# Manual for SOA Exam MLC.

## Chapter 4. Life insurance.
## Actuarial problems.



Extract from:
"Arcones' Manual for SOA Exam MLC. Fall 2009 Edition",
available at http://www.actexmadriver.com/

(#1, Exam M, Fall 2005) For a special whole life insurance on $(x)$, you are given:

(i) $Z$ is the present value random variable for this insurance.

(ii) Death benefits are paid at the moment of death.

(iii) $\mu_x(t) = 0.02,\ t \geq 0$

(iv) $\delta = 0.08$

(v) $b_t = e^{0.03t},\ t \geq 0$

Calculate $\mathrm{Var}(Z)$.

(A) 0.075 (B) 0.080 (C) 0.085 (D) 0.090 (E) 0.095

 

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(iv) $\delta = 0.08$

(v) $b_t = e^{0.03t}$, $t \geq 0$

Calculate $\mathrm{Var}(Z)$.

(A) 0.075 (B) 0.080 (C) 0.085 (D) 0.090 (E) 0.095

(C) We have that $Z = b_{T_x} v^{T_x} = e^{0.03T_x} e^{-0.08T_x} = e^{-0.05T_x}$, which is the present value of a unit whole life insurance with $\delta = 0.05$. Hence,

$$E[Z] = \frac{0.02}{0.02 + 0.05} = \frac{2}{7},$$

$$E[Z^2] = \frac{0.02}{0.02 + (2)0.05} = \frac{1}{6},$$

$$\mathrm{Var}(Z) = \frac{1}{6} - \left(\frac{2}{7}\right)^2 = \frac{25}{294} = 0.08503401361.$$

 

(#7, Exam M, Spring 2005) $Z$ is the present-value random variable for a whole life insurance of $b$ payable at the moment of death of $(x)$. You are given:

(i) $\delta = 0.04$. (ii) $\mu_x(t) = 0.02$, $t \geq 0$. (iii) The single benefit premium for this insurance is equal to $\mathrm{Var}(Z)$. Calculate $b$.

(A) 2.75 (B) 3.00 (C) 3.25 (D) 3.50 (E) 3.75

 

(#7, Exam M, Spring 2005) $Z$ is the present-value random variable for a whole life insurance of $b$ payable at the moment of death of $(x)$. You are given:
(i) $\delta = 0.04$. (ii) $\mu_x(t) = 0.02$, $t \geq 0$. (iii) The single benefit premium for this insurance is equal to $\mathrm{Var}(Z)$. Calculate $b$.
(A) 2.75 (B) 3.00 (C) 3.25 (D) 3.50 (E) 3.75
(E) We have that $Z_x = be^{-\delta T_x} = be^{-(0.04)T_x}$. The density of $T_x$ is $f_{T_x}(t) = 0.02e^{-0.02t}$, $t \geq 0$. We know that $E[Z] = \mathrm{Var}(Z)$. We have that

$$E[Z] = \frac{b(0.02)}{0.02 + 0.04} = \frac{b}{3},$$

$$E[Z^2] = \frac{b^2(0.02)}{+0.02 + (2)(0.04)} = \frac{b^2}{5},$$

$$\mathrm{Var}(Z) = E[Z^2] - (E[Z])^2 = \frac{b^2}{5} - \frac{b^2}{9} = \frac{4b^2}{45}$$

Hence, $\frac{b}{3} = \frac{4b^2}{45}$ and $b = \frac{15}{4} = 3.75$.

 

(#15, Exam M, Spring 2005) For an increasing 10–year term insurance, you are given:

(i) $b_{k+1} = 100,000(1+k)$, $k = 0, 1, \dots, 9$

(ii) Benefits are payable at the end of the year of death.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) The single benefit premium for this insurance on (41) is 16,736.

Calculate the single benefit premium for this insurance on (40).

(A) 12,700 (B) 13,600 (C) 14,500 (D) 15,500 (E) 16,300

 

We have to find the APV of the cashflow $(100000)\,(IA)_{40:\overline{10}|^{1}}$. We know the value of the cashflow $(100000)\,(IA)^{1}_{41:\overline{10}|}$.

| | | | | | | |
|---|---|---|---|---|---|---|
| Cashflow of $(IA)^{1}_{41:\overline{10}|}$ | 0 | 1 | 2 | $\cdots$ | 9 | 10 |
| Cashflow of $(IA)^{1}_{40:\overline{10}|}$ | 1 | 2 | 3 | $\cdots$ | 10 | 0 |
| Time | 41 | 42 | 43 | $\cdots$ | 50 | 51 |

