

Created by T. Madas

**EXPONENTIALS  
&  
LOGARITHMS  
practice**

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**Question 1**

Solve each of the following equations.

a)  $e^{x+1} = 17$

b)  $e^{4-3y} = 20$

c)  $5e^{2z} + 3 = 38$

d)  $2e^{1-w} + 7 = 23$

e)  $7e^{2t+3} - 2 = 47$

$$x = -1 + \ln 17 \approx 1.83, \quad y = \frac{1}{3}(4 - \ln 20) \approx 0.335, \quad z = \frac{1}{2} \ln 7 \approx 0.973,$$

$$w = 1 - \ln 8 \approx -1.08, \quad t = \frac{1}{2}(-3 + \ln 7) \approx -0.527$$

Handwritten solutions for the five equations:

- a)  $e^{x+1} = 17$   
 $\Rightarrow \ln(e^{x+1}) = \ln 17$   
 $\Rightarrow x+1 = \ln 17$   
 $\Rightarrow x = -1 + \ln 17$
- b)  $e^{4-3y} = 20$   
 $\Rightarrow \ln(e^{4-3y}) = \ln 20$   
 $\Rightarrow 4-3y = \ln 20$   
 $\Rightarrow 4 - \ln 20 = 3y$   
 $\Rightarrow y = \frac{1}{3}(4 - \ln 20)$
- c)  $5e^{2z} + 3 = 38$   
 $\Rightarrow 5e^{2z} = 35$   
 $\Rightarrow e^{2z} = 7$   
 $\Rightarrow \ln(e^{2z}) = \ln 7$   
 $\Rightarrow 2z = \ln 7$   
 $\Rightarrow z = \frac{1}{2} \ln 7$
- d)  $2e^{1-w} + 7 = 23$   
 $\Rightarrow 2e^{1-w} = 16$   
 $\Rightarrow e^{1-w} = 8$   
 $\Rightarrow \ln(e^{1-w}) = \ln 8$   
 $\Rightarrow 1-w = \ln 8$   
 $\Rightarrow w = 1 - \ln 8$
- e)  $7e^{2t+3} - 2 = 47$   
 $\Rightarrow 7e^{2t+3} = 49$   
 $\Rightarrow e^{2t+3} = 7$   
 $\Rightarrow \ln(e^{2t+3}) = \ln 7$   
 $\Rightarrow 2t+3 = \ln 7$   
 $\Rightarrow 2t = \ln 7 - 3$   
 $\Rightarrow t = \frac{1}{2}(\ln 7 - 3)$

## Question 2

Solve each of the following equations.

a)  $e^{x-2} = 10$

b)  $e^{1-2y} = 40$

c)  $3e^{-2z} - 5 = 28$

d)  $4e^{2-w} + 5 = 33$

e)  $2e^{2t+2} + 2 = 22$

$$x = 2 + \ln 10 \approx 4.30, \quad y = \frac{1}{2}(1 - \ln 40) \approx -1.34, \quad z = -\frac{1}{2} \ln 11 \approx -1.20,$$

$$w = 2 - \ln 7 \approx 0.0541, \quad t = \frac{1}{2}(-2 + \ln 10) \approx 0.151$$

Handwritten solutions for Question 2:

(a)  $e^{x-2} = 10$   
 $\Rightarrow \ln(e^{x-2}) = \ln 10$   
 $\Rightarrow x-2 = \ln 10$   
 $\Rightarrow x = 2 + \ln 10$   
 $\Rightarrow x \approx 4.30$

(b)  $e^{1-2y} = 40$   
 $\Rightarrow \ln(e^{1-2y}) = \ln 40$   
 $\Rightarrow 1-2y = \ln 40$   
 $\Rightarrow -2y = \ln 40 - 1$   
 $\Rightarrow y = \frac{1}{2}(1 - \ln 40)$   
 $\Rightarrow y \approx -1.34$

(c)  $3e^{-2z} - 5 = 28$   
 $\Rightarrow 3e^{-2z} = 33$   
 $\Rightarrow e^{-2z} = 11$   
 $\Rightarrow \ln(e^{-2z}) = \ln 11$   
 $\Rightarrow -2z = \ln 11$   
 $\Rightarrow z = -\frac{1}{2} \ln 11$   
 $\Rightarrow z \approx -1.20$

(d)  $4e^{2-w} + 5 = 33$   
 $\Rightarrow 4e^{2-w} = 28$   
 $\Rightarrow e^{2-w} = 7$   
 $\Rightarrow \ln(e^{2-w}) = \ln 7$   
 $\Rightarrow 2-w = \ln 7$   
 $\Rightarrow -w = \ln 7 - 2$   
 $\Rightarrow w = 2 - \ln 7$   
 $\Rightarrow w \approx 0.0541$

(e)  $2e^{2t+2} + 2 = 22$   
 $\Rightarrow 2e^{2t+2} = 20$   
 $\Rightarrow e^{2t+2} = 10$   
 $\Rightarrow \ln(e^{2t+2}) = \ln 10$   
 $\Rightarrow 2t+2 = \ln 10$   
 $\Rightarrow 2t = \ln 10 - 2$   
 $\Rightarrow t = \frac{1}{2}(\ln 10 - 2)$   
 $\Rightarrow t \approx 0.151$

