LOGARITHMS PRACTICE

SIMPLIFYING EXPRESSIONS

Question 1

Simplify each of the following logarithmic expressions, giving the final answer as a single logarithm.

a) $\log_2 7 + \log_2 2$

b) $\log_2 20 - \log_2 4$

c) $3\log_5 2 + \log_5 8$

d) $2\log_6 8 - 5\log_6 2$

e) $\log_{10} 8 + \log_{10} 5 - \log_{10} 0.5$

$\boxed{\log_2 14}$, $\boxed{\log_2 5}$, $\boxed{\log_5 64}$, $\boxed{\log_6 2}$, $\boxed{\log_{10} 80}$

(a) $\log_2 7 + \log_2 2 = \log_2(7\times 2) = \log_2 14$

(b) $\log_2 20 - \log_2 4 = \log_2\left(\frac{20}{4}\right) = \log_2 5$

(c) $3\log_5 2 + \log_5 8 = \log_5 2^3 + \log_5 8 = \log_5 8 + \log_5 8 = \log_5 64$

(d) $2\log_6 8 - 5\log_6 2 = \log_6 8^2 - \log_6 2^5 = \log_6 64 - \log_6 32 = \log_6\left(\frac{64}{32}\right)$
$= \log_6 2$

(e) $\log_{10} 8 + \log_{10} 5 - \log_{10} 0\cdot 5 = \log_{10}\left(\frac{8\times 5}{0\cdot 5}\right) = \log_{10}(80)$

Question 2

Simplify each of the following logarithmic expressions, giving the final answer as a single logarithm.

a) $\log_3 5 + \log_3 2$

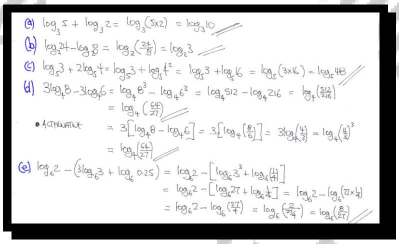
b) $\log_2 24 - \log_2 8$

c) $\log_5 3 + 2\log_5 4$

d) $3\log_4 8 - 3\log_4 6$

e) $\log_6 2 - (3\log_6 3 + \log_6 0.25)$

$\boxed{\log_2 10}$, $\boxed{\log_2 3}$, $\boxed{\log_5 48}$, $\boxed{\log_4\left(\frac{64}{27}\right)}$, $\boxed{\log_6\left(\frac{8}{27}\right)}$



(a) $\log_3 5 + \log_3 2 = \log_3(5\times 2) = \log_3 10$

(b) $\log_2 24 - \log_2 8 = \log_2\left(\frac{24}{8}\right) = \log_2 3$

(c) $\log_5 3 + 2\log_5 4 = \log_5 3 + \log_5 4^2 = \log_5 3 + \log_5 16 = \log_5(3\times 16) = \log_5 48$

(d) $3\log_4 8 - 3\log_4 6 = \log_4 8^3 - \log_4 6^3 = \log_4 512 - \log_4 216 = \log_4\left(\frac{512}{216}\right)$
$= \log_4\left(\frac{64}{27}\right)$

• ALTERNATIVE $= 3\left[\log_4 8 - \log_4 6\right] = 3\left[\log_4\left(\frac{8}{6}\right)\right] = 3\log_4\left(\frac{4}{3}\right) = \log_4\left(\frac{4}{3}\right)^3$
$= \log_4\left(\frac{64}{27}\right)$

(e) $\log_6 2 - (3\log_6 3 + \log_6 0.25) = \log_6 2 - \left[\log_6 3^3 + \log_6\left(\frac{1}{4}\right)\right]$
$= \log_6 2 - \left[\log_6 27 + \log_6 \frac{1}{4}\right] = \log_6 2 - \log_6\left(27\times\frac{1}{4}\right)$
$= \log_6 2 - \log_6\left(\frac{27}{4}\right) = \log_6\left(\frac{2}{27/4}\right) = \log_6\left(\frac{8}{27}\right)$

Question 3

Simplify each of the following logarithmic expressions, giving the final answer as a number not involving a logarithm.

a) $\log_2 4 - \log_2 0.5$

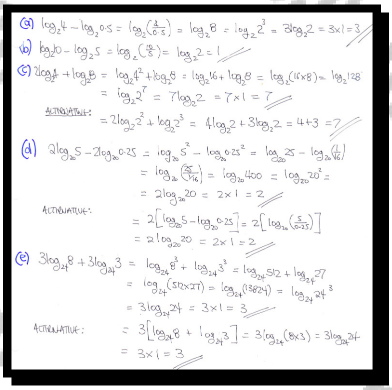
b) $\log_2 10 - \log_2 5$

c) $2\log_2 4 + \log_2 8$

d) $2\log_{20} 5 - 2\log_{20} 0.25$

e) $3\log_{24} 8 + 3\log_{24} 3$

$\boxed{3}$, $\boxed{1}$, $\boxed{7}$, $\boxed{2}$, $\boxed{3}$



(a) $\log_2 4 - \log_2 0.5 = \log_2\left(\frac{4}{0.5}\right) = \log_2 8 = \log_2 2^3 = 3\log_2 2 = 3\times 1 = 3$

(b) $\log_2 10 - \log_2 5 = \log_2\left(\frac{10}{5}\right) = \log_2 2 = 1$

(c) $2\log_2 4 + \log_2 8 = \log_2 4^2 + \log_2 8 = \log_2 16 + \log_2 8 = \log_2(16\times 8) = \log_2 128$
$= \log_2 2^7 = 7\log_2 2 = 7\times 1 = 7$

ALTERNATIVE: $= 2\log_2 2^2 + \log_2 2^3 = 4\log_2 2 + 3\log_2 2 = 4+3 = 7$

(d) $2\log_{20} 5 - 2\log_{20} 0.25 = \log_{20} 5^2 - \log_{20} 0.25^2 = \log_{20} 25 - \log_{20}\left(\frac{1}{16}\right)$
$= \log_{20}\left(\frac{25}{1/16}\right) = \log_{20} 400 = \log_{20} 20^2 =$
$= 2\log_{20} 20 = 2\times 1 = 2$

