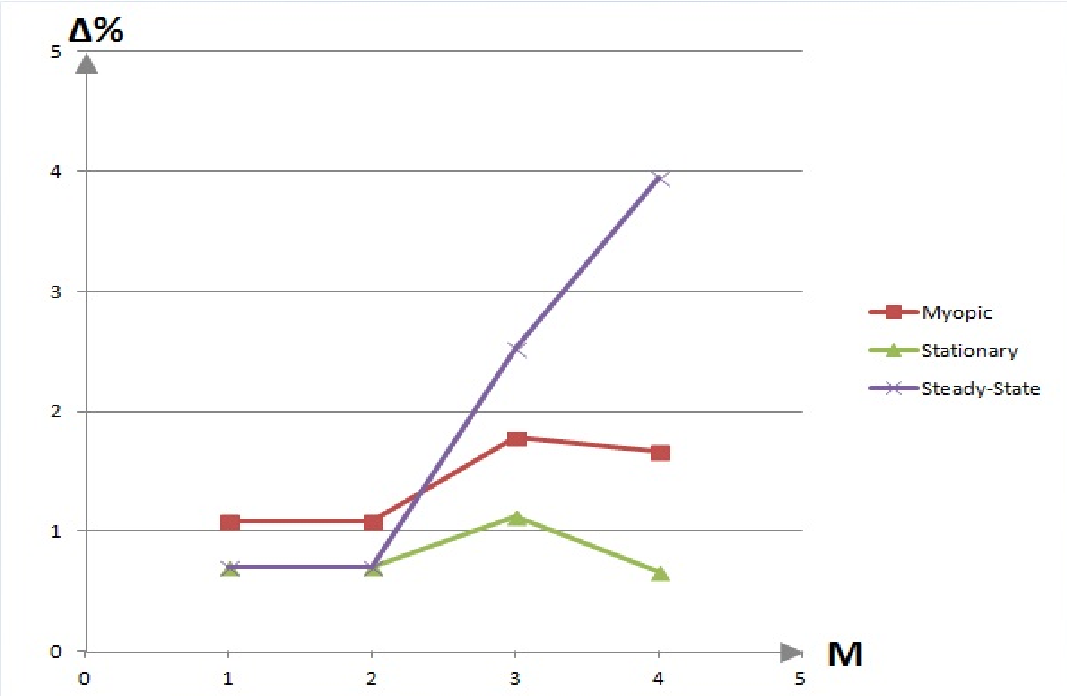

Figure 4.8: % cost deviations when M increases

4.2 Analysis of Full Factorial Experiment

A full factorial experiment is performed with the parameter values given in Table 4.14. Each problem is solved using all approaches. In total, 324 problem instances are considered.

The aim of the full factorial experiment is to investigate the problem instances that give the best and the worst $\%\Delta$ values of the proposed heuristics and the individual effects of the parameters on the $\%\Delta$ values of the heuristics. Average $\%\Delta$ values with respect to each level of the parameters are provided in Table 4.15.

Table 4.14: Parameter Values Used in Full Factorial Experiment

Factor	c_s	c_f	c_r	c_p	c_h	N	T	M	$\vec{A}_1$	I_1
Levels	20	10	2	5	0.2	5	5	3	(2,3,4)	0
	50	20	5	10	0.5		10			
	100			15	1		20			

Table 4.15: Average $\%\Delta$'s of The Heuristic Approaches

	Myopic	Stationary	Steady-State
c_s			
20	1.390	0.460	1.703
50	1.429	0.544	0.957
100	1.370	0.438	0.586
c_f			
10	0.572	0.329	0.834
20	2.221	0.632	1.330
c_r			
2	1.488	0.525	1.133
5	1.305	0.436	1.031
c_p			
5	2.081	0.631	1.518
10	1.551	0.390	0.792
15	0.557	0.421	0.936
c_h			
0.2	1.495	0.428	0.569
0.5	1.389	0.459	0.880
1	1.306	0.554	1.797
T			
5	1.357	0.734	1.486
10	1.459	0.439	0.974
20	1.373	0.269	0.786