## Question 3

Solve each of the following equations.

a)  $e^{2x} = 9$

b)  $5e^{1-y} = 30$

c)  $4 - 3e^{2z} = 3$

d)  $13 - e^{4-w} = 10$

e)  $2e^{3t+2} + 2 = 20$

$$x = \ln 3 \approx 1.10, \quad y = 1 - \ln 6 \approx -0.792, \quad z = -\frac{1}{2} \ln 3 \approx -0.549,$$

$$w = 4 - \ln 3 \approx 2.90, \quad t = \frac{1}{3}(-2 + \ln 9) \approx 0.0657$$

Handwritten solutions for Question 3:

a)  $e^{2x} = 9$   
 $\Rightarrow \ln(e^{2x}) = \ln 9$   
 $\Rightarrow 2x = \ln 9$   
 $\Rightarrow 2x = \ln 3^2$   
 $\Rightarrow 2x = 2 \ln 3$   
 $\Rightarrow x = \ln 3$   
 $\Rightarrow x \approx 1.10$

b)  $5e^{1-y} = 30$   
 $\Rightarrow e^{1-y} = 6$   
 $\Rightarrow \ln(e^{1-y}) = \ln 6$   
 $\Rightarrow 1-y = \ln 6$   
 $\Rightarrow 1 - \ln 6 = y$   
 $\Rightarrow y \approx -0.792$

c)  $4 - 3e^{2z} = 3$   
 $\Rightarrow 1 = 3e^{2z}$   
 $\Rightarrow \frac{1}{3} = e^{2z}$   
 $\Rightarrow \ln \frac{1}{3} = \ln(e^{2z})$   
 $\Rightarrow \ln \frac{1}{3} = 2z$   
 $\Rightarrow -\ln 3 = 2z$   
 $\Rightarrow z = -\frac{1}{2} \ln 3$   
 $\Rightarrow z \approx -0.549$

d)  $13 - e^{4-w} = 10$   
 $\Rightarrow 3 = e^{4-w}$   
 $\Rightarrow \ln 3 = \ln(e^{4-w})$   
 $\Rightarrow \ln 3 = 4 - w$   
 $\Rightarrow w = 4 - \ln 3$   
 $\Rightarrow w \approx 2.90$

e)  $2e^{3t+2} + 2 = 20$   
 $\Rightarrow 2e^{3t+2} = 18$   
 $\Rightarrow e^{3t+2} = 9$   
 $\Rightarrow \ln(e^{3t+2}) = \ln 9$   
 $\Rightarrow 3t+2 = \ln 9$   
 $\Rightarrow 3t = \ln 9 - 2$   
 $\Rightarrow t = \frac{1}{3}(\ln 9 - 2)$   
 $\Rightarrow t \approx 0.0657$

## Question 4

Solve each of the following equations, leaving your final answers as expressions involving natural logarithms in their simplest form.

a)  $e^x = 4$

b)  $e^{2y} = 9$

c)  $2e^{-z} + 1 = 9$

d)  $4e^{2w} - 7 = 57$

e)  $2e^{-3t} - 7 = 243$

$$x = 2\ln 2, \quad y = \ln 3, \quad z = -2\ln 2, \quad w = 2\ln 2, \quad t = -\ln 5$$

Handwritten solutions for Question 4:

a)  $e^x = 4$   
 $\Rightarrow x = \ln 4$   
 $\Rightarrow x = \ln(2^2)$   
 $\Rightarrow x = 2\ln 2$

b)  $e^{2y} = 9$   
 $\Rightarrow 2y = \ln 9$   
 $\Rightarrow 2y = \ln 3^2$   
 $\Rightarrow 2y = 2\ln 3$   
 $\Rightarrow y = \ln 3$

c)  $2e^{-z} + 1 = 9$   
 $\Rightarrow 2e^{-z} = 8$   
 $\Rightarrow e^{-z} = 4$   
 $\Rightarrow -z = \ln 4$   
 $\Rightarrow -z = 2\ln 2$   
 $\Rightarrow z = -2\ln 2$

d)  $4e^{2w} - 7 = 57$   
 $\Rightarrow 4e^{2w} = 64$   
 $\Rightarrow e^{2w} = 16$   
 $\Rightarrow 2w = \ln 16$   
 $\Rightarrow 2w = \ln 2^4$   
 $\Rightarrow 2w = 4\ln 2$   
 $\Rightarrow w = 2\ln 2$

e)  $2e^{-3t} - 7 = 243$   
 $\Rightarrow 2e^{-3t} = 250$   
 $\Rightarrow e^{-3t} = 125$   
 $\Rightarrow -3t = \ln 125$   
 $\Rightarrow -3t = \ln 5^3$   
 $\Rightarrow -3t = 3\ln 5$   
 $\Rightarrow t = -\ln 5$

## Question 5

Solve each of the following equations, leaving your final answers as expressions involving natural logarithms in their simplest form.

a)  $e^{2x} = 16$

b)  $e^{-2y} - 1 = 8$

c)  $3e^{2z} - 20 = 88$

d)  $e^{-3w} + 5 = 32$

e)  $2e^{2t-2} + 2 = 10$

$$\boxed{x = 2\ln 2}, \quad \boxed{y = -\ln 3}, \quad \boxed{z = \ln 6}, \quad \boxed{w = -\ln 3}, \quad \boxed{t = 1 + \ln 2 = \ln(2e)}$$