ALTERNATIVE:
$= 2\left[\log_{20} 5 - \log_{20} 0.25\right] = 2\left[\log_{20}\left(\frac{5}{0.25}\right)\right]$
$= 2\log_{20} 20 = 2\times 1 = 2$

(e) $3\log_{24} 8 + 3\log_{24} 3 = \log_{24} 8^3 + \log_{24} 3^3 = \log_{24} 512 + \log_{24} 27$
$= \log_{24}(512\times 27) = \log_{24}(13824) = \log_{24} 24^3$
$= 3\log_{24} 24 = 3\times 1 = 3$

ALTERNATIVE: $= 3\left[\log_{24} 8 + \log_{24} 3\right] = 3\log_{24}(8\times 3) = 3\log_{24} 24$
$= 3\times 1 = 3$

I.Y.G.B. madasmaths.com

Question 4

Simplify each of the following logarithmic expressions, giving the final answer as a number not involving a logarithm.

a) $\log_2 24 - \log_2 3$

b) $\log_3 96 - 3\log_3 2 - \log_3 4$

c) $\log_5 500 + \log_5\left(\frac{1}{10}\right) - \log_5\left(\frac{2}{5}\right)$

d) $2\log_3 54 - \log_3 0.25 - 4\log_3 2$

e) $8\log_6 2 - \left(\log_6 4 - 3\log_6 9\right)$

$\boxed{3}$, $\boxed{1}$, $\boxed{3}$, $\boxed{6}$, $\boxed{6}$

(a) $\log_2 24 - \log_2 3 = \log_2\left(\frac{24}{3}\right) = \log_2 8 = \log_2 2^3 = 3\log_2 2 = 3\times 1 = 3$

(b) $\log_3 96 - 3\log_3 2 - \log_3 4 = \log_3 96 - \log_3 8 - \log_3 4 = \log_3\left(\frac{96}{8\times 4}\right)$
$= \log_3 3 = 1$

(c) $\log_5 500 + \log_5\left(\frac{1}{10}\right) - \log_5\left(\frac{2}{5}\right) = \log_5\left(\frac{500\times\frac{1}{10}}{2/5}\right) = \log_5 125 = \log_5 5^3$
$= 3\log_5 5 = 3\times 1 = 3$

(d) $2\log_3 54 - \log_3 0.25 - 4\log_3 2 = \log_3 54^2 - \log_3\left(\frac{1}{4}\right) - \log_3 2^4$
$= \log_3 2916 - \log_3\left(\frac{1}{4}\right) - \log_3 16$
$= \log_3\left(\frac{2916}{\frac{1}{4}\times 16}\right) = \log_3 729$
$= \log_3 3^6 = 6\log_3 3 = 6\times 1 = 6$

(e) $8\log_6 2 - \left[\log_6 4 - 3\log_6 9\right] = \log_6 2^8 - \left[\log_6 4 - \log_6 9^3\right]$
$= \log_6 256 - \left[\log_6 4 - \log_6 729\right]$
$= \log_6 256 - \log_6\left(\frac{4}{729}\right) = \log_6\left(\frac{256}{4/729}\right)$
$= \log_6(46656) = \log_6 6^6 = 6\log_6 6$
$= 6\times 1 = 6$

I.Y.G.B. madasmaths.com

Question 5

Simplify each of the following logarithmic expressions, giving the final answer as a number not involving a logarithm.

a) $2\log_{12} 3 + 4\log_{12} 2$

b) $\log_8 25 + \log_8 10 - 3\log_8 5$

c) $2\log_{10} 20 - (\log_{10} 5 + \log_{10} 8)$

d) $4\log_3 2 - \log_3 4 - 2\log_3 \sqrt{3} - \log_3 12$

e) $4\log_2\left(\frac{1}{4}\right) - 3\log_{\frac{1}{2}}(32)$

$\boxed{2}$, $\boxed{\frac{1}{3}}$, $\boxed{1}$, $\boxed{-2}$, $\boxed{7}$

(a) $2\log_{12}3 + 4\log_{12}2 = \log_{12}3^2 + \log_{12}2^4 = \log_{12}9 + \log_{12}16 = \log_{12}144$

$= \log_{12}12^2 = 2\log_{12}12 = 2\times 1 = 2$

OR

$= 2\left[\log_{12}3 + 2\log_{12}2\right] = 2\left[\log_{12}3 + \log_{12}2^2\right]$

$= 2\left[\log_{12}3 + \log_{12}4\right] = 2\log_{12}(3\times 4) = 2\log_{12}12 = 2\times 1 = 2$

(b) $\log_8 25 + \log_8 10 - 3\log_8 5 = \log_8(25\times 10) - \log_8 5^3 = \log_8 250 - \log_8 125$

$= \log_8\left(\frac{250}{125}\right) = \log_8 2$

$= \log_8 8^{\frac{1}{3}}$

$= \frac{1}{3}\log_8 8$

$= \frac{1}{3}$

$= \frac{1}{\log_2 8}$

$= \frac{1}{\log_2 2^3}$

$= \frac{1}{3\log_2 2} = \frac{1}{3\times 1} = \frac{1}{3}$

(c) $2\log_{10}20 - (\log_{10}5 + \log_{10}8) = \log_{10}20^2 - \log_{10}(5\times 8) = \log_{10}400 - \log_{10}40$

$= \log_{10}\left(\frac{400}{40}\right) = \log_{10}10 = 1$

(d) $4\log_3 2 - \log_3 4 - 2\log_3\sqrt{3} - \log_3 12 = \log_3 2^4 - \log_3 4 - \log_3\sqrt{3}^2 - \log_3 12$

$= \log_3 16 - \log_3 4 - \log_3 3 - \log_3 12$

$= \log_3\left(\frac{16}{4\times 3\times 12}\right) = \log_3\left(\frac{16}{144}\right)$

