Exponentials and logarithms 14C

1 $V = 20\ 000\mathrm{e}^{-\frac{1}{12}}$

a The new value is when $t = 0$

$$\Rightarrow V = 20\ 000 \times \mathrm{e}^{-\frac{0}{12}}$$
$$= 20\ 000 \times 1$$
$$= 20\ 000$$

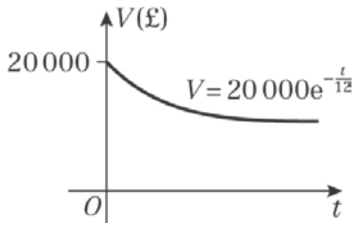
New value = £20 000

b Value after 4 years is given when $t = 4$

$$\Rightarrow V = 20\ 000 \times \mathrm{e}^{-\frac{4}{12}}$$
$$= 20\ 000 \times \mathrm{e}^{-\frac{1}{3}}$$
$$= 14\ 330.63$$

Value after 4 years is £14 331 (to nearest £).

c



2 $P = 20 + 10\mathrm{e}^{\frac{t}{50}}$

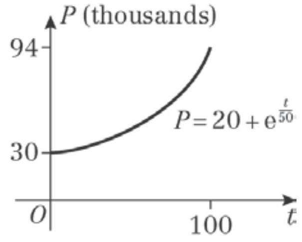
a The year 2000 corresponds to $t = 0$.

Substitute $t = 0$ into $P = 20 + 10\mathrm{e}^{\frac{t}{50}}$

$P = 20 + 10 \times \mathrm{e}^0 = 20 + 10 \times 1 = 30$
Population = 30 thousand

b $P = 20 + 10\mathrm{e}^{\frac{30}{50}}$
P = 38.221... thousand

2 c



Year 2100 is $t = 100$

$$P = 20 + 10\mathrm{e}^{\frac{100}{50}}$$
$$= 20 + 10\mathrm{e}^2$$
$$= 93.891 \text{ thousand}$$

d $P = 20 + 10\mathrm{e}^{\frac{500}{50}}$
$= 220\ 284.6579\ldots$ thousand
The model predicts the population of the country to be over 220 million. This is highly unlikely and by 2500 new factors are likely to affect population growth. Therefore, the model is not valid for predictions that far into the future.

3 $N = 300 - 100\mathrm{e}^{-0.5t}$

a The number first diagnosed means when $t = 0$.

Substitute $t = 0$ in $N = 300 - 100\mathrm{e}^{-0.5t}$

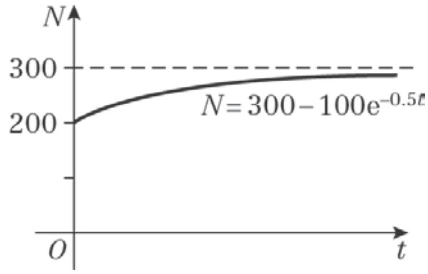
$$N = 300 - 100 \times \mathrm{e}^{-0.5 \times 0}$$
$$= 300 - 100 \times 1$$
$$= 200$$

b The long term prediction suggests $t \to \infty$.

As $t \to \infty, \mathrm{e}^{-0.5t} \to 0$

So $N \to 300 - 100 \times 0 = 300$

3 c



4 a i $R = 12e^{0.2m}$
$R = 12e^{0.2 \times 1}$
$= 14.6568...$
15 rabbits

ii $R = 12e^{0.2m}$
$R = 12e^{0.2 \times 12}$
$= 132.278...$
132 rabbits

b When $m = 0$, $R = 12$
12 is the initial number of rabbits in the population.

c $\frac{dR}{dm} = 0.2 \times 12e^{0.2m}$
$= 2.4e^{0.2m}$
When $m = 6$,
$\frac{dR}{dm} = 2.4e^{0.2 \times 6}$
$= 7.9682...$
≈ 8

d This model will stop giving valid results for large enough values of t as new factors are likely to affect population growth, such as the rabbits running out of food or space.

5 a $p = e^{-0.13h}$
$p = e^{-0.13 \times 4.394}$
$= 0.5648359...$
0.565 bars

5 b $\frac{dp}{dh} = -0.13e^{-0.13h}$
As $p = e^{-0.13h}$
$\frac{dp}{dh} = -0.13p$
$k = -0.13$

c The atmospheric pressure decreases exponentially as the altitude increases.

d $\frac{dp}{dh} = -0.13e^{-0.13h}$
When $h = 0$,
$\frac{dp}{dh} = -0.13e^{-0.13 \times 0}$
$= -0.13$
When $h = 1$,
$\frac{dp}{dh} = -0.13e^{-0.13 \times 1}$
$= -0.114152406...$

Percentage change
$= \frac{-0.11415206 + 0.13}{-0.13} \times 100$
$= 12.19...$
$= 12\%$

6 a Model 1: $T = 20\,000e^{-0.24t}$
When $t = 1$, $T = 20\,000e^{-0.24 \times 1}$
$= 15\,732.557...$
$= £15\,733$

Model 2: $T = 19\,000e^{-0.255t} + 1000$
When $t = 1$, $T = 19\,000e^{-0.255 \times 1} + 1000$
$= 15\,723.413...$
$= £15\,723$

b Model 1: $T = 20\,000e^{-0.24t}$
When $t = 10$, $T = 20\,000e^{-0.24 \times 10}$
$= 1814.359...$
$= £1814$

6 b Model 2: $T = 19\,000e^{-0.255t} + 1000$

When $t = 10$,

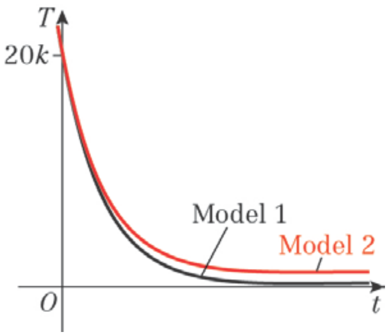
$T = 19\,000e^{-0.255 \times 10} + 1000$

$= 2483.5516...$

$= £2484$

So, Model 2 predicts a larger value.

c



d In Model 2 the tractor will always be worth at least £1000. This could be the value of the tractor as scrap metal.