Table 4.16: The Best and Worst $\%\Delta$ Values of Myopic Approach

		Problem Instances						
	$\%\Delta$	c_s	c_f	c_r	c_p	c_h	T	M
Best	0.000	50	10	5	10	0.2	5	3
Best	0.000	100	10	5	10	0.2	5	3
Best	0.000	100	10	5	10	0.5	5	3
Best	0.000	20	10	5	10	1	5	3
Best	0.000	50	10	2	15	0.2	5	3
Best	0.000	100	10	2	15	0.2	5	3
Best	0.000	50	10	5	15	0.2	5	3
Best	0.000	100	10	5	15	0.2	5	3
Best	0.000	100	10	2	15	0.5	5	3
Best	0.000	100	10	5	15	0.5	5	3
Best	0.000	20	10	2	15	1	5	3
Best	0.000	20	10	5	15	1	5	3
Best	0.000	50	10	2	15	0.2	10	3
Best	0.000	100	10	2	15	0.2	10	3
Best	0.000	50	10	5	15	0.2	10	3
Best	0.000	100	10	5	15	0.2	10	3
Best	0.000	100	10	2	15	0.5	10	3
Best	0.000	100	10	5	15	0.5	10	3
Best	0.000	50	10	2	15	0.2	20	3
Best	0.000	100	10	2	15	0.2	20	3
Best	0.000	50	10	5	15	0.2	20	3
Best	0.000	100	10	5	15	0.2	20	3
Best	0.000	100	10	2	15	0.5	20	3
Best	0.000	100	10	5	15	0.5	20	3
Worst	5.402	20	20	5	5	0.2	10	3
Avg	1.397							

The average $\%\Delta$ over all problem instances and the parameter instances that give the best and the worst $\%\Delta$ values of MA are provided in Table 4.16. For 24 instances out of 324, MA gives the optimal solution. Its average $\%\Delta$ value over all problem instances is 1.397. As stated in the sensitivity analysis, MA performs better under high values of c_p (Table 4.15 and Table 4.16).

The number of the problem instances that $\%\Delta$ values of MA is better (or equal or worse) compared to other heuristics are provided in Table 4.17. MA performs better than SP for 112 instances out of 324 and equal to SP for 60 instances out of 324. MA performs better than SSA for 164 instances out of 324 and equal to SSA for 36 instances out of 324. It performs better than both SP and SSA for 112 instances out of 324 and three heuristics have same $\%\Delta$ value for 36 instances out of 324.

Table 4.17: The Number of Problem Instances That MA Performs Better (or Equal or Worse) Compared to Other Heuristics

	Myopic Approach		
	Better Than	Equal To	Worse Than
Stationary Policy	112	60	152
Steady-State Approximation	164	36	124
Both Stationary Policy and Steady-State Approximation	112	36	124

Table 4.18: The Best and Worst $\%\Delta$ Values Stationary Policy

	Problem Instances							
	$\%\Delta$	c_s	c_f	c_r	c_p	c_h	T	M
Best	0.000	50	10	5	10	0.2	5	3
Best	0.000	100	10	5	10	0.2	5	3
Best	0.000	100	10	5	10	0.5	5	3
Best	0.000	50	10	2	15	0.2	5	3
Best	0.000	100	10	2	15	0.2	5	3
Best	0.000	50	10	5	15	0.2	5	3
Best	0.000	100	10	5	15	0.2	5	3
Best	0.000	100	10	2	15	0.5	5	3
Best	0.000	100	10	5	15	0.5	5	3
Best	0.000	50	10	2	15	0.2	10	3
Best	0.000	100	10	2	15	0.2	10	3
Best	0.000	50	10	5	15	0.2	10	3
Best	0.000	100	10	5	15	0.2	10	3
Best	0.000	100	10	2	15	0.5	10	3
Best	0.000	100	10	5	15	0.5	10	3
Best	0.000	50	10	2	15	0.2	20	3
Best	0.000	100	10	2	15	0.2	20	3
Best	0.000	50	10	5	15	0.2	20	3
Best	0.000	100	10	5	15	0.2	20	3
Best	0.000	100	10	2	15	0.5	20	3
Best	0.000	100	10	5	15	0.5	20	3
Worst	1.723	50	10	5	5	1	5	3
Avg	0.481							