We obtain the cashflow of $(IA)^{1}_{40:\overline{10}|}$, by adding the cashflows of $(IA)^{1}_{41:\overline{10}|}$ and $A^{1}_{40:\overline{10}|}$ and subtracting $(10)\cdot{}_{10|}A^{1}_{40:\overline{1}|}$. Notice that the first three cashflows combine to the fourth cashflow

| | | | | | | |
|---|---|---|---|---|---|---|
| Cashflow of $(IA)^{1}_{41:\overline{10}|}$ | 0 | 1 | 2 | $\cdots$ | 9 | 10 |
| Cashflow of $A^{1}_{40:\overline{10}|}$ | 1 | 1 | 1 | $\cdots$ | 1 | 0 |
| Cashflow of $-(10)\cdot{}_{10|1}A_{40}$ | 0 | 0 | 0 | $\cdots$ | 0 | $-10$ |
| Cashflow of $(IA)^{1}_{40:\overline{10}|}$ | 1 | 2 | 3 | $\cdots$ | 10 | 0 |
| Time | 41 | 42 | 43 | $\cdots$ | 50 | 51 |

 

Hence,

$$\begin{aligned}
&(100000)\,(IA)^{1}_{40:\overline{10}|} = vp_{40}(100000)\,(IA)^{1}_{41:\overline{10}|} + (100000)A^{1}_{40:\overline{10}|} \\
&\quad - (100000)(10)A_{40:\overline{11}|}^{\ \ \frac{1}{}} \\
&= vp_{40}(100000)\,(IA)^{1}_{41:\overline{10}|} + (100000)(A_{40} - {}_{10}E_{40}A_{50}) \\
&\quad - (100000)(10)\cdot {}_{10}|_{1}A_{40} \\
&= 16736(1-0.00278)(1.06)^{-1} \\
&\quad + (100000)(0.16132 - (0.53667)(0.24905)) \\
&\quad - (10)(100000)(1.06)^{-11}\frac{8950901 - 8897913}{9313166} \\
&= 15513.8207685362.
\end{aligned}$$

(#25, Exam M, Fall 2005) For a special 3–year term insurance on $(x)$, you are given:
(i) $Z$ is the present–value random variable for this insurance.
(ii) $q_{x+k} = 0.02(k+1)$, $k = 0, 1, 2$
(iii) The following benefits are payable at the end of the year of death:

| $k$ | $b_{k+1}$ |
|---|---|
| 0 | 300 |
| 1 | 350 |
| 2 | 400 |

(iv) $i = 0.06$
Calculate $\mathrm{Var}(Z)$.
(A) 9,600 (B) 10,000 (C) 10,400 (D) 10,800 (E) 11,200

 

(C) Let $K_x$ be the time interval of death. We have that

$$Z = (300)vI(K_x = 1) + (350)v^2 I(K_x = 2) + (400)v^3 I(K_x = 3)$$

and

$$\mathbb{P}\{K_x = 1\} = q_x = 0.02,$$
$$\mathbb{P}\{K_x = 2\} = p_x q_{x+1} = (1 - 0.02)(0.04) = 0.0392,$$
$$\mathbb{P}\{K_x = 3\} = p_x p_{x+1} q_{x+2} = (1 - 0.02)(1 - 0.04)(0.06) = 0.056448.$$

 

Hence,

$$
\begin{aligned}
& E[Z] \\
=&(300)v\mathbb{P}\{K_x = 1\} + (350)v^2\mathbb{P}\{K_x = 2\} + (400)v^3\mathbb{P}\{K_x = 3\} \\
=&(300)(1.06)^{-1}(0.02) + (350)(1.06)^{-2}(0.0392) + (400)(1.06)^{-3}(0.056448) = \\
& E[Z^2] \\
=&(300)^2v^2\mathbb{P}\{K_x = 1\} + (350)^2v^4\mathbb{P}\{K_x = 2\} + (400)v^6\mathbb{P}\{K_x = 3\} \\
=&(300)^2(1.06)^{-2}(0.02) + (350)^2(1.06)^{-4}(0.0392) \\
& + (400)^2(1.06)^{-6}(0.056448) = 11772.60538, \\
& \mathrm{Var}(Z) = 11772.60538 - (36.82906023)^2 = 10416.2257.
\end{aligned}
$$

 

(#12, Exam M, Fall 2006) For a whole life insurance of 1 on $(x)$ with benefits payable at the moment of death, you are given:

$$(i) \quad \delta_t = \begin{cases} 0.02 & \text{if } t < 12 \\ 0.03 & \text{if } t \geq 12 \end{cases}$$

$$(ii) \quad \mu_x(t) = \begin{cases} 0.04 & \text{if } t < 5 \\ 0.05 & \text{if } t \geq 5 \end{cases}$$

Calculate the actuarial present value of this insurance.