Handwritten solutions for the five equations:

(a)  $e^{2x} = 16$   
 $\Rightarrow 2x = \ln 16$   
 $\Rightarrow 2x = \ln 2^4$   
 $\Rightarrow 2x = 4\ln 2$   
 $\Rightarrow x = 2\ln 2$

(b)  $e^{-2y} - 1 = 8$   
 $\Rightarrow e^{-2y} = 9$   
 $\Rightarrow -2y = \ln 9$   
 $\Rightarrow -2y = \ln 3^2$   
 $\Rightarrow -2y = 2\ln 3$   
 $\Rightarrow y = -\ln 3$

(c)  $3e^{2z} - 20 = 88$   
 $\Rightarrow 3e^{2z} = 108$   
 $\Rightarrow e^{2z} = 36$   
 $\Rightarrow 2z = \ln 36$   
 $\Rightarrow z = \frac{1}{2}\ln 36$   
 $\Rightarrow z = \ln 3e^{\frac{1}{2}}$   
 $\Rightarrow z = \ln 6$

(d)  $e^{-3w} + 5 = 32$   
 $\Rightarrow e^{-3w} = 27$   
 $\Rightarrow -3w = \ln 27$   
 $\Rightarrow -3w = \ln 3^3$   
 $\Rightarrow -3w = 3\ln 3$   
 $\Rightarrow w = -\ln 3$

(e)  $2e^{2t} + 2 = 10$   
 $\Rightarrow e^{2t} + 1 = 5$   
 $\Rightarrow e^{2t} = 4$   
 $\Rightarrow 2t = \ln 4$   
 $\Rightarrow 2t = 2\ln 2$   
 $\Rightarrow t = \ln 2$

## Question 6

Solve each of the following equations, leaving your final answers as expressions involving natural logarithms in their simplest form.

a)  $e^{4x} = 16$

b)  $2e^{3y} - 1 = 127$

c)  $3e^{\frac{z}{2}} + 5 = 14$

d)  $1 - 25e^{-4w} = \frac{24}{25}$

e)  $\frac{7 + 16807e^{-2t}}{35} = 10$

$$x = \ln 2, \quad y = 2 \ln 2, \quad z = 2 \ln 3, \quad w = \ln 5, \quad t = \ln 7$$

Handwritten solutions for Question 6:

a)  $e^{4x} = 16$   
 $\Rightarrow 4x = \ln 16$   
 $\Rightarrow 4x = \ln 2^4$   
 $\Rightarrow 4x = 4 \ln 2$   
 $\Rightarrow x = \ln 2$

b)  $2e^{3y} - 1 = 127$   
 $\Rightarrow 2e^{3y} = 128$   
 $\Rightarrow e^{3y} = 64$   
 $\Rightarrow 3y = \ln 64$   
 $\Rightarrow 3y = \ln 2^6$   
 $\Rightarrow 3y = 6 \ln 2$   
 $\Rightarrow y = 2 \ln 2$

c)  $3e^{\frac{z}{2}} + 5 = 14$   
 $\Rightarrow 3e^{\frac{z}{2}} = 9$   
 $\Rightarrow e^{\frac{z}{2}} = 3$   
 $\Rightarrow \frac{z}{2} = \ln 3$   
 $\Rightarrow z = 2 \ln 3$

d)  $1 - 25e^{-4w} = \frac{24}{25}$   
 $\Rightarrow \frac{1}{25} = 25e^{-4w}$   
 $\Rightarrow \frac{1}{25} = e^{4w}$   
 $\Rightarrow 4w = \ln 25$   
 $\Rightarrow 4w = \ln 5^2$   
 $\Rightarrow 4w = 2 \ln 5$   
 $\Rightarrow w = \ln 5$

e)  $\frac{7 + 16807e^{-2t}}{35} = 10$   
 $\Rightarrow 7 + 16807e^{-2t} = 350$   
 $\Rightarrow 16807e^{-2t} = 343$   
 $\Rightarrow \frac{e^{-2t}}{e^t} = \frac{1}{49}$   
 $\Rightarrow e^{-3t} = \frac{1}{49}$   
 $\Rightarrow -3t = \ln \frac{1}{49}$   
 $\Rightarrow -3t = -\ln 49$   
 $\Rightarrow 3t = \ln 49$   
 $\Rightarrow 3t = \ln 7^2$   
 $\Rightarrow 3t = 2 \ln 7$   
 $\Rightarrow t = \ln 7$

**Question 7**

Simplify each of the following expressions, giving the answer to the required form.