The average $\%\Delta$ over all problem instances and the parameter instances that give the best and the worst $\%\Delta$ values of SP are provided in Table 4.18. For 21 instances out of 324, SP gives the optimal solution. Its average $\%\Delta$ value over all problem instances is 0.481. In addition, its $\%\Delta$ value is only 1.723 even in the worst case. SP gives nearly the optimal solution in most instances however as stated before it is computationally intractable for a large number of machines and long planning horizons. When the level of T increases, the improvement is observed easily (Table 4.15).

The number of the problem instances that $\%\Delta$ values of SP is better (or equal or worse) are provided in Table 4.19. SP performs better than MA for 152 instances out of 324 and equal to MA for 60 instances out of 324. SP performs better than SSA for 143 instances out of 324 and equal to SSA for 181 instances out of 324. It performs better than both MA and SSA for 58 instances out of 324. Its performance is not worst under any parameter instance considered.

Table 4.19: The Number of Problem Instances That SP Performs Better (or Equal or Worse) Compared to Other Heuristics

	Stationary Policy		
	Better Than	Equal To	Worse Than
Myopic Approach	152	60	112
Steady-State Approximation	143	181	0
Both Myopic Approach and Steady-State Approximation	58	36	0

Table 4.20: The Best and Worst %Δ Values Steady-State Approximation

	Problem Instances							
	$\%\Delta$	c_s	c_f	c_r	c_p	c_h	T	M
Best	0.000	50	10	5	10	0.2	5	3
Best	0.000	100	10	5	10	0.2	5	3
Best	0.000	50	10	2	15	0.2	5	3
Best	0.000	100	10	2	15	0.2	5	3
Best	0.000	50	10	5	15	0.2	5	3
Best	0.000	100	10	5	15	0.2	5	3
Best	0.000	100	10	5	10	0.5	5	3
Best	0.000	100	10	2	15	0.5	5	3
Best	0.000	100	10	5	15	0.5	5	3
Best	0.000	50	10	2	15	0.2	10	3
Best	0.000	100	10	2	15	0.2	10	3
Best	0.000	50	10	5	15	0.2	10	3
Best	0.000	100	10	5	15	0.2	10	3
Best	0.000	100	10	2	15	0.5	10	3
Best	0.000	100	10	5	15	0.5	10	3
Best	0.000	50	10	2	15	0.2	20	3
Best	0.000	100	10	2	15	0.2	20	3
Best	0.000	50	10	5	15	0.2	20	3
Best	0.000	100	10	5	15	0.2	20	3
Best	0.000	100	10	2	15	0.5	20	3
Best	0.000	100	10	5	15	0.5	20	3
Worst	5.188	20	10	2	5	1	5	3
Avg	1.082							

The average $\%\Delta$ over all problem instances and the parameter instances that give the best and the worst $\%\Delta$ values of SSA are provided in Table 4.20. For 21 instances out of 324, SSA gives the optimal solution. Its $\%\Delta$ is 5.188 in the worst case and 1.082 on the average. It is observed in Table 4.15 that under high values of c_s, it gives better results as stated in Section 4.1. Under high values of T, it performs better. As opposed to T, under high values of c_h, it performs worse.

The number of the problem instances that $\%\Delta$ values of SSA is better (or equal or worse) are provided in Table 4.21. SSA performs better than MA for 124 instances out of 324 and equal to MA for 36 instances out of 324. SSA does not perform better than SP for any instance considered and equal to SSA for 181 instances out of 324. That is, SSA is weakly dominated by SP. It performs worse than both MA and SSA for 113 instances out of 324.