(A) 0.59 (B) 0.61 (C) 0.64 (D) 0.66 (E) 0.68

 

(#12, Exam M, Fall 2006) For a whole life insurance of 1 on $(x)$ with benefits payable at the moment of death, you are given:

$$(i) \quad \delta_t = \begin{cases} 0.02 & \text{if } t < 12 \\ 0.03 & \text{if } t \geq 12 \end{cases}$$

$$(ii) \quad \mu_x(t) = \begin{cases} 0.04 & \text{if } t < 5 \\ 0.05 & \text{if } t \geq 5 \end{cases}$$

Calculate the actuarial present value of this insurance.
(A) 0.59 (B) 0.61 (C) 0.64 (D) 0.66 (E) 0.68

$$\overline{A}_x = \overline{A}^{1}_{x:\overline{5}|} + {}_5|_7\overline{A}_x + {}_{12}|\overline{A}_x = \overline{A}^{1}_{x:\overline{5}|} + {}_5E_x\overline{A}^{1}_{x+5:\overline{7}|} + {}_{12}E_x\overline{A}_{x+12}$$

$$= (1 - e^{-(5)(0.04+0.02)})\frac{0.04}{0.04 + 0.02}$$

$$+ e^{-(5)(0.04+0.02)}(1 - e^{-(7)(0.05+0.02)})\frac{0.05}{0.05 + 0.02}$$

$$+ e^{-(5)(0.04+0.02)}e^{-(7)(0.05+0.02)}\frac{0.05}{0.05 + 0.03} = 0.661421868072728.$$

 

(#35, Fall 2006) For a whole life insurance of 1000 on (80), with death benefits payable at the end of the year of death, you are given:
(i) Mortality follows a select and ultimate mortality table with a one-year select period.
(ii) $q_{[80]} = 0.5q_{80}$
(iii) $i = 0.06$
(iv) $1000A_{80} = 679.80$
(v) $1000A_{81} = 689.52$
Calculate $1000A_{[80]}$.
(A) 655 (B) 660 (C) 665 (D) 670 (E) 675

 

(#35, Fall 2006) For a whole life insurance of 1000 on (80), with death benefits payable at the end of the year of death, you are given:
(i) Mortality follows a select and ultimate mortality table with a one-year select period.
(ii) $q_{[80]} = 0.5q_{80}$
(iii) $i = 0.06$
(iv) $1000A_{80} = 679.80$
(v) $1000A_{81} = 689.52$
Calculate $1000A_{[80]}$.
(A) 655 (B) 660 (C) 665 (D) 670 (E) 675
(D) Using that $A_{80} = vq_{80} + vp_{80}A_{81}$, we get that

$$0.67980 = (1.06)^{-1}q_{80} + (1.06)^{-1}(1 - q_{80})0.68952.$$

and $q_{80} = \frac{0.67980-(1.06)^{-1}0.68952}{(1.06)^{-1}(1-0.68952)} = 0.1$. Hence, $q_{[80]} = 0.05$ and

$$1000A_{[80]} = 1000vq_{[80]} + vp_{[80]}1000A_{81}$$
$$=1000(1.06)^{-1}(0.05) + (1.06)^{-1}(1 - 0.05)689.52 = 665.1453$$

 

(#10, Exam MLC, Spring 2007) For whole life insurances of 1000 on (65) and (66):
(i) Death benefits are payable at the end of the year of death.
(ii) The interest rate is 0.10 for 2008 and 0.06 for 2009 and thereafter.
(iii) $q_{65} = 0.010$ and $q_{66} = 0.012$
(iv) The actuarial present value on December $31^{\mathrm{st}}$ 2007 of the insurance on (66) is 300.
Calculate the actuarial present value on December $31^{\mathrm{st}}$ 2007 of the insurance on (65).
(A) 279 (B) 284 (C) 289 (D) 293 (E) 298

(C) We know that

$$300 = (1000)\sum_{k=1}^{\infty}(1.1)^{-1}(1.06)^{-(k-1)} \cdot {}_{k-1}p_{66} \cdot q_{66+k-1}.$$

We need to find

$$(1000)\sum_{k=1}^{\infty}(1.1)^{-1}(1.06)^{-(k-1)} \cdot {}_{k-1}p_{65} \cdot q_{65+k-1}$$

$$=(1000)(1.1)^{-1}q_{65} + (1000)\sum_{k=2}^{\infty}(1.1)^{-1}(1.06)^{-(k-1)}p_{65} \cdot {}_{k-2}p_{66} \cdot q_{65+k-1}$$

$$=(1000)(1.1)^{-1}q_{65} + (1000)\sum_{k=1}^{\infty}(1.1)^{-1}(1.06)^{-k}p_{65} \cdot {}_{k-1}p_{66} \cdot q_{66+k-1}$$

$$=(1000)(1.1)^{-1}q_{65} + p_{65}(1.06)^{-1}(1000)\sum_{k=1}^{\infty}(1.1)^{-1}(1.06)^{-(k-1)} \cdot {}_{k-1}p_{66} \cdot q_{6}$$

$$=(1000)(1.1)^{-1}(0.01) + (0.99)(1.06)^{-1}(300) = 289.2796.$$

 