a)  $2\ln 9 - \ln 6 - 4\ln \sqrt{3} + \ln 2 \equiv a \ln 3$

b)  $2\ln 54 - \ln 12 \equiv b \ln 3$

c)  $\frac{7}{4}\ln 16 - \frac{2}{3}\ln 8 \equiv c \ln 2$

d)  $2\ln 56 - \left[ \ln 168 - \ln\left(\frac{3}{7}\right) \right] \equiv k \ln 2$

e)  $2\ln 108 - 3\ln 48 \equiv p \ln 3 + q \ln 2$

$a=1$ ,  $b=5$ ,  $c=5$ ,  $k=3$ ,  $p=3, q=-8$

Handwritten solutions for Question 7:

a)  $2\ln 9 - \ln 6 - 4\ln \sqrt{3} + \ln 2 = \ln 81 - \ln 6 - \ln(3^4) + \ln 2$   
 $= \ln 81 - \ln 6 - \ln 81 + \ln 2$   
 $= \ln\left(\frac{81 \times 2}{6 \times 81}\right) = \ln\left(\frac{2}{6}\right) = \ln\left(\frac{1}{3}\right) = -\ln 3$

b)  $2\ln 54 - \ln 12 = \ln(54^2) - \ln 12 = \ln\left(\frac{2916}{12}\right) = \ln(243)$   
 $= \ln(3^5) = 5\ln 3$

c)  $\frac{7}{4}\ln 16 - \frac{2}{3}\ln 8 = \frac{7}{4}\ln(2^4) - \frac{2}{3}\ln(2^3) = 7\ln 2 - 2\ln 2 = 5\ln 2$

d)  $2\ln 56 - \left(\ln 168 - \ln\left(\frac{3}{7}\right)\right) = \ln 56^2 - \ln 168 + \ln\left(\frac{3}{7}\right)$   
 $= \ln\left(\frac{56^2 \times 3}{168 \times 7}\right) = \ln\left(\frac{9184}{1176}\right) = \ln 8 = 3\ln 2$

e)  $2\ln 108 - 3\ln 48 = \ln 108^2 - \ln 48^3 = \ln\left(\frac{11664}{110592}\right) = \ln\left(\frac{1}{9.48}\right)$   
 $= \ln 27 - \ln 288 = \ln 3^3 - \ln 2^8 = 3\ln 3 - 8\ln 2$

**Question 8**

Solve each of the following logarithmic equations.

a)  $\ln(x+1) = 2$

b)  $\ln(4-y) = 2$

c)  $\ln(3z-1) + 6 = 7$

d)  $2\ln(1-2w) + 2 = 6$

e)  $\ln(3-2t) + 4 = -2$

$$x = e^2 - 1 \approx 6.39, \quad y = 4 - e^2 \approx -3.39, \quad z = \frac{1}{3}(e+1) \approx 1.24,$$

$$w = \frac{1}{2}(1 - e^2) \approx -3.19, \quad t = \frac{1}{2}(3 - e^{-6}) \approx 1.50$$

Handwritten solutions for the logarithmic equations:

- a)  $\ln(x+1) = 2$   
 $\Rightarrow e^{\ln(x+1)} = e^2$   
 $\Rightarrow x+1 = e^2$   
 $\Rightarrow x = e^2 - 1$   
 $\Rightarrow x \approx 6.39$
- b)  $\ln(4-y) = 2$   
 $\Rightarrow e^{\ln(4-y)} = e^2$   
 $\Rightarrow 4-y = e^2$   
 $\Rightarrow 4 - e^2 = y$   
 $\Rightarrow y \approx -3.39$
- c)  $\ln(3z-1) + 6 = 7$   
 $\Rightarrow \ln(3z-1) = 1$   
 $\Rightarrow e^{\ln(3z-1)} = e^1$   
 $\Rightarrow 3z-1 = e$   
 $\Rightarrow 3z = e+1$   
 $\Rightarrow z = \frac{1}{3}(e+1) \approx 1.24$
- d)  $2\ln(1-2w) + 2 = 6$   
 $\Rightarrow 2\ln(1-2w) = 4$   
 $\Rightarrow \ln(1-2w) = 2$   
 $\Rightarrow e^{\ln(1-2w)} = e^2$   
 $\Rightarrow 1-2w = e^2$   
 $\Rightarrow -2w = e^2 - 1$   
 $\Rightarrow w = \frac{1}{2}(1 - e^2) \approx -3.19$
- e)  $\ln(3-2t) + 4 = -2$   
 $\Rightarrow \ln(3-2t) = -6$   
 $\Rightarrow e^{\ln(3-2t)} = e^{-6}$   
 $\Rightarrow 3-2t = e^{-6}$   
 $\Rightarrow -2t = e^{-6} - 3$   
 $\Rightarrow t = \frac{1}{2}(3 - e^{-6}) \approx 1.50$

**Question 9**

Solve each of the following logarithmic equations.

a)  $\ln(x-3) = 2$

b)  $1 - \ln(3-y) = -1$

c)  $2\ln(2z-1) + 5 = 3$

d)  $\ln(1-2w) = 1 + \ln w$

e)  $\ln(2t-3) = 2 + \ln(2t+3)$

$$x = e^2 + 3 \approx 10.4, \quad y = 3 - e^2 \approx -4.39, \quad z = \frac{1}{2}(e^{-1} + 1) \approx 0.684,$$

$$w = \frac{1}{e+2} \approx 0.212, \quad t = \frac{3(1+e^2)}{2(1-e^2)} \approx -1.97$$