Table 4.21: The Number of Problem Instances That SSA Performs Better (or Equal or Worse) Compared to Other Heuristics

	Steady-State Approximation		
	Better Than	Equal To	Worse Than
Myopic Approach	124	36	164
Stationary Policy	0	181	143
Both Myopic Approach and Stationary Policy	0	36	113

4.3 General Evaluation of the Heuristic Approaches

According to results of the sensitivity analysis and the full factorial experiment, we observe some explicit features of the heuristics. Our findings are as follows:

- MA performs better than the other heuristics when c_r or c_p is considerably high. In addition, MA is easy to implement for any problem instance.
- The performance of SP weakly dominates SSA under most problem instances considered since it makes inventory pooling as opposed to SSA. In addition, it dominates MA in many problem instances. Its performance gets worse as c_r increases in contrast to the other two heuristics. However, it is computationally intractable for a large number of machines and long planning horizons as DP formulation.
- The SSA performs worse when c_s is low or c_h is high. In addition, its performance gets worse when there is a large number of machines. It gives moderate

quality solution under most problem instances. It is easy to implement for any problem instance.

- Under considerably high values of c_s and long planning horizons, all heuristics give closer results to the optimal solution. However, under high values of c_s, SSA performs better than MA and for long planning horizons, MA performs a little bit better than SSA.

CHAPTER 5

CONCLUSIONS

The machine in any production environment is subject to failure. Firms need to carry spare parts inventory to cope with failure and ensure smooth operations through preventive maintenance. In other words, preventive maintenance and uncertain failures can be consider as the major reasons of spare part inventory. Spare parts are usually expensive pieces and there is a risk of deterioration or obsolescence during the waiting period in inventory. In addition, shortages of spare parts may cause extra cost due to loss of production. Therefore, planning preventive maintenance activities and managing spare part inventory should be handled together.

In this study, we focus on the problem of integrated planning for preventive maintenance and spare part inventory. We consider a system that consists of multiple machines that are identical and independent. Each machine has a non-repairable critical part to operate. If the critical part fails, the corresponding machine stops operating until the critical part is replaced. The failure probability increases with the age of the critical part.

In the literature, there are several papers that consider the joint problem of preventive maintenance and spare part inventory planning. These studies generally consider a given policy then try to find best policy parameters. We do not impose any predetermined preventive maintenance or inventory control policy as in the literature.

We propose a Dynamic Programming formulation for the integrated planning for preventive replacement and spare part inventory planning. The objective is to minimize total expected cost over a finite planning horizon. Decisions that we make in each period are the amount of order quantity and whether to replace or not decisions for each part. This formulation provides optimal solution over a finite planning horizon. However, it is hard to obtain the optimal solution for a long planning horizon and for systems with large number of operating units. Hence, we propose three heuristic approaches. Those are (i) Myopic approach, (ii) Stationary policy, (iii) Steady state approximation. MA ignores the impacts of the current decisions on the future events. Under SP, we restrict our attention to age-based replacement and base stock policy. The third heuristic approach is based on approximating the finite horizon problem by an infinite horizon for a single machine.

By computational study, we investigate the performances of the proposed heuristic approaches under different problem parameters with respect to the solution quality. Our analyses reveal the following findings:

- MA is a favorable approach when c_r or c_p is considerably high since it performs better than the other heuristics and it is easy to implement. In addition, SP performs better than the other heuristics under long planning horizons and large number of the machines but it is computationally intractable for a large number of machines and long planning horizons. Hence MA is favorable since it gives better solutions than SSA.
- SP performs better than the other heuristics under most problem instances. It is computationally intractable for a large number of machines and long planning horizons. Hence it is a favorable approach for systems with a few number of machine and small planning horizons.
- The policy under SSA is easy to obtain but inventory pooling is not taken into consideration and infinite horizon problem is solved. Hence, it is not favorable for systems under long planning horizons and large number of the machines. However, under high c_s values, such an approach is favorable.

This thesis can be extended by considering replenishment lead time and relaxing instantaneous replacements of the critical parts. In such a setting, we need to define two more state variables to keep track of the replenishment lead time for outstanding order and equipment pieces in the process of replacement. Hence, the state space and the complexity of the problem increase. In addition, unit replacement cost can be taken based on the age of the corresponding unit. Furthermore, a better solution approach for Stationary policy can be sought rather than complete enumeration.

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