(#22, Exam MLC, Spring 2007) For a special whole life insurance on (40), you are given:
(i) The death benefit is 1000 for the first 10 years and 2500 thereafter.
(ii) Death benefits are payable at the moment of death.
(iii) $Z$ is the present–value random variable.
(iv) Mortality follows DeMoivre's law with $\omega = 100$.
(v) $\delta = 0.10$
Calculate $\mathbb{P}(Z > 700)$.
(A) 0.059 (B) 0.079 (C) 0.105 (D) 0.169 (E) 0.212

 

(C) We have that

$$Z = (1000)e^{-\delta T_x} I(T_x < 10) + (2500)e^{-\delta T_x} I(T_x \geq 10)$$

and

$$\begin{aligned}
&\mathbb{P}\{Z > 700\} = \mathbb{P}\{(1000)e^{-0.1 T_x} > 700, T_x < 10\} \\
&+ \mathbb{P}\{(2500)e^{-0.1 T_x} > 700, T_x \geq 10\} \\
=&\mathbb{P}\{T_x < \frac{\ln(0.7)}{-0.1}, T_x < 10\} + \mathbb{P}\{T_x < \frac{\ln(7/25)}{-0.1}, T_x \geq 10\} \\
=&\mathbb{P}\{T_x < 3.566749, T_x < 10\} + \mathbb{P}\{T_x < 12.72966, T_x \geq 10\} \\
=&\mathbb{P}\{T_x < 3.566749\} + \mathbb{P}\{10 \leq T_x < 12.72966\} \\
=&\frac{3.566749}{60} + \frac{12.72966 - 10}{60} = 0.1049402.
\end{aligned}$$

 

(#27, Exam MLC, Spring 2007) For a special whole life insurance, you are given:
(i) $b_t = e^{-t}$, $t > 0$
(ii) $\mu$ is constant.
(iii) $\delta = 0.06$
(iv) $Z = e^{-T}v^T$, where $T$ is the future lifetime random variable.
(v) $E[Z] = 0.03636$
Calculate $\mathrm{Var}[Z]$.
(A) 0.017 (B) 0.021 (C) 0.025 (D) 0.029 (E) 0.033

 

(A) We have that $Z = e^{-T}e^{-\delta T} = e^{-1.06T}$, which is the present value of a unit whole life insurance with $\delta = 1.06$. So, we may assume that we have a unit whole life insurance and $\delta = 1.06$. Hence,

$$0.03636 = E[Z] = \frac{\mu}{1.06 + \mu}$$

and $\mu = \frac{(0.03636)(1.06)}{1-0.03636} = 0.04$. Thus

$$E[Z^2] = \frac{0.04}{(2)(1.06) + 0.04} = 0.0185185185185185$$

and

$$\mathrm{Var}(Z) = 0.0185185185185185 - (0.03636)^2 = 0.0171964689185185.$$

 

(#2, MLC–09–08) For a whole life insurance of 1000 on $(x)$ with benefits payable at the moment of death:

(i) $\delta_t = \begin{cases} 0.04 & \text{if } 0 < t \leq 10 \\ 0.05 & \text{if } t < 10 \end{cases}$

(ii) $\mu_x(t) = \begin{cases} 0.06 & \text{if } 0 < t \leq 10 \\ 0.07 & \text{if } t < 10 \end{cases}$

Calculate the single benefit premium for this insurance.

(A) 379 (B) 411 (C) 444 (D) 519 (E) 594

 

(#2, MLC–09–08) For a whole life insurance of 1000 on $(x)$ with benefits payable at the moment of death:

(i) $\delta_t = \begin{cases} 0.04 & \text{if } 0 < t \leq 10 \\ 0.05 & \text{if } t < 10 \end{cases}$

(ii) $\mu_x(t) = \begin{cases} 0.06 & \text{if } 0 < t \leq 10 \\ 0.07 & \text{if } t < 10 \end{cases}$

Calculate the single benefit premium for this insurance.

(A) 379 (B) 411 (C) 444 (D) 519 (E) 594

(E) We have that

$$\begin{aligned} (1000)A_x &= (1000)\left(A^1_{x:10} + {}_{10}E_x A_{x+10}\right) \\ &= (1000)(1 - e^{-(10)(0.04+0.06)})\frac{0.06}{0.04+0.06} \\ &\quad + (1000)e^{-(10)(0.04+0.06)}\frac{0.07}{0.07+0.05} \\ &= (600)(1-e^{-1}) + e^{-1}\frac{7000}{12} = 593.868676 \end{aligned}$$

 

(#3, MLC–09–08) For a special whole life insurance on $(x)$, payable at the moment of death:

(i) $\mu_x(t) = 0.05,\ t > 0$

(ii) $\delta = 0.08$

(iii) The death benefit at time $t$ is $b_t = e^{0.06t},\ t > 0$.