$= \log_3\left(\frac{1}{9}\right) = \log_3 3^{-2} = -2\log_3 3$

$= -2\times 1 = -2$

(e) $4\log_2\left(\frac{1}{4}\right) - 3\log_{\frac{1}{2}}(32) = 4\log_2 2^{-2} - 3\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-5} = -8\log_2 2 + 15\log_{\frac{1}{2}}\left(\frac{1}{2}\right)$

$= -8\times 1 + 15\times 1 = -8 + 15 = 7$

ALTERNATIVE

$= -4\log_2 4 - 3\times\frac{\log_2 32}{\log_2\left(\frac{1}{2}\right)}$ (BY USING RULES)

$= -4\log_2 2^2 - 3\times\frac{\log_2 2^5}{\log_2 2^{-1}} = -8\log_2 2 - 3\times\frac{5\cancel{\log_2 2}}{-1\cancel{\log_2 2}}$

$= -8\times 1 - 3(-5) = -8 + 15 = 7$

Question 6

Simplify each of the following logarithmic expressions, giving the final answer as a number not involving a logarithm.

a) $\log_2 40 - \log_2 5$

b) $\log_6 4 + \log_6 9$

c) $\log_2\left(\frac{5}{2}\right) + \log_2\left(\frac{4}{3}\right) - \log_2\left(\frac{5}{3}\right)$

d) $\frac{1}{3}\log_{\frac{3}{2}}\left(\frac{8}{27}\right) + \frac{1}{2}\log_{\frac{3}{2}}\left(\frac{4}{9}\right)$

e) $\frac{1}{2}\log_{\frac{2}{3}}\left(\frac{4}{9}\right) - 2\log_{\frac{2}{3}}\left(\frac{9}{4}\right)$

$\boxed{3}$, $\boxed{2}$, $\boxed{1}$, $\boxed{-2}$, $\boxed{5}$

(a) $\log_2 40 - \log_2 5 = \log_2\left(\frac{40}{5}\right) = \log_2 8 = \log_2 2^3 = 3\log_2 2$
$= 3\times 1 = 3$

(b) $\log_6 4 + \log_6 9 = \log_6(4\times 9) = \log_6 6^2 = 2\log_6 6$
$= 2\times 1 = 2$

(c) $\log_2\left(\frac{5}{2}\right) + \log_2\left(\frac{4}{3}\right) - \log_2\left(\frac{5}{3}\right) = \log_2\left(\frac{\frac{5}{2}\times\frac{4}{3}}{\frac{5}{3}}\right) = \log_2\left(\frac{\frac{10}{3}}{\frac{5}{3}}\right)$
$= \log_2 2 = 1$

(d) $\frac{1}{3}\log_{\frac{3}{2}}\left(\frac{8}{27}\right) + \frac{1}{2}\log_{\frac{3}{2}}\left(\frac{4}{9}\right) = \log_{\frac{3}{2}}\left(\frac{8}{27}\right)^{\frac{1}{3}} + \log_{\frac{3}{2}}\left(\frac{4}{9}\right)^{\frac{1}{2}}$
$= \log_{\frac{3}{2}}\left(\frac{2}{3}\right) + \log_{\frac{3}{2}}\left(\frac{2}{3}\right)$
$= 2\log_{\frac{3}{2}}\left(\frac{2}{3}\right) = -2\log_{\frac{3}{2}}\left(\frac{3}{2}\right)$
$= -2\times 1 = -2$

(e) $\frac{1}{2}\log_{\frac{2}{3}}\left(\frac{4}{9}\right) - 2\log_{\frac{2}{3}}\left(\frac{9}{4}\right) = \frac{1}{2}\log_{\frac{2}{3}}\left(\frac{4}{9}\right) + 2\log_{\frac{2}{3}}\left(\frac{4}{9}\right)$
$= \frac{5}{2}\log_{\frac{2}{3}}\left(\frac{4}{9}\right) = \frac{5}{2}\log_{\frac{2}{3}}\left(\frac{2}{3}\right)^2$
$= \frac{5}{2}\times 2\log_{\frac{2}{3}}\left(\frac{2}{3}\right) = 5$

I.Y.G.B. madasmaths.com

Question 7

Simplify each of the following logarithmic expressions, giving the final answer as a single fraction.

a) $\log_4 2$

b) $\log_4 8$

c) $\log_4 \left(2\sqrt{2}\right)$

d) $\log_5 \left(\dfrac{1}{\sqrt{125}}\right)$

$$\boxed{\frac{1}{2}}, \boxed{\frac{3}{2}}, \boxed{\frac{3}{4}}, \boxed{-\frac{3}{2}}$$

(a) • $\log_4 2 = \log_4 4^{\frac{1}{2}} = \frac{1}{2}\log_4 4 = \frac{1}{2}\times 1 = \frac{1}{2}$

• $\log_4 2 = \dfrac{\log_2 2}{\log_2 4} = \dfrac{1}{\log_2 2^2} = \dfrac{1}{2\log_2 2} = \dfrac{1}{2}$

(b) • $\log_4 8 = \log_4 4^{\frac{3}{2}} = \frac{3}{2}\log_4 4 = \frac{3}{2}$

• $\log_4 8 = \dfrac{\log_2 8}{\log_2 4} = \dfrac{\log_2 2^3}{\log_2 2^2} = \dfrac{3\cancel{\log_2 2}}{2\cancel{\log_2 2}} = \dfrac{3}{2}$

(c) $\log_4(2\sqrt{2}) = \log_4(2^1 \times 2^{\frac{1}{2}}) = \log_4 2^{\frac{3}{2}} = \frac{3}{2}(\log_4 2)$ part(a)

$= \frac{3}{2}\times\frac{1}{2} = \frac{3}{4}$

(d) $\log_5\left(\frac{1}{\sqrt{125}}\right) = \log_5\left(125^{-\frac{1}{2}}\right) = -\frac{1}{2}\log_5 125 = -\frac{1}{2}\log_5 5^3$

$= -\frac{3}{2}\log_5 5 = -\frac{3}{2}$

SOLVING EQUATIONS USING LOGARITHMS

Question 1

Solve each of the following exponential equations.

