EXPONENTIALS
&
LOGARITHMS
practice

Question 1

Solve each of the following equations.

a) $e^{x+1} = 17$

b) $e^{4-3y} = 20$

c) $5e^{2z} + 3 = 38$

d) $2e^{1-w} + 7 = 23$

e) $7e^{2t+3} - 2 = 47$

$$\boxed{x = -1 + \ln 17 \approx 1.83}, \quad \boxed{y = \frac{1}{3}(4 - \ln 20) \approx 0.335}, \quad \boxed{z = \frac{1}{2}\ln 7 \approx 0.973},$$

$$\boxed{w = 1 - \ln 8 \approx -1.08}, \quad \boxed{t = \frac{1}{2}(-3 + \ln 7) \approx -0.527}$$

Question 2

Solve each of the following equations.

a) $e^{x-2} = 10$

b) $e^{1-2y} = 40$

c) $3e^{-2z} - 5 = 28$

d) $4e^{2-w} + 5 = 33$

e) $2e^{2t+2} + 2 = 22$

$$\boxed{x = 2 + \ln 10 \approx 4.30}, \quad \boxed{y = \frac{1}{2}(1 - \ln 40) \approx -1.34}, \quad \boxed{z = -\frac{1}{2}\ln 11 \approx -1.20},$$

$$\boxed{w = 2 - \ln 7 \approx 0.0541}, \quad \boxed{t = \frac{1}{2}(-2 + \ln 10) \approx 0.151}$$

Question 3

Solve each of the following equations.

a) $e^{2x} = 9$

b) $5e^{1-y} = 30$

c) $4 - 3e^{2z} = 3$

d) $13 - e^{4-w} = 10$

e) $2e^{3t+2} + 2 = 20$

$$\boxed{x = \ln 3 \approx 1.10}, \quad \boxed{y = 1 - \ln 6 \approx -0.792}, \quad \boxed{z = -\frac{1}{2}\ln 3 \approx -0.549},$$

$$\boxed{w = 4 - \ln 3 \approx 2.90}, \quad \boxed{t = \frac{1}{3}(-2 + \ln 9) \approx 0.0657}$$

Question 4

Solve each of the following equations, leaving your final answers as expressions involving natural logarithms in their simplest form.

a) $e^x = 4$

b) $e^{2y} = 9$

c) $2e^{-z} + 1 = 9$

d) $4e^{2w} - 7 = 57$

e) $2e^{-3t} - 7 = 243$

$\boxed{x = 2\ln 2}$, $\boxed{y = \ln 3}$, $\boxed{z = -2\ln 2}$, $\boxed{w = 2\ln 2}$, $\boxed{t = -\ln 5}$

Question 5

Solve each of the following equations, leaving your final answers as expressions involving natural logarithms in their simplest form.

a) $e^{2x} = 16$

b) $e^{-2y} - 1 = 8$

c) $3e^{2z} - 20 = 88$

d) $e^{-3w} + 5 = 32$

e) $2e^{2t-2} + 2 = 10$

$$\boxed{x = 2\ln 2}, \ \boxed{y = -\ln 3}, \ \boxed{z = \ln 6}, \ \boxed{w = -\ln 3}, \ \boxed{t = 1 + \ln 2 = \ln(2e)}$$

Question 6

Solve each of the following equations, leaving your final answers as expressions involving natural logarithms in their simplest form.

a) $e^{4x} = 16$

b) $2e^{3y} - 1 = 127$

c) $3e^{\frac{z}{2}} + 5 = 14$

d) $1 - 25e^{-4w} = \dfrac{24}{25}$

e) $\dfrac{7 + 16807e^{-2t}}{35} = 10$

$\boxed{x = \ln 2}$, $\boxed{y = 2\ln 2}$, $\boxed{z = 2\ln 3}$, $\boxed{w = \ln 5}$, $\boxed{t = \ln 7}$

Question 7

Simplify each of the following expressions, giving the answer to the required form.

a) $2\ln 9 - \ln 6 - 4\ln\sqrt{3} + \ln 2 \equiv a\ln 3$

b) $2\ln 54 - \ln 12 \equiv b\ln 3$

c) $\frac{7}{4}\ln 16 - \frac{2}{3}\ln 8 \equiv c\ln 2$

d) $2\ln 56 - \left[\ln 168 - \ln\left(\frac{3}{7}\right)\right] \equiv k\ln 2$

e) $2\ln 108 - 3\ln 48 \equiv p\ln 3 + q\ln 2$

$\boxed{a=1}$, $\boxed{b=5}$, $\boxed{c=5}$, $\boxed{k=3}$, $\boxed{p=3, q=-8}$

Question 8

Solve each of the following logarithmic equations.

a) $\ln(x+1) = 2$

b) $\ln(4-y) = 2$

c) $\ln(3z-1) + 6 = 7$

d) $2\ln(1-2w) + 2 = 6$

e) $\ln(3-2t) + 4 = -2$

$$\boxed{x = e^2 - 1 \approx 6.39}, \quad \boxed{y = 4 - e^2 \approx -3.39}, \quad \boxed{z = \tfrac{1}{3}(e+1) \approx 1.24},$$

$$\boxed{w = \tfrac{1}{2}(1 - e^2) \approx -3.19}, \quad \boxed{t = \tfrac{1}{2}(3 - e^{-6}) \approx 1.50}$$

