

Exponentials and logarithms 14D

1 a $\log_4 256 = 4$

b $\log_3 \left(\frac{1}{9}\right) = -2$

c $\log_{10} 1\,000\,000 = 6$

d $\log_{11} 11 = 1$

e $\log_{0.2} 0.008 = 3$

2 a $2^4 = 16$

b $5^2 = 25$

c $9^{\frac{1}{2}} = 3$

d $5^{-1} = 0.2$

e $10^5 = 100\,000$

3 a If $\log_2 8 = x$ then $2^x = 8$, so $x = 3$

b If $\log_5 25 = x$ then $5^x = 25$, so $x = 2$

c If $\log_{10} 10\,000\,000 = x$
then $10^x = 10\,000\,000$, so $x = 7$

d If $\log_{12} 12 = x$ then $12^x = 12$, so $x = 1$

e If $\log_3 729 = x$ then $3^x = 729$, so $x = 6$

f If $\log_{10} \sqrt{10} = x$
then $10^x = \sqrt{10}$, so $x = \frac{1}{2}$
(Power $\frac{1}{2}$ means 'square root'.)

3 g If $\log_4 (0.25) = x$ then $4^x = 0.25 = \frac{1}{4}$,
so $x = -1$
(Negative power means 'reciprocal'.)

h $\log_{0.25} 16 = x$
 $\Rightarrow 0.25^x = 16$
 $\Rightarrow \left(\frac{1}{4}\right)^x = 16$, so $x = -2$
 $\left(\left(\frac{1}{4}\right)^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2} = \frac{1}{\left(\frac{1}{16}\right)} = 16\right)$

i $\log_a (a^{10}) = x$
 $\Rightarrow a^x = a^{10}$, so $x = 10$

j $\log_{\left(\frac{2}{3}\right)} \left(\frac{9}{4}\right) = x$
 $\Rightarrow \left(\frac{2}{3}\right)^x = \frac{9}{4} = \frac{1}{\left(\frac{2}{3}\right)^2} = \frac{1}{\left(\frac{4}{9}\right)} = \frac{9}{4} \Rightarrow x = -2$

4 a Using a power, $5^4 = x$
So $x = 625$

b Using a power, $x^2 = 81$
So $x = 9$
(The base of a logarithm cannot be negative, so $x = -9$ is not possible.)

c Using a power, $7^1 = x$
So $x = 7$

d $2^3 = x - 1$
 $x = 2^3 + 1$
 $= 9$

$$\begin{aligned}
 4 \text{ e } \quad 3^4 &= 4x + 1 \\
 4x &= 3^4 - 1 \\
 x &= \frac{1}{4}(3^4 - 1) \\
 &= 20
 \end{aligned}$$

f Using a power,

$$\begin{aligned}
 x^2 &= 2x \\
 x^2 - 2x &= 0 \\
 x(x - 2) &= 0 \\
 x &= 2
 \end{aligned}$$

(The base of a logarithm cannot be 0, so $x = 0$ is not possible)

$$5 \text{ a } \log_9 230 = 2.475$$

$$b \log_5 33 = 2.173$$

$$c \log_{10} 1020 = 3.009$$

$$d \log_e 3 = 1.099$$

$$6 \text{ a } \text{ Let } \log_2 50 = x$$

$$2^x = 50$$

$$\text{As } 2^5 = 32 \text{ and } 2^6 = 64,$$

$$32 < 2^x < 64$$

$$2^5 < 2^x < 2^6$$

$$\text{So } 5 < x < 6$$

$$b \log_2 50 = 5.644$$

$$7 \text{ a i } \log_2 2 = 1$$

$$\text{ii } \log_3 3 = 1$$

$$\text{iii } \log_{17} 17 = 1$$

$$b \text{ Let } \log_a a = x$$

$$a^x = a$$

$$x = 1$$

$$\therefore \log_a a = 1$$

$$8 \text{ a i } \log_2 1 = 0$$

$$\text{ii } \log_3 1 = 0$$

$$\text{iii } \log_{17} 1 = 0$$

$$b \text{ Let } \log_a 1 = x$$

$$a^x = 1$$

$$x = 0$$

$$\therefore \log_a 1 = 0$$