a) $3^x = 11$

b) $4^{y-1} = 18$

c) $2^{2z+1} = 80$

d) $5^{3w} = 30$

e) $6^{7-2t} = 7776$

$\boxed{x \approx 2.18}$, $\boxed{y \approx 3.08}$, $\boxed{z \approx 2.66}$, $\boxed{w \approx 0.704}$, $\boxed{t = 1}$

(a) $3^x = 11$
$\Rightarrow \log 3^x = \log 11$
$\Rightarrow x \log 3 = \log 11$
$\Rightarrow x = \dfrac{\log 11}{\log 3}$
$\Rightarrow x \approx 2.18$ // 3sf

(b) $4^{y-1} = 18$
$\Rightarrow \log 4^{y-1} = \log 18$
$\Rightarrow (y-1)\log 4 = \log 18$
$\Rightarrow y-1 = \dfrac{\log 18}{\log 4}$
$\Rightarrow y-1 \approx 2.08\ldots$
$\Rightarrow y = 3.08$ // (3 sf)

(c) $2^{2z+1} = 80$
$\Rightarrow \log 2^{2z+1} = \log 80$
$\Rightarrow (2z+1)\log 2 = \log 80$
$\Rightarrow 2z+1 = \dfrac{\log 80}{\log 2}$
$\Rightarrow 2z+1 \simeq 6.3219\ldots$
$\Rightarrow z \approx 2.66$ // (3 sf)

(d) $5^{3w} = 30$
$\Rightarrow \log 5^{3w} = \log 30$
$\Rightarrow (3w)\log 5 = \log 30$
$\Rightarrow 3w = \dfrac{\log 30}{\log 5}$
$\Rightarrow 3w \simeq 2.11328\ldots$
$\Rightarrow w \simeq 0.704$ //

(e) $6^{7-2t} = 7776$
$\Rightarrow \log 6^{7-2t} = \log 7776$
$\Rightarrow (7-2t)\log_6 = \log 7776$
$\Rightarrow 7-2t = \dfrac{\log 7776}{\log 6}$
$\Rightarrow 7-2t = 5$
$\Rightarrow t = 1$ //

Question 2

Solve each of the following exponential equations.

a) $4^x = 200$

b) $2^{3y} = 84$

c) $6^{2-z} = 400$

d) $3^{2w-1} = 4$

e) $12^{3t+4} = 75$

$\boxed{x \approx 3.82}$, $\boxed{y \approx 2.13}$, $\boxed{z \approx -1.34}$, $\boxed{w \approx 1.13}$, $\boxed{t \approx -0.754}$

(a) $4^x = 200$
$\Rightarrow \log 4^x = \log 200$
$\Rightarrow x \log 4 = \log 200$
$\Rightarrow x = \frac{\log 200}{\log 4}$
$\Rightarrow x \approx 3.82$

(b) $2^{3y} = 84$
$\Rightarrow \log 2^{3y} = \log 84$
$\Rightarrow (3y)\log 2 = \log 84$
$\Rightarrow 3y = \frac{\log 84}{\log 2}$
$\Rightarrow 3y \simeq 6.3923...$
$\Rightarrow y \approx 2.13$

(c) $6^{2-z} = 400$
$\Rightarrow \log 6^{2-z} = \log 400$
$\Rightarrow (2-z)\log 6 = \log 400$
$\Rightarrow 2-z = \frac{\log 400}{\log 6}$
$\Rightarrow 2-z \approx 3.3439$
$\Rightarrow -1.34 \approx z$

(d) $3^{2w-1} = 4$
$\Rightarrow \log 3^{2w-1} = \log 4$
$\Rightarrow (2w-1)\log 3 = \log 4$
$\Rightarrow 2w-1 = \frac{\log 4}{\log 3}$
$\Rightarrow 2w-1 \simeq 1.2619...$
$\Rightarrow w \simeq 1.13$

(e) $12^{3t+4} = 75$
$\Rightarrow \log 12^{3t+4} = \log 75$
$\Rightarrow (3t+4)\log 12 = \log 75$
$\Rightarrow 3t+4 = \frac{\log 75}{\log 12}$
$\Rightarrow 3t+4 \simeq 1.7375...$
$\Rightarrow t \simeq -0.754$

Question 3

Solve each of the following exponential equations.

a) $5 \times 3^{x} = 200$

b) $5 \times 4^{y-2} = 165$

c) $4 \times 2^{2z+1} = 1000$

d) $5 \times 5^{3w} = 10000$

e) $200(0.7)^{t-1} = 6000$

$\boxed{x \approx 3.36}$, $\boxed{y \approx 4.52}$, $\boxed{z \approx 3.48}$, $\boxed{w \approx 1.57}$, $\boxed{t \approx -8.54}$

(a) $5 \times 3^{x} = 200$
$\Rightarrow 3^{x} = 40$
$\Rightarrow \log 3^{x} = \log 40$
$\Rightarrow x \log 3 = \log 40$
$\Rightarrow x = \frac{\log 40}{\log 3}$
$\Rightarrow x \simeq 3.36$

(b) $5 \times 4^{y-2} = 165$
$\Rightarrow 4^{y-2} = 33$
$\Rightarrow \log 4^{y-2} = \log 33$
$\Rightarrow (y-2)\log 4 = \log 33$
$\Rightarrow y-2 = \frac{\log 33}{\log 4}$
$\Rightarrow y-2 \simeq 2.5222\ldots$
$\Rightarrow y \simeq 4.52$

(c) $4 \times 2^{2z+1} = 1000$
$\Rightarrow 2^{2z+1} = 250$
$\Rightarrow \log 2^{2z+1} = \log 250$
$\Rightarrow (2z+1)\log 2 = \log 250$
$\Rightarrow 2z+1 = \frac{\log 250}{\log 2}$
$\Rightarrow 2z+1 = 7.9658\ldots$
$\Rightarrow z \simeq 3.48$