Handwritten solutions for the logarithmic equations:

a)  $\ln(x-3) = 2$   
 $\Rightarrow e^{\ln(x-3)} = e^2$   
 $\Rightarrow x-3 = e^2$   
 $\Rightarrow x = 3 + e^2$   
 $\Rightarrow x \approx 10.4$

b)  $1 - \ln(3-y) = -1$   
 $\Rightarrow 2 = \ln(3-y)$   
 $\Rightarrow e^2 = e^{\ln(3-y)}$   
 $\Rightarrow 3-y = e^2$   
 $\Rightarrow 3 - e^2 = y$   
 $\Rightarrow y \approx -4.39$

c)  $2\ln(2z-1) + 5 = 3$   
 $\Rightarrow 2\ln(2z-1) = -2$   
 $\Rightarrow \ln(2z-1) = -1$   
 $\Rightarrow e^{\ln(2z-1)} = e^{-1}$   
 $\Rightarrow 2z-1 = e^{-1}$   
 $\Rightarrow 2z = 1 + e^{-1}$   
 $\Rightarrow z = \frac{1}{2}(1 + e^{-1})$   
 $\Rightarrow z \approx 0.684$

d)  $\ln(1-2w) = 1 + \ln w$   
 $\Rightarrow \ln(1-2w) - \ln w = 1$   
 $\Rightarrow \ln\left(\frac{1-2w}{w}\right) = 1$   
 $\Rightarrow e^{\ln\left(\frac{1-2w}{w}\right)} = e^1$   
 $\Rightarrow \frac{1-2w}{w} = e$   
 $\Rightarrow 1-2w = ew$   
 $\Rightarrow 1 = 2w + ew$   
 $\Rightarrow 1 = w(2+e)$   
 $\Rightarrow w = \frac{1}{2+e}$   
 $\Rightarrow w \approx 0.212$

e)  $\ln(2t-3) = 2 + \ln(2t+3)$   
 $\Rightarrow \ln(2t-3) - \ln(2t+3) = 2$   
 $\Rightarrow \ln\left(\frac{2t-3}{2t+3}\right) = 2$   
 $\Rightarrow e^{\ln\left(\frac{2t-3}{2t+3}\right)} = e^2$   
 $\Rightarrow \frac{2t-3}{2t+3} = e^2$   
 $\Rightarrow 2t-3 = 2e^2t + 3e^2$   
 $\Rightarrow 2t - 2e^2t = 3 + 3e^2$   
 $\Rightarrow t(2-2e^2) = 3 + 3e^2$   
 $\Rightarrow t = \frac{3 + 3e^2}{2 - 2e^2}$   
 $\Rightarrow t = \frac{3(1+e^2)}{2(1-e^2)} \approx -1.97$

**Question 10**

Solve each of the following logarithmic equations.

a)  $\ln(x+3) = 2$

b)  $2\ln(1-y) + 1 = 3$

c)  $3 - 2\ln(2z-1) = 5$

d)  $\ln(2w+1) = 1 + \ln(w-1)$

e)  $\ln(t+1) = 2 + \ln(3t)$

$$x = e^2 - 3 \approx 4.39, \quad y = 1 - e \approx -1.72, \quad z = \frac{1}{2}(e^{-1} + 1) \approx 0.684, \quad w = \frac{e+1}{e-2} \approx 5.18,$$

$$t = \frac{1}{3e^2 - 1} \approx 0.0472$$

(a)  $\ln(x+3) = 2$   
 $\Rightarrow e^{\ln(x+3)} = e^2$   
 $\Rightarrow x+3 = e^2$   
 $\Rightarrow x = e^2 - 3$   
 $\Rightarrow x \approx 4.39$

(b)  $2\ln(1-y) + 1 = 3$   
 $\Rightarrow 2\ln(1-y) = 2$   
 $\Rightarrow \ln(1-y) = 1$   
 $\Rightarrow e^{\ln(1-y)} = e^1$   
 $\Rightarrow 1-y = e$   
 $\Rightarrow 1-e = y$   
 $\Rightarrow y = 1-e$   
 $\Rightarrow y \approx -1.72$

(c)  $3 - 2\ln(2z-1) = 5$   
 $\Rightarrow -2 = 2\ln(2z-1)$   
 $\Rightarrow -1 = \ln(2z-1)$   
 $\Rightarrow \frac{e^{-1}}{e} = e^{\ln(2z-1)}$   
 $\Rightarrow e^{-1} = 2z-1$   
 $\Rightarrow 1+e^{-1} = 2z$   
 $\Rightarrow z = \frac{1}{2}(1+e^{-1})$   
 $\Rightarrow z \approx 0.684$

(d)  $\ln(2w+1) = 1 + \ln(w-1)$   
 $\Rightarrow \ln(2w+1) - \ln(w-1) = 1$   
 $\Rightarrow \ln\left(\frac{2w+1}{w-1}\right) = 1$   
 $\Rightarrow e^{\ln\left(\frac{2w+1}{w-1}\right)} = e^1$   
 $\Rightarrow \frac{2w+1}{w-1} = e$   
 $\Rightarrow 2w+1 = ew - e$   
 $\Rightarrow 1+e = ew - 2w$   
 $\Rightarrow 1+e = w(e-2)$   
 $\Rightarrow w = \frac{e+1}{e-2}$   
 $\Rightarrow w \approx 5.18$