Question 9

Solve each of the following logarithmic equations.

a) $\ln(x-3) = 2$

b) $1 - \ln(3-y) = -1$

c) $2\ln(2z-1) + 5 = 3$

d) $\ln(1-2w) = 1 + \ln w$

e) $\ln(2t-3) = 2 + \ln(2t+3)$

$$\boxed{x = e^2 + 3 \approx 10.4}, \; \boxed{y = 3 - e^2 \approx -4.39}, \; \boxed{z = \tfrac{1}{2}(e^{-1}+1) \approx 0.684},$$

$$\boxed{w = \frac{1}{e+2} \approx 0.212}, \; \boxed{t = \frac{3(1+e^2)}{2(1-e^2)} \approx -1.97}$$

Question 10

Solve each of the following logarithmic equations.

a) $\ln(x+3) = 2$

b) $2\ln(1-y)+1 = 3$

c) $3-2\ln(2z-1) = 5$

d) $\ln(2w+1) = 1+\ln(w-1)$

e) $\ln(t+1) = 2+\ln(3t)$

$$\boxed{x = e^2 - 3 \approx 4.39}, \quad \boxed{y = 1-e \approx -1.72}, \quad \boxed{z = \tfrac{1}{2}\left(e^{-1}+1\right) \approx 0.684}, \quad \boxed{w = \frac{e+1}{e-2} \approx 5.18},$$

$$\boxed{t = \frac{1}{3e^2-1} \approx 0.0472}$$

Question 11

Solve each of the following logarithmic equations.

a) $\ln(3x) - \ln(x-1) = 1$

b) $2\ln y = \ln(2y+8)$

c) $2\ln z = \ln(4z+12)$

d) $\ln(2w) = 2 + \ln(w+3)$

e) $\ln(t+7) = 1 + \ln(t-2)$

$$x = \frac{e}{e-3} \approx -9.65 \ , \quad \boxed{y = 4, \ y \neq -2} \ , \quad \boxed{z = 6, \ z \neq -2} \ , \quad w = \frac{3e^2}{2-e^2} \approx -4.11 \ ,$$

$$t = \frac{2e+7}{e-1} \approx 7.24$$

Question 12

Solve each of the following logarithmic equations.

a) $\ln(2x) - \ln(x+2) = 1$

b) $2\ln y = \ln(y+20)$

c) $2\ln z = \ln(5z-6)$

d) $\ln(2w-1) = 2 + \ln(w-2)$

e) $\ln(4t+1) = 1 + \ln(3t-2)$

$$x = \frac{2e}{2-e} \approx -7.57 \quad , \quad y = 5, \ y \neq -4 \quad , \quad z = 2, \ z = 3 \quad , \quad w = \frac{2e^2-1}{e^2-2} \approx 2.56 \quad ,$$

$$t = \frac{2e+1}{3e-4} \approx 1.55$$

Question 13

Solve each of the following exponential equations.

a) $e^x + 8e^{-x} = 6$

b) $3e^y + 2e^{-y} = 7$

c) $3e^z - 2e^{-z} + 5 = 0$

d) $2e^w + 5e^{-w} = 7$

e) $2e^t + 5 = 3e^{-t}$

$$\boxed{x = \ln 2, 2\ln 2}, \boxed{y = \ln 2, -\ln 3}, \boxed{z = -\ln 3}, \boxed{w = 0, \ln\left(\frac{5}{2}\right)}, \boxed{t = -\ln 2}$$

Question 14

Solve each of the following exponential equations.

a) $e^x + 3e^{-x} = 4$

b) $3e^y + 20e^{-y} = 19$

c) $3e^z + 5e^{-z} = 16$

d) $e^w + e^{-w} = \dfrac{10}{3}$

e) $e^{2t} + 15 = 8e^t$

$$\boxed{x = 0, \ln 3}, \quad \boxed{y = \ln 5, \ln\left(\frac{4}{3}\right)}, \quad \boxed{z = \ln 5, -\ln 3}, \quad \boxed{w = \pm\ln 3}, \quad \boxed{t = \ln 3, \ln 5}$$

Question 15

Solve each of the following exponential equations.

a) $e^x - 8e^{-x} = 2$

b) $2\left(e^y + e^{-y}\right) = 5$

c) $\dfrac{8e^z}{e^{2z} - 1} = 3$

d) $e^w - 3 = \dfrac{8}{e^w - 1}$

e) $e^{2t} + e^{-2t} = 4$

$$\boxed{x = 2\ln 2}, \quad \boxed{y = \pm\ln 2,} \quad \boxed{z = \ln 3}, \quad \boxed{w = \ln 5}, \quad \boxed{t = \tfrac{1}{2}\ln\left(2 \pm \sqrt{3}\right) = \pm\tfrac{1}{2}\ln\left(2 + \sqrt{3}\right)}$$