(d) $5 \times 5^{3w} = 10000$
$\Rightarrow 5^{3w} = 2000$
$\Rightarrow \log 5^{3w} = \log 2000$
$\Rightarrow (3w)\log 5 = \log 2000$
$\Rightarrow 3w = \frac{\log 2000}{\log 5}$
$\Rightarrow 3w \simeq 4.7227\ldots$
$\Rightarrow w \simeq 1.57$

(e) $200(0.7)^{t-1} = 6000$
$\Rightarrow (0.7)^{t-1} = 30$
$\Rightarrow \log(0.7)^{t-1} = \log 30$
$\Rightarrow (t-1)\log(0.7) = \log 30$
$\Rightarrow t-1 = \frac{\log 30}{\log 0.7}$
$\Rightarrow t-1 \simeq -9.5358\ldots$
$\Rightarrow t \simeq -8.54$

Question 4

Solve each of the following exponential equations.

a) $3\times\left(\frac{1}{3}\right)^{x}=0.0042$

b) $\frac{1}{3}\times 2^{y-1}=84$

c) $5(0.4)^{2-z}=20$

d) $10\times 2^{w}=8\times 5^{w+1}$

e) $5\times 2^{t-1}=2\times 5^{2t}$

$\boxed{x\approx 5.98}$, $\boxed{y\approx 8.98}$, $\boxed{z\approx 3.51}$, $\boxed{w\approx -1.51}$, $\boxed{t\approx 0.0883}$

(a) $3\times\left(\frac{1}{3}\right)^{x}=0.0042$

$\Rightarrow \left(\frac{1}{3}\right)^{x}=0.0014$

$\Rightarrow \log\left(\frac{1}{3}\right)^{x}=\log(0.0014)$

$\Rightarrow x\log\left(\frac{1}{3}\right)=\log(0.0014)$

$\Rightarrow x=\frac{\log(0.0014)}{\log\left(\frac{1}{3}\right)}$

$\Rightarrow x\approx 5.98$

(b) $\frac{1}{3}\times 2^{y-1}=84$

$\Rightarrow 2^{y-1}=252$

$\Rightarrow \log 2^{y-1}=\log 252$

$\Rightarrow (y-1)\log 2=\log 252$

$\Rightarrow y-1=\frac{\log 252}{\log 2}$

$\Rightarrow y-1\approx 7.98\ldots$

$\Rightarrow y\approx 8.98$

(c) $5(0.4)^{2-z}=20$

$\Rightarrow (0.4)^{2-z}=4$

$\Rightarrow \log(0.4)^{2-z}=\log 4$

$\Rightarrow (2-z)\log(0.4)=\log 4$

$\Rightarrow 2-z=\frac{\log 4}{\log(0.4)}$

$\Rightarrow 2-z\simeq -1.51\ldots$

$\Rightarrow z\simeq 3.51$

(d) $10\times 2^{w}=8\times 5^{w+1}$

$\Rightarrow \log(10\times 2^{w})=\log(8\times 5^{w+1})$

$\Rightarrow \log 10+\log 2^{w}=\log 8+\log 5^{w+1}$

$\Rightarrow \log 10+w\log 2=\log 8+(w+1)\log 5$

$\Rightarrow \log 10+w\log 2=\log 8+w\log 5+\log 5$

$\Rightarrow w\log 2-w\log 5=\log 8+\log 5-\log 10$

$\Rightarrow w(\log 2-\log 5)=\log 40-\log 10$

$\Rightarrow w=\frac{\log 4}{\log 2-\log 5}$

$\Rightarrow w=1.51$

ALTERNATIVE

$10\times 2^{w}=8\times 5^{w+1}$

$\Rightarrow 10\times 2^{w}=8\times 5^{1}\times 5^{w}$

$\Rightarrow 2^{w}=4\times 5^{w}$

$\Rightarrow \frac{2^{w}}{5^{w}}=4$

$\Rightarrow \left(\frac{2}{5}\right)^{w}=4$

$\Rightarrow \log(0.4)^{w}=\log 4$

$\Rightarrow w\log(0.4)=\log 4$

$\Rightarrow w=\frac{\log 4}{\log(0.4)}$

$\Rightarrow w\approx 1.51$

(e) $5\times 2^{t-1}=2\times 5^{2t}$

$\Rightarrow \log(5\times 2^{t-1})=\log(2\times 5^{2t})$

$\Rightarrow \log 5+\log 2^{t-1}=\log 2+\log 5^{2t}$

$\Rightarrow \log 5+(t-1)\log 2=\log 2+2t\log 5$

$\Rightarrow \log 5+t\log 2-\log 2=\log 2+2t\log 5$

$\Rightarrow t\log 2-2t\log 5=\log 2+\log 2-\log 5$

$\Rightarrow t(\log 2-2\log 5)=2\log 2-\log 5$

$\Rightarrow t=\frac{\log 4-\log 5}{\log 2-\log 25}$

$\Rightarrow t=\frac{\log 4/5}{\log 2/25}$

$\Rightarrow t=\frac{\log(0.8)}{\log(0.08)}$

$\Rightarrow t\approx 0.0883$

Question 5

Solve each of the following exponential equations.

a) $2^x = 3^{x+1}$

b) $3^{y-1} = 2^{2y}$

c) $2^{2z+1} = 7^z$

d) $4^{3w} = 3^{2w-1}$

e) $6^{1-t} = 5^{3t+1}$

$\boxed{x \approx -2.71}$, $\boxed{y \approx -3.82}$, $\boxed{z \approx 1.24}$, $\boxed{w \approx -0.560}$, $\boxed{t \approx 0.0275}$