(iv) $Z$ is the present value random variable for this insurance at issue.

Calculate $\mathrm{Var}(Z)$.

(A) 0.038 (B) 0.041 (C) 0.043 (D) 0.045 (E) 0.048

 

(#3, MLC–09–08) For a special whole life insurance on $(x)$, payable at the moment of death:
(i) $\mu_x(t) = 0.05,\ t > 0$
(ii) $\delta = 0.08$
(iii) The death benefit at time $t$ is $b_t = e^{0.06t},\ t > 0$.
(iv) $Z$ is the present value random variable for this insurance at issue.
Calculate $\mathrm{Var}(Z)$.
(A) 0.038 (B) 0.041 (C) 0.043 (D) 0.045 (E) 0.048
(D) We have that $Z = e^{0.06t}e^{-0.08t} = e^{-0.02t}$, which is the present value of a unit whole life insurance under $\delta = 0.02$. So, we may assume that we have a unit whole life insurance and $\delta = 0.02$. We have that

$$\mathrm{Var}(Z_x) = {}^2A_x - (A_x)^2 = \frac{0.05}{0.05 + (2)(0.02)} - \left(\frac{0.05}{0.05 + 0.02}\right)^2$$
$$= \frac{5}{9} - \frac{25}{49} = 0.04535147392.$$

 

(#4, MLC–09–08) For a group of individuals all age $x$, you are given:
(i) 25% are smokers (s); 75% are nonsmokers (ns).
(ii)

| $k$ | $q^{s}_{x+k}$ | $q^{ns}_{x+k}$ |
|---|---|---|
| 0 | 0.10 | 0.05 |
| 1 | 0.20 | 0.10 |
| 2 | 0.30 | 0.15 |

(iii) $i = 0.02$
Calculate $10{,}000 A^{1}_{x:\overline{2}|}$ for an individual chosen at random from this group.
(A) 1690 (B) 1710 (C) 1730 (D) 1750 (E) 1770

 

(C) For smokers,

$$10000A^{1}_{x:\overline{2}|} = 10000\left((0.1)(1.02)^{-1} + (0.9)(0.2)(1.02)^{-2}\right) = 2710.495963.$$

For nonsmokers,

$$10000A^{1}_{x:\overline{2}|} = 10000\left((0.05)(1.02)^{-1} + (0.95)(0.1)(1.02)^{-2}\right) = 1403.306421.$$

For the group,

$$10000A^{1}_{x:\overline{2}|} = (0.25)(2710.495963) + (0.75)(1403.306421) = 1730.103806.$$

(#8, MLC–09–08) For a sequence, $u(k)$ is defined by the following recursion formula

$$u(k) = \alpha(k) + \beta(k) \times u(k-1) \text{ for } k = 1, 2, 3, \ldots$$

(i) $\alpha(k) = -\frac{q_{k-1}}{p_{k-1}}$.
(ii) $\beta(k) = \frac{1+i}{p_{k-1}}$.
(iii) $u(70) = 1.0$
Which of the following is equal to $u(40)$?
(A) $A_{30}$ (B) $A_{40}$ (C) $A_{40:\overline{30}|}$ (D) $A^{1}_{40:\overline{30}|}$ (E) $A_{40:\overset{1}{\overline{30}|}}$

 

(C) The iterative equation is $u(k)vp_{k-1} + vq_{k-1} = u(k-1)$. From $u(70) = 1.0$, we get that

$$u(69) = vp_{69} + vq_{69},$$
$$u(68) = (vp_{69} + vq_{69})vp_{68} + vq_{68} = v^2 p_{68} p_{69} + v^2 p_{68} q_{69} + vq_{68} = A_{68:\overline{2}|},$$
$$u(67) = (v^2 p_{68} p_{69} + v^2 p_{68} q_{69} + vq_{68})vp_{67} + vq_{67}$$
$$= v^3 p_{67} p_{68} p_{69} + v^3 p_{67} p_{68} q_{69} + v^2 p_{67} q_{68} + vq_{67} = A_{67:\overline{3}|},$$

$\cdots$

$$u(40) = (A_{41:\overline{29}|})vp_{40} + vq_{40} = A_{40:\overline{30}|},$$

(#17, MLC–09–08) For a whole life insurance of 1 on (41) with death benefit payable at the end of year of death, you are given:

(i) $i = 0.05$
(ii) $p_{40} = 0.9972$
(iii) $A_{41} - A_{40} = 0.00822$
(iv) ${}^2A_{41} - {}^2A_{40} = 0.00433$
(v) $Z$ is the present-value random variable for this insurance.

Calculate $\mathrm{Var}(Z)$.