(e)  $\ln(t+1) = 2 + \ln(3t)$   
 $\Rightarrow \ln(t+1) - \ln(3t) = 2$   
 $\Rightarrow \ln\left(\frac{t+1}{3t}\right) = 2$   
 $\Rightarrow e^{\ln\left(\frac{t+1}{3t}\right)} = e^2$   
 $\Rightarrow \frac{t+1}{3t} = e^2$   
 $\Rightarrow t+1 = 3te^2$   
 $\Rightarrow 1 = 3te^2 - t$   
 $\Rightarrow 1 = t(3e^2 - 1)$   
 $\Rightarrow t = \frac{1}{3e^2 - 1}$   
 $\Rightarrow t \approx 0.0472$

**Question 11**

Solve each of the following logarithmic equations.

a)  $\ln(3x) - \ln(x-1) = 1$

b)  $2\ln y = \ln(2y+8)$

c)  $2\ln z = \ln(4z+12)$

d)  $\ln(2w) = 2 + \ln(w+3)$

e)  $\ln(t+7) = 1 + \ln(t-2)$

$$x = \frac{e}{e-3} \approx -9.65, \quad y = 4, \quad y \neq -2, \quad z = 6, \quad z \neq -2, \quad w = \frac{3e^2}{2-e^2} \approx -4.11,$$

$$t = \frac{2e+7}{e-1} \approx 7.24$$

Handwritten solutions for the logarithmic equations:

(a)  $\ln(3x) - \ln(x-1) = 1$   
 $\Rightarrow \ln\left(\frac{3x}{x-1}\right) = 1$   
 $\Rightarrow e^{\ln\left(\frac{3x}{x-1}\right)} = e^1$   
 $\Rightarrow \frac{3x}{x-1} = e$   
 $\Rightarrow 3x = ex - e$   
 $\Rightarrow e = ex - 3x$   
 $\Rightarrow e = x(e-3)$   
 $\Rightarrow x = \frac{e}{e-3} \approx -9.65$

(b)  $2\ln y = \ln(2y+8)$   
 $\Rightarrow \ln y^2 = \ln(2y+8)$   
 $\Rightarrow y^2 = 2y+8$   
 $\Rightarrow y^2 - 2y - 8 = 0$   
 $\Rightarrow (y-4)(y+2) = 0$   
 $\Rightarrow y = 4$  (since  $y \neq -2$ )

(c)  $2\ln z = \ln(4z+12)$   
 $\Rightarrow \ln z^2 = \ln(4z+12)$   
 $\Rightarrow z^2 = 4z+12$   
 $\Rightarrow z^2 - 4z - 12 = 0$   
 $\Rightarrow (z+2)(z-6) = 0$   
 $\Rightarrow z = 6$  (since  $z \neq -2$ )

(d)  $\ln(2w) = 2 + \ln(w+3)$   
 $\Rightarrow \ln(2w) - \ln(w+3) = 2$   
 $\Rightarrow \ln\left(\frac{2w}{w+3}\right) = 2$   
 $\Rightarrow \frac{2w}{w+3} = e^2$   
 $\Rightarrow 2w = e^2w + 3e^2$   
 $\Rightarrow 2w - e^2w = 3e^2$   
 $\Rightarrow w(2-e^2) = 3e^2$   
 $\Rightarrow w = \frac{3e^2}{2-e^2} \approx -4.11$

(e)  $\ln(t+7) = 1 + \ln(t-2)$   
 $\Rightarrow \ln(t+7) - \ln(t-2) = 1$   
 $\Rightarrow \ln\left(\frac{t+7}{t-2}\right) = 1$   
 $\Rightarrow e^{\ln\left(\frac{t+7}{t-2}\right)} = e^1$   
 $\Rightarrow \frac{t+7}{t-2} = e$   
 $\Rightarrow t+7 = et-2e$   
 $\Rightarrow 7+2e = et-t$   
 $\Rightarrow 7+2e = t(e-1)$   
 $\Rightarrow t = \frac{7+2e}{e-1} \approx 7.24$

**Question 12**

Solve each of the following logarithmic equations.

a)  $\ln(2x) - \ln(x+2) = 1$

b)  $2\ln y = \ln(y+20)$

c)  $2\ln z = \ln(5z-6)$

d)  $\ln(2w-1) = 2 + \ln(w-2)$

e)  $\ln(4t+1) = 1 + \ln(3t-2)$

$$\boxed{x = \frac{2e}{2-e} \approx -7.57}, \quad \boxed{y = 5, y \neq -4}, \quad \boxed{z = 2, z = 3}, \quad \boxed{w = \frac{2e^2-1}{e^2-2} \approx 2.56},$$

$$\boxed{t = \frac{2e+1}{3e-4} \approx 1.55}$$

**Question 13**

Solve each of the following exponential equations.

a)  $e^x + 8e^{-x} = 6$

b)  $3e^y + 2e^{-y} = 7$

c)  $3e^z - 2e^{-z} + 5 = 0$

d)  $2e^w + 5e^{-w} = 7$

e)  $2e^t + 5 = 3e^{-t}$

$x = \ln 2, 2 \ln 2$ ,  $y = \ln 2, -\ln 3$ ,  $z = -\ln 3$ ,  $w = 0, \ln\left(\frac{5}{2}\right)$ ,  $t = -\ln 2$