(a) $2^x = 3^{x+1}$

$\Rightarrow \log 2^x = \log 3^{x+1}$

$\Rightarrow x\log 2 = (x+1)\log 3$

$\Rightarrow x\log 2 = x\log 3 + \log 3$

$\Rightarrow x\log 2 - x\log 3 = \log 3$

$\Rightarrow x(\log 2 - \log 3) = \log 3$

$\Rightarrow x = \dfrac{\log 3}{\log 2 - \log 3}$

$\Rightarrow x \simeq -2.71$

(b) $3^{y-1} = 2^{2y}$

$\Rightarrow \log 3^{y-1} = \log 2^{2y}$

$\Rightarrow (y-1)\log 3 = (2y)\log 2$

$\Rightarrow y\log 3 - \log 3 = 2y\log 2$

$\Rightarrow y\log 3 - 2y\log 2 = \log 3$

$\Rightarrow y(\log 3 - 2\log 2) = \log 3$

$\Rightarrow y = \dfrac{\log 3}{\log 3 - 2\log 2}$

$\Rightarrow y = -3.82$

(c) $2^{2z+1} = 7^z$

$\Rightarrow \log 2^{2z+1} = \log 7^z$

$\Rightarrow (2z+1)\log 2 = z\log 7$

$\Rightarrow 2z\log 2 + \log 2 = z\log 7$

$\Rightarrow 2z\log 2 - z\log 7 = -\log 2$

$\Rightarrow z(2\log 2 - \log 7) = -\log 2$

$\Rightarrow z = \dfrac{-\log 2}{2\log 2 - \log 7}$

$\Rightarrow z \simeq 1.24$

(d) $4^{3w} = 3^{2w-1}$

$\Rightarrow \log 4^{3w} = \log 3^{2w-1}$

$\Rightarrow (3w)\log 4 = (2w-1)\log 3$

$\Rightarrow 3w\log 4 = 2w\log 3 - \log 3$

$\Rightarrow 3w\log 4 - 2w\log 3 = -\log 3$

$\Rightarrow w(3\log 4 - 2\log 3) = -\log 3$

$\Rightarrow w = \dfrac{-\log 3}{3\log 4 - 2\log 3}$

$\Rightarrow w \simeq -0.560$

(e) $6^{1-t} = 5^{3t+1}$

$\Rightarrow \log 6^{1-t} = \log 5^{3t+1}$

$\Rightarrow (1-t)\log 6 = (3t+1)\log 5$

$\Rightarrow \log 6 - t\log 6 = 3t\log 5 + \log 5$

$\Rightarrow \log 6 - \log 5 = 3t\log 5 + t\log 6$

$\Rightarrow \log 6 - \log 5 = t(3\log 5 + \log 6)$

$\Rightarrow t = \dfrac{\log 6 - \log 5}{3\log 5 + \log 6}$

$\Rightarrow t \simeq 0.0275$

Question 6

Solve each of the following exponential equations.

a) $2^{x+3} = 6^{x-1}$

b) $3^{2y} = 2^{y+1}$

c) $6^{z} = 2^{2z-1}$

d) $8^{4-3w} = 7^{w}$

e) $3^{2t+1} = 5^{200}$

$\boxed{x \approx 3.52}$, $\boxed{y \approx 0.461}$, $\boxed{z \approx -1.71}$, $\boxed{w \approx 1.02}$, $\boxed{t \approx 146}$

(a) $2^{x+3} = 6^{x-1}$
$\Rightarrow \log 2^{x+3} = \log 6^{x-1}$
$\Rightarrow (x+3)\log 2 = (x-1)\log 6$
$\Rightarrow x\log 2 + 3\log 2 = x\log 6 - \log 6$
$\Rightarrow x\log 2 - x\log 6 = -\log 6 - 3\log 2$
$\Rightarrow x(\log 2 - \log 6) = -(\log 6 + 3\log 2)$
$\Rightarrow x = -\dfrac{\log 6 + 3\log 2}{\log 2 - \log 6} \approx 3.52$

(b) $3^{2y} = 2^{y+1}$
$\Rightarrow \log 3^{2y} = \log 2^{y+1}$
$\Rightarrow (2y)\log 3 = (y+1)\log 2$
$\Rightarrow 2y\log 3 = y\log 2 + \log 2$
$\Rightarrow 2y\log 3 - y\log 2 = \log 2$
$\Rightarrow y(2\log 3 - \log 2) = \log 2$
$\Rightarrow y = \dfrac{\log 2}{2\log 3 - \log 2} \approx 0.461$

(c) $6^{z} = 2^{2z-1}$
$\Rightarrow \log 6^{z} = \log 2^{2z-1}$
$\Rightarrow z\log 6 = (2z-1)\log 2$
$\Rightarrow z\log 6 = 2z\log 2 - \log 2$
$\Rightarrow \log 2 = 2z\log 2 - z\log 6$
$\Rightarrow \log 2 = z(2\log 2 - \log 6)$
$\Rightarrow z = \dfrac{\log 2}{2\log 2 - \log 6} \approx -1.71$

(d) $8^{4-3w} = 7^{w}$
$\Rightarrow \log 8^{4-3w} = \log 7^{w}$
$\Rightarrow (4-3w)\log 8 = w\log 7$
$\Rightarrow 4\log 8 - 3w\log 8 = w\log 7$
$\Rightarrow 4\log 8 = 3w\log 8 + w\log 7$
$\Rightarrow 4\log 8 = w(3\log 8 + \log 7)$
$\Rightarrow w = \dfrac{4\log 8}{3\log 8 + \log 7}$
$\Rightarrow w \simeq 1.02$

(e) $3^{2t+1} = 5^{200}$
$\Rightarrow \log 3^{2t+1} = \log 5^{200}$
$\Rightarrow (2t+1)\log 3 = 200\log 5$
$\Rightarrow 2t+1 = \dfrac{200\log 5}{\log 3}$
$\Rightarrow 2t+1 \simeq 292.9947\ldots$
$\Rightarrow t \simeq 145.947\ldots$
$\Rightarrow t \simeq 146$

Question 7

Solve each of the following equations giving your answer correct to 3 s.f.

a) $2^{2x} - 2^x - 6 = 0$

b) $4^y - 3\left(2^y\right) - 10 = 0$

c) $3^{2z+1} - 14 \times \left(3^z\right) + 8 = 0$

d) $4^w - 3\left(2^{w+1}\right) = 0$

e) $3^{t+1} = 6 + 3^{2t-1}$

$\boxed{x \approx 1.58}$, $\boxed{y \approx 2.32}$, $\boxed{z \approx 1.26 \text{ or } -0.369}$, $\boxed{w \approx 2.58}$, $\boxed{t = 1 \text{ or } t \approx 1.63}$