(A) 0.023 (B) 0.024 (C) 0.025 (D) 0.026 (E) 0.027

(#17, MLC–09–08) For a whole life insurance of 1 on (41) with death benefit payable at the end of year of death, you are given:
(i) $i = 0.05$
(ii) $p_{40} = 0.9972$
(iii) $A_{41} - A_{40} = 0.00822$
(iv) ${}^2A_{41} - {}^2A_{40} = 0.00433$
(v) $Z$ is the present-value random variable for this insurance.
Calculate $\mathrm{Var}(Z)$.
(A) 0.023 (B) 0.024 (C) 0.025 (D) 0.026 (E) 0.027
(C) We have that

$$A_{41} - 0.00822 = A_{40} = vq_{40} + vp_{40}A_{41} = v(1 - 0.9972) + v(0.9972)A_{41},$$

$$A_{41} = \frac{0.00822 + (1.05)^{-1}(1 - 0.9972)}{1 - (1.05)^{-1}(0.9972)} = 0.2164962121,$$

$${}^2A_{41} = \frac{0.00433 + (1.05)^{-2}(1 - 0.9972)}{1 - (1.05)^{-2}(0.9972)} = 0.07192616334,$$

$$\mathrm{Var}(Z) = 0.07192616334 - (0.2164962121)^2 = 0.02505555349.$$

 

(#34, MLC–09–08) You are given:
(i) the following select-and-ultimate mortality table with 3-year select period:

| $x$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ | $q_{x+3}$ | $x+3$ |
|---|---|---|---|---|---|
| 60 | 0.09 | 0.11 | 0.13 | 0.15 | 63 |
| 61 | 0.10 | 0.12 | 0.14 | 0.16 | 64 |
| 62 | 0.11 | 0.13 | 0.15 | 0.17 | 65 |
| 63 | 0.12 | 0.14 | 0.16 | 0.18 | 66 |
| 64 | 0.13 | 0.15 | 0.17 | 0.19 | 67 |

(ii) $i = 0.03$
Calculate ${}_{2|2}A_{[60]}$, the actuarial present value of a 2-year deferred 2-year term insurance on 60.
(A) 0.156 (B) 0.160 (C) 0.186 (D) 0.190 (E) 0.195

 

(#34, MLC–09–08) You are given:
(i) the following select-and-ultimate mortality table with 3-year select period:

| $x$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ | $q_{x+3}$ | $x+3$ |
|---|---|---|---|---|---|
| 60 | 0.09 | 0.11 | 0.13 | 0.15 | 63 |
| 61 | 0.10 | 0.12 | 0.14 | 0.16 | 64 |
| 62 | 0.11 | 0.13 | 0.15 | 0.17 | 65 |
| 63 | 0.12 | 0.14 | 0.16 | 0.18 | 66 |
| 64 | 0.13 | 0.15 | 0.17 | 0.19 | 67 |

(ii) $i = 0.03$
Calculate ${}_{2|2}A_{[60]}$, the actuarial present value of a 2-year deferred 2-year term insurance on 60.
(A) 0.156 (B) 0.160 (C) 0.186 (D) 0.190 (E) 0.195
**Solution:** (D)

$$ {}_{2|2}A_{[60]} = (0.91)(0.89)(0.13)v^3 + (0.91)(0.89)(0.87)(0.15)v^4 $$
$$ = 0.1902584485 $$

(#56, MLC–09–08) For a continuously increasing whole life insurance on $(x)$, you are given:
(i) The force of mortality is constant.
(ii) $\delta = 0.06$
(iii) ${}^2\overline{A}_x = 0.25$
Calculate $(\overline{IA})_x$.
(A) 2.889 (B) 3.125 (C) 4.000 (D) 4.667 (E) 5.500

 

(#56, MLC–09–08) For a continuously increasing whole life insurance on $(x)$, you are given:
(i) The force of mortality is constant.
(ii) $\delta = 0.06$
(iii) ${}^2\overline{A}_x = 0.25$
Calculate $(\overline{IA})_x$.
(A) 2.889 (B) 3.125 (C) 4.000 (D) 4.667 (E) 5.500
(C) We have that

$$\frac{1}{4} = 0.25 = {}^2\overline{A}_x = \frac{\mu}{\mu + 2\delta} = \frac{\mu}{\mu + 0.12}.$$

So, $\mu + 0.12 = 4\mu$ and $\mu = \frac{0.12}{3} = 0.04$. We have that

$$(\overline{IA})_x = \int_0^\infty te^{-\delta t}\mu e^{-\mu t}\,dt = \int_0^\infty t(0.04)e^{-(0.1)t}\,dt = \frac{(0.04)}{(0.1)^2} = 4.$$

 

(#69, MLC–09–08) For a fully discrete 2-year term insurance of 1 on (x):
(i) 0.95 is the lowest premium such that there is a 0% chance of loss in year 1.
(ii) $p_x = 0.75$
(iii) $p_{x+1} = 0.80$
(iv) $Z$ is the random variable for the present value at issue of future benefits.
Calculate $\mathrm{Var}(Z)$.
(A) 0.15 (B) 0.17 (C) 0.19 (D) 0.21 (E) 0.23