Handwritten solutions for the exponential equations in question 13:

a)  $e^x + 8e^{-x} = 6$   
 $\Rightarrow e^x + \frac{8}{e^x} = 6$   
 $\Rightarrow y + \frac{8}{y} = 6 \quad (y=e^x)$   
 $\Rightarrow y^2 + 8 = 6y$   
 $\Rightarrow y^2 - 6y + 8 = 0$   
 $\Rightarrow (y-2)(y-4) = 0$   
 $\Rightarrow y = 2 \text{ or } 4$   
 $\therefore x = \ln 2 \text{ or } \ln 4$

b)  $3e^y + 2e^{-y} = 7$   
 $\Rightarrow 3e^y + \frac{2}{e^y} = 7$   
 $\Rightarrow 3a + \frac{2}{a} = 7 \quad (a=e^y)$   
 $\Rightarrow 3a^2 + 2 = 7a$   
 $\Rightarrow 3a^2 - 7a + 2 = 0$   
 $\Rightarrow (3a-7)(a-2) = 0$   
 $\Rightarrow a = \frac{7}{3} \text{ or } 2$   
 $\therefore y = \ln \frac{7}{3} \text{ or } \ln 2$

c)  $3e^z - 2e^{-z} + 5 = 0$   
 $\Rightarrow 3e^z - \frac{2}{e^z} + 5 = 0$   
 $\Rightarrow 3a^2 - 2 + 5a = 0 \quad (a=e^z)$   
 $\Rightarrow 3a^2 + 5a - 2 = 0$   
 $\Rightarrow (3a-1)(a+2) = 0$   
 $\Rightarrow a = \frac{1}{3} \text{ or } -2$   
 $\therefore z = \ln \frac{1}{3} = -\ln 3$

d)  $2e^w + 5e^{-w} = 7$   
 $\Rightarrow 2e^w + \frac{5}{e^w} = 7$   
 $\Rightarrow 2a + \frac{5}{a} = 7 \quad (a=e^w)$   
 $\Rightarrow 2a^2 + 5 = 7a$   
 $\Rightarrow 2a^2 - 7a + 5 = 0$   
 $\Rightarrow (2a-5)(a-1) = 0$   
 $\Rightarrow a = \frac{5}{2} \text{ or } 1$   
 $\therefore w = \ln \frac{5}{2} \text{ or } 0$

e)  $2e^t + 5 = 3e^{-t}$   
 $\Rightarrow 2e^t + 5 = \frac{3}{e^t}$   
 $\Rightarrow 2a + 5 = \frac{3}{a} \quad (a=e^t)$   
 $\Rightarrow 2a^2 + 5a = 3$   
 $\Rightarrow 2a^2 + 5a - 3 = 0$   
 $\Rightarrow (2a-1)(a+3) = 0$   
 $\Rightarrow a = \frac{1}{2} \text{ or } -3$   
 $\therefore t = \ln \frac{1}{2} = -\ln 2$

**Question 14**

Solve each of the following exponential equations.

a)  $e^x + 3e^{-x} = 4$

b)  $3e^y + 20e^{-y} = 19$

c)  $3e^z + 5e^{-z} = 16$

d)  $e^w + e^{-w} = \frac{10}{3}$

e)  $e^{2t} + 15 = 8e^t$

$x = 0, \ln 3$ ,  $y = \ln 5, \ln\left(\frac{4}{3}\right)$ ,  $z = \ln 5, -\ln 3$ ,  $w = \pm \ln 3$ ,  $t = \ln 3, \ln 5$

Handwritten solutions for the exponential equations:

a)  $e^x + 3e^{-x} = 4$   
 $\Rightarrow e^x + \frac{3}{e^x} = 4$   
 $\Rightarrow a + \frac{3}{a} = 4 \quad (a = e^x)$   
 $\Rightarrow a^2 + 3 = 4a$   
 $\Rightarrow a^2 - 4a + 3 = 0$   
 $\Rightarrow (a-3)(a-1) = 0$   
 $\Rightarrow a = \frac{1}{3} \quad e^x = \frac{1}{3}$   
 $\Rightarrow x = \ln \frac{1}{3} = -\ln 3$   
 $\Rightarrow x = \ln 3$

b)  $3e^y + 20e^{-y} = 19$   
 $\Rightarrow 3e^y + \frac{20}{e^y} = 19$   
 $\Rightarrow 3a + \frac{20}{a} = 19 \quad (a = e^y)$   
 $\Rightarrow 3a^2 + 20 = 19a$   
 $\Rightarrow 3a^2 - 19a + 20 = 0$   
 $\Rightarrow (3a-4)(a-5) = 0$   
 $\Rightarrow a = \frac{4}{3} \quad e^y = \frac{4}{3}$   
 $\Rightarrow y = \ln \frac{4}{3}$   
 $\Rightarrow y = \ln 5$

c)  $3e^z + 5e^{-z} = 16$   
 $\Rightarrow 3e^z + \frac{5}{e^z} = 16$   
 $\Rightarrow 3a + \frac{5}{a} = 16 \quad (a = e^z)$   
 $\Rightarrow 3a^2 + 5 = 16a$   
 $\Rightarrow 3a^2 - 16a + 5 = 0$   
 $\Rightarrow (3a-1)(a-5) = 0$   
 $\Rightarrow a = \frac{1}{3} \quad e^z = \frac{1}{3}$   
 $\Rightarrow z = \ln \frac{1}{3} = -\ln 3$   
 $\Rightarrow z = \ln 5$