(a) $2^{2x} - 2^x - 6 = 0$

$\Rightarrow (2^x)^2 - (2^x) - 6 = 0$

$\Rightarrow a^2 - a - 6 = 0 \quad (a = 2^x)$

$\Rightarrow (a-3)(a+2) = 0$

$\Rightarrow a = \begin{cases} 3 \\ -2 \end{cases}$

$\Rightarrow 2^x = \begin{cases} 3 \\ \cancel{-2} \end{cases}$

$\Rightarrow \log 2^x = \log 3$

$\Rightarrow x \log 2 = \log 3$

$\Rightarrow x = \frac{\log 3}{\log 2} \approx 1.58$

(b) $4^y - 3(2^y) - 10 = 0$

$\Rightarrow (2^2)^y - 3(2^y) - 10 = 0$

$\Rightarrow (2^y)^2 - 3(2^y) - 10 = 0$

$\Rightarrow a^2 - 3a - 10 = 0 \quad (a = 2^y)$

$\Rightarrow (a-5)(a+2) = 0$

$\Rightarrow a = \begin{cases} 5 \\ -2 \end{cases}$

$\Rightarrow 2^y = \begin{cases} 5 \\ \cancel{-2} \end{cases}$

$\Rightarrow \log 2^y = \log 5$

$\Rightarrow y \log 2 = \log 5$

$\Rightarrow y = \frac{\log 5}{\log 2} \approx 2.32$

(c) $3^{2z+1} - 14(3^z) + 8 = 0$

$\Rightarrow 3^{2z} \times 3^1 - 14(3^z) + 8 = 0$

$\Rightarrow 3(3^z)^2 - 14(3^z) + 8 = 0$

$\Rightarrow 3a^2 - 14a + 8 = 0 \quad (a = 3^z)$

$\Rightarrow (3a-2)(a-4) = 0$

$\Rightarrow a = \begin{cases} \frac{2}{3} \\ 4 \end{cases}$

$\Rightarrow 3^z = \begin{cases} \frac{2}{3} \\ 4 \end{cases}$

$\Rightarrow \log 3^z = \begin{cases} \log\left(\frac{2}{3}\right) \\ \log(4) \end{cases}$

$\Rightarrow z \log 3 = \begin{cases} \log(2/3) \\ \log 4 \end{cases}$

$\Rightarrow z = \begin{cases} \frac{\log(2/3)}{\log 3} \approx -0.369 \\ \frac{\log 4}{\log 3} \approx 1.26 \end{cases}$

(d) $4^w - 3(2^{w+1}) = 0$

$\Rightarrow (2^2)^w - 3(2^w \times 2^1) = 0$

$\Rightarrow (2^w)^2 - 6(2^w) = 0$

$\Rightarrow a^2 - 6a = 0 \quad (a = 2^w)$

$\Rightarrow a(a-6) = 0$

$\Rightarrow a = \begin{cases} 0 \\ 6 \end{cases}$

$\Rightarrow 2^w = \begin{cases} \cancel{0} \\ 6 \end{cases}$

$\Rightarrow \log 2^w = \log 6$

$\Rightarrow w \log 2 = \log 6$

$\Rightarrow w = \frac{\log 6}{\log 2} \approx 2.58$

(e) $3^{t+1} = 6 + 3^{2t-1}$

$\Rightarrow 3^t \times 3^1 = 6 + 3^{2t} \times 3^{-1}$

$\Rightarrow 3(3^t) = 6 + (3^t)^2 \times \frac{1}{3}$

$\Rightarrow 3a = 6 + a^2 \times \frac{1}{3} \quad (a = 3^t)$

$\Rightarrow 9a = 18 + a^2$

$\Rightarrow a^2 - 9a + 18 = 0$

$\Rightarrow (a-6)(a-3) = 0$

$a = \begin{cases} 6 \\ 3 \end{cases}$

$\Rightarrow 3^t = \begin{cases} 6 \\ 3 \end{cases}$

$\Rightarrow \log 3^t = \begin{cases} \log 6 \\ \log 3 \end{cases}$

$\Rightarrow t \log 3 = \begin{cases} \log 6 \\ \log 3 \end{cases}$

$\Rightarrow t = \begin{cases} \frac{\log 6}{\log 3} \approx 1.63 \\ 1 \end{cases}$

madasmaths.com I.Y.G.B.

Question 8

Solve each of the following equations giving your answer correct to 3 s.f.

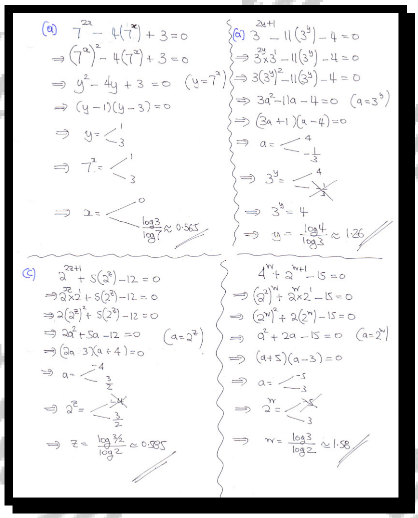
a) $7^{2x} - 4\left(7^x\right) + 3 = 0$

b) $3^{2y+1} - 11\left(3^y\right) - 4 = 0$

c) $2^{2z+1} + 5\left(2^z\right) - 12 = 0$

d) $4^w + 2^{w+1} - 15 = 0$

$\boxed{x \approx 0.565 \text{ or } x = 0}$, $\boxed{y \approx 1.26}$, $\boxed{z \approx 0.585}$, $\boxed{w \approx 1.58}$