 

(#69, MLC–09–08) For a fully discrete 2-year term insurance of 1 on (x):
(i) 0.95 is the lowest premium such that there is a 0% chance of loss in year 1.
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(iii) $p_{x+1} = 0.80$
(iv) $Z$ is the random variable for the present value at issue of future benefits.
Calculate $\mathrm{Var}(Z)$.
(A) 0.15 (B) 0.17 (C) 0.19 (D) 0.21 (E) 0.23
(C) Since the present value of death benefit at time one is $v$, we have that $v = 0.95$. Hence,

$$A^{1}_{x:\overline{2}|} = vq_x + v^2 p_x q_{x+1} = (0.95)(0.25) + (0.95)^2(0.75)(0.2) = 0.372875,$$
$$^2A^{1}_{x:\overline{2}|} = (0.95)^2(0.25) + (0.95)^4(0.75)(0.2) = 0.3478009375,$$
$$\mathrm{Var}(Z) = 0.3478009375 - (0.372875)^2 = 0.2087651719$$

 

(#72, MLC–09–08) Each of 100 independent lives purchase a single premium 5-year deferred whole life insurance of 10 payable at the moment of death. You are given:
(i) $\mu = 0.04$
(ii) $\delta = 0.06$
(iii) $F$ is the aggregate amount the insurer receives from the 100 lives.
Using the normal approximation, calculate $F$ such that the probability the insurer has sufficient funds to pay all claims is 0.95.
(A) 280 (B) 390 (C) 500 (D) 610 (E) 720

 

(A) Let $Z$ the be present value of the aggregate benefits.

$$E[{}_5|\overline{Z}_x] = e^{-(5)(0.04+0.06)}\frac{0.04}{0.04+0.06} = e^{-0.5}(0.4),$$
$$E[{}^2{}_5|\overline{Z}_x] = e^{-(5)(0.04+(2)0.06)}\frac{0.04}{0.04+(2)0.06} = e^{-0.8}(0.25),$$
$$\mathrm{Var}({}_5|\overline{Z}_x) = e^{-0.8}(0.25) - e^{-1}(0.16) = 0.05347153044,$$
$$E[Z] = (100)(10)e^{-0.5}(0.4) = 242.6122639,$$
$$\sqrt{\mathrm{Var}(Z)} = (10)(10)\sqrt{0.05347153044} = 23.12391196,$$
$$F = 242.6122639 + (1.645)(23.12391196) = 280.6510991.$$

 

(#76, MLC–09–08) A fund is established by collecting an amount $P$ from each of 100 independent lives age 70.
The fund will pay the following benefits:

- 10, payable at the end of the year of death, for those who die before age 72, or
- $P$, payable at age 72, to those who survive.

You are given:
Mortality follows the Illustrative Life Table.
(i) $i = 0.08$
Calculate $P$, using the equivalence principle.
(A) 2.33 (B) 2.38 (C) 3.02 (D) 3.07 (E) 3.55

(C) We have that

$$P = (10)vq_{70} + (10)v^2 p_{70} q_{71} + Pv^2 p_{70} p_{71},$$

$$P = \frac{(10)vq_{70} + (10)v^2 p_{70} q_{71}}{1 - v^2 p_{70} p_{71}}$$

$$= \frac{(10)v(0.03318) + (10)v^2(1 - 0.03318)(0.03626)}{1 - v^2(1 - 0.03318)(1 - 0.03626)} = 3.021319121.$$

 

(#107, MLC–09–08) Z is the present value random variable for a 15-year pure endowment of 1 on $(x)$:
(i) The force of mortality is constant over the 15-year period.
(ii) $v = 0.9$.
(iii) $\mathrm{Var}(Z) = 0.065E[Z]$
Calculate $q_x$.
(A) 0.020 (B) 0.025 (C) 0.030 (D) 0.035 (E) 0.040

(#107, MLC–09–08) Z is the present value random variable for a 15-year pure endowment of 1 on $(x)$:
(i) The force of mortality is constant over the 15-year period.
(ii) $v = 0.9$.
(iii) $\mathrm{Var}(Z) = 0.065E[Z]$
Calculate $q_x$.
(A) 0.020 (B) 0.025 (C) 0.030 (D) 0.035 (E) 0.040
(B) We have that $Z = v^{15}I(T_x > 15)$, $E[Z] = v^{15}{}_{15}p_x$ and $\mathrm{Var}(Z) = v^{30}{}_{15}p_x(1 - {}_{15}p_x)$. From

$$v^{30}{}_{15}p_x(1 - {}_{15}p_x) = \mathrm{Var}(Z) = 0.065E[Z] = (0.065)v^{15}{}_{15}p_x,$$

we get that $1 - {}_{15}p_x = (0.065)(0.9)^{-15}$ and $e^{-15\mu} = {}_{15}p_x = 1 - (0.065)(0.9)^{-15} = 0.6842991763$. Hence, $q_x = 1 - e^{-\mu} = 1 - (0.6842991763)^{1/15} = 0.02497354106$.