d)  $e^w + e^{-w} = \frac{10}{3}$   
 $\Rightarrow e^w + \frac{1}{e^w} = \frac{10}{3}$   
 $\Rightarrow a + \frac{1}{a} = \frac{10}{3} \quad (a = e^w)$   
 $\Rightarrow a^2 + 1 = \frac{10}{3}a$   
 $\Rightarrow 3a^2 + 3 = 10a$   
 $\Rightarrow 3a^2 - 10a + 3 = 0$   
 $\Rightarrow (3a-1)(a-3) = 0$   
 $\Rightarrow a = \frac{1}{3} \quad e^w = \frac{1}{3}$   
 $\Rightarrow w = \ln \frac{1}{3} = -\ln 3$   
 $\Rightarrow w = \ln 3$

e)  $e^{2t} + 15 = 8e^t$   
 $\Rightarrow e^{2t} - 8e^t + 15 = 0$   
 $\Rightarrow (e^t)^2 - 8e^t + 15 = 0$   
 $\Rightarrow a^2 - 8a + 15 = 0 \quad (a = e^t)$   
 $\Rightarrow (a-3)(a-5) = 0$   
 $\Rightarrow a = \frac{3}{8} \quad e^t = \frac{3}{8}$   
 $\Rightarrow t = \ln \frac{3}{8} = \ln 3 - \ln 8$   
 $\Rightarrow t = \ln 5$

**Question 15**

Solve each of the following exponential equations.

a)  $e^x - 8e^{-x} = 2$

b)  $2(e^y + e^{-y}) = 5$

c)  $\frac{8e^z}{e^{2z}-1} = 3$

d)  $e^w - 3 = \frac{8}{e^w - 1}$

e)  $e^{2t} + e^{-2t} = 4$

$x = 2 \ln 2$ ,  $y = \pm \ln 2$ ,  $z = \ln 3$ ,  $w = \ln 5$ ,  $t = \frac{1}{2} \ln(2 \pm \sqrt{3}) = \pm \frac{1}{2} \ln(2 + \sqrt{3})$

Handwritten solutions for the exponential equations in question 15:

a)  $e^x - 8e^{-x} = 2$   
 $\Rightarrow a - \frac{8}{a} = 2$  ( $a = e^x$ )  
 $\Rightarrow a^2 - 8 = 2a$   
 $\Rightarrow a^2 - 2a - 8 = 0$   
 $\Rightarrow (a-4)(a+2) = 0$   
 $\Rightarrow a = 4$  or  $a = -2$  (reject)  
 $\therefore x = \ln 4 = 2 \ln 2$

b)  $2(e^y + e^{-y}) = 5$   
 $\Rightarrow 2e^y + \frac{2}{e^y} = 5$  ( $a = e^y$ )  
 $\Rightarrow 2a^2 + 2 = 5a$   
 $\Rightarrow 2a^2 - 5a + 2 = 0$   
 $\Rightarrow (2a-1)(a-2) = 0$   
 $\Rightarrow a = \frac{1}{2}$  or  $a = 2$   
 $y = \ln \frac{1}{2}$  or  $y = \ln 2$   
 $y = -\ln 2$  or  $y = \ln 2$

c)  $\frac{8e^z}{e^{2z}-1} = 3$   
 $\Rightarrow 8e^z = 3(e^{2z}-1)$   
 $\Rightarrow 0 = 3e^{2z} - 8e^z - 3$   
 $\Rightarrow 3(e^z)^2 - 8e^z - 3 = 0$  ( $a = e^z$ )  
 $\Rightarrow (3a+1)(a-3) = 0$

d)  $e^w - 3 = \frac{8}{e^w - 1}$   
 $\Rightarrow (e^w - 3)(e^w - 1) = 8$   
 $\Rightarrow e^{2w} - 4e^w + 3 = 8$   
 $\Rightarrow e^{2w} - 4e^w - 5 = 0$  ( $a = e^w$ )  
 $\Rightarrow (a+1)(a-5) = 0$   
 $\Rightarrow a = -1$  (reject) or  $a = 5$   
 $\therefore w = \ln 5$

e)  $e^{2t} + e^{-2t} = 4$   
 $\Rightarrow e^{2t} + \frac{1}{e^{2t}} = 4$  ( $a = e^{2t}$ )  
 $\Rightarrow a + \frac{1}{a} = 4$   
 $\Rightarrow a^2 - 4a + 1 = 0$   
 $\Rightarrow (a-2)^2 - 4 + 1 = 0$   
 $\Rightarrow (a-2)^2 = 3$   
 $\Rightarrow a - 2 = \pm \sqrt{3}$   
 $\Rightarrow a = 2 \pm \sqrt{3}$   
 $\Rightarrow e^{2t} = 2 \pm \sqrt{3}$   
 $\Rightarrow 2t = \ln(2 \pm \sqrt{3})$   
 $\Rightarrow t = \frac{1}{2} \ln(2 \pm \sqrt{3})$