 

(#121, MLC–09–08) Lee, age 63, considers the purchase of a single premium whole life insurance of 10,000 with death benefit payable at the end of the year of death.
The company calculates benefit premiums using:
(i) mortality based on the Illustrative Life Table,
(ii) $i = 0.05$
The company calculates contract premiums as 112% of benefit premiums.
The single contract premium at age 63 is 5233.
Lee decides to delay the purchase for two years and invests the 5233.
Calculate the minimum annual rate of return that the investment must earn to accumulate to an amount equal to the single contract premium at age 65.
(A) 0.030 (B) 0.035 (C) 0.040 (D) 0.045 (E) 0.050

 

(A) We have that $A_{63} = \frac{5233}{(10000)(1.12)} = 0.4672321429$. From the iterative formula

$$A_{63} = vq_{63} + v^2 p_{63} q_{64} + v^2 p_{63} p_{64} A_{65}.$$

Hence,

$$\begin{aligned} &A_{65} \\ =&\frac{(0.4672321429)(1.05)^2 - (0.01788)(1.05) - (1 - 0.01788)(0.01952)}{(1 - 0.01788)(1 - 0.01952)} \\ =&0.5146725616. \end{aligned}$$

From $(0.4672321429)(1 + i)^2 = 0.5146725616$, we get that $i = 0.02984629318$.

 

(#148, MLC–09–08) A decreasing term life insurance on (80) pays $(20 - k)$ at the end of the year of death if (80) dies in year $k + 1$, for $k = 0, 1, 2, \dots, 19$. You are given:

(i) $i = 0.06$

(ii) For a certain mortality table with $q_{80} = 0.2$, the single benefit premium for this insurance is 13.

(iii) For this same mortality table, except that $q_{80} = 0.1$, the single benefit premium for this insurance is $P$.

Calculate $P$.

(A) 11.1 (B) 11.4 (C) 11.7 (D) 12.0 (E) 12.3

(#148, MLC–09–08) A decreasing term life insurance on (80) pays $(20 - k)$ at the end of the year of death if (80) dies in year $k + 1$, for $k = 0, 1, 2, \dots, 19$. You are given:
(i) $i = 0.06$
(ii) For a certain mortality table with $q_{80} = 0.2$, the single benefit premium for this insurance is 13.
(iii) For this same mortality table, except that $q_{80} = 0.1$, the single benefit premium for this insurance is $P$.
Calculate $P$.
(A) 11.1 (B) 11.4 (C) 11.7 (D) 12.0 (E) 12.3
(E) We have that

$$13 = (DA)^{1}_{80:\overline{20}|} = (20)(0.2)v + (0.8)v\,(DA)^{1}_{81:\overline{19}|},$$

$$(DA)^{1}_{81:\overline{19}|} = \frac{(13 - (20)(0.2)v}{0.8v} = 12.25.$$

Hence, for the new table

$$(DA)^{1}_{80:\overline{20}|} = (20)(0.1)v + (0.9)v(12.225) = 12.26650943.$$

 

(#175, MLC–09–08) A group of 1000 lives each age 30 sets up a fund to pay 1000 at the end of the first year for each member who dies in the first year, and 500 at the end of the second year for each member who dies in the second year. Each member pays into the fund an amount equal to the single benefit premium for a special 2-year term insurance, with:

(i) Benefits:

| $k$ | $b_{k+1}$ |
|---|---|
| 0 | 1000 |
| 1 | 500 |

(ii) Mortality follows the Illustrative Life Table. (iii) $i = 0.06$

The actual experience of the fund is as follows:

| $k$ | Interest Rate Earned | Number of Deaths |
|---|---|---|
| 0 | 0.070 | 1 |
| 1 | 0.069 | 1 |

Calculate the difference, at the end of the second year, between the expected size of the fund as projected at time 0 and the actual fund.

(A) 840 (B) 870 (C) 900 (D) 930 (E) 960

 

(C) From the table $q_{30} = 0.00153$ and $q_{31} = 0.00161$. The amount which member pays is

$$A^{1}_{x:\overline{2}|} = (1000)(1.06)^{-1}q_{30} + (500)(1.06)^{-2}(1 - q_{30})q_{31} = 2.158747197.$$

The expected size of the fund at the end of the second year as projected at time 0 is zero. The actual fund at the end of the second year is

$$(1000)(2.158747197)(1.07)(1.069) - (1000)(1.069) - 500 = 900.2398058.$$

 