(a) $7^{2x} - 4(7^x) + 3 = 0$

$\Rightarrow (7^x)^2 - 4(7^x) + 3 = 0$

$\Rightarrow y^2 - 4y + 3 = 0 \quad (y = 7^x)$

$\Rightarrow (y-1)(y-3) = 0$

$\Rightarrow y = 1, 3$

$\Rightarrow 7^x = 1, 3$

$\Rightarrow x = 0, \ \frac{\log 3}{\log 7} \approx 0.565$

(a) $3^{2y+1} - 11(3^y) - 4 = 0$

$\Rightarrow 3^{2y} \times 3^1 - 11(3^y) - 4 = 0$

$\Rightarrow 3(3^y)^2 - 11(3^y) - 4 = 0$

$\Rightarrow 3a^2 - 11a - 4 = 0 \quad (a = 3^y)$

$\Rightarrow (3a+1)(a-4) = 0$

$\Rightarrow a = 4, -\frac{1}{3}$

$\Rightarrow 3^y = 4, \cancel{-\frac{1}{3}}$

$\Rightarrow 3^y = 4$

$\Rightarrow y = \frac{\log 4}{\log 3} \approx 1.26$

(c) $2^{2z+1} + 5(2^z) - 12 = 0$

$\Rightarrow 2^{2z} \times 2^1 + 5(2^z) - 12 = 0$

$\Rightarrow 2(2^z)^2 + 5(2^z) - 12 = 0$

$\Rightarrow 2a^2 + 5a - 12 = 0 \quad (a = 2^z)$

$\Rightarrow (2a-3)(a+4) = 0$

$\Rightarrow a = -4, \frac{3}{2}$

$\Rightarrow 2^z = \cancel{-4}, \frac{3}{2}$

$\Rightarrow z = \frac{\log 3/2}{\log 2} \approx 0.585$

$4^w + 2^{w+1} - 15 = 0$

$\Rightarrow (2^2)^w + 2^w \times 2^1 - 15 = 0$

$\Rightarrow (2^w)^2 + 2(2^w) - 15 = 0$

$\Rightarrow a^2 + 2a - 15 = 0 \quad (a = 2^w)$

$\Rightarrow (a+5)(a-3) = 0$

$\Rightarrow a = -5, 3$

$\Rightarrow 2^w = \cancel{-5}, 3$

$\Rightarrow w = \frac{\log 3}{\log 2} \approx 1.58$

SOLVING EQUATIONS INVOLVING LOGARITHMS

Question 1

Solve each of the following logarithmic equations.

a) $\log_2(x+1) - \log_2 x = \log_2 3$

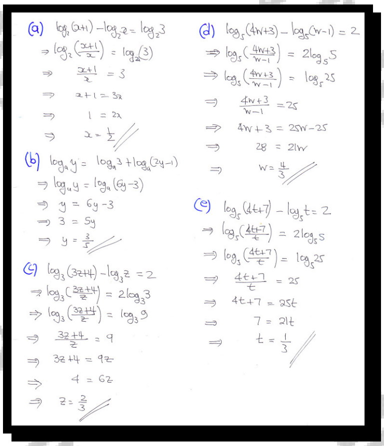
b) $\log_a y = \log_a 3 + \log_a(2y-1)$

c) $\log_3(3z+4) - \log_3 z = 2$

d) $\log_5(4w+3) - \log_5(w-1) = 2$

e) $\log_5(4t+7) - \log_5 t = 2$

$\boxed{x=\frac{1}{2}}$, $\boxed{y=\frac{3}{5}}$, $\boxed{z=\frac{2}{3}}$, $\boxed{w=\frac{4}{3}}$, $\boxed{t=\frac{1}{3}}$



(a) $\log_2(x+1) - \log_2 x = \log_2 3$

$\Rightarrow \log_2\left(\frac{x+1}{x}\right) = \log_2(3)$

$\Rightarrow \frac{x+1}{x} = 3$

$\Rightarrow x+1 = 3x$

$\Rightarrow 1 = 2x$

$\Rightarrow x = \frac{1}{2}$

(b) $\log_a y = \log_a 3 + \log_a(2y-1)$

$\Rightarrow \log_a y = \log_a(6y-3)$

$\Rightarrow y = 6y-3$

$\Rightarrow 3 = 5y$

$\Rightarrow y = \frac{3}{5}$

(c) $\log_3(3z+4) - \log_3 z = 2$

$\Rightarrow \log_3\left(\frac{3z+4}{z}\right) = 2\log_3 3$

$\Rightarrow \log_3\left(\frac{3z+4}{z}\right) = \log_3 9$

$\Rightarrow \frac{3z+4}{z} = 9$

$\Rightarrow 3z+4 = 9z$

$\Rightarrow 4 = 6z$

$\Rightarrow z = \frac{2}{3}$

(d) $\log_5(4w+3) - \log_5(w-1) = 2$

$\Rightarrow \log_5\left(\frac{4w+3}{w-1}\right) = 2\log_5 5$

$\Rightarrow \log_5\left(\frac{4w+3}{w-1}\right) = \log_5 25$

$\Rightarrow \frac{4w+3}{w-1} = 25$

$\Rightarrow 4w+3 = 25w-25$

$\Rightarrow 28 = 21w$

$\Rightarrow w = \frac{4}{3}$

(e) $\log_5(4t+7) - \log_5 t = 2$

$\Rightarrow \log_5\left(\frac{4t+7}{t}\right) = 2\log_5 5$

$\Rightarrow \log_5\left(\frac{4t+7}{t}\right) = \log_5 25$

$\Rightarrow \frac{4t+7}{t} = 25$

$\Rightarrow 4t+7 = 25t$

$\Rightarrow 7 = 21t$

$\Rightarrow t = \frac{1}{3}$

Question 2

Solve each of the following logarithmic equations.

a) $\log_a(2x+7) = \log_a x + 2\log_a 3$

b) $\log_a(3y+10) - \log_a y = 2\log_a 3$

c) $\log_2(2z+1) = 2 + \log_2 z$

d) $\log_3(4w+1) - \log_3(w-1) = 2$

e) $\log_2(3t+4) - \log_2 t = 3$

$\boxed{x=1}$, $\boxed{y=\frac{5}{3}}$, $\boxed{z=\frac{1}{2}}$, $\boxed{w=2}$, $\boxed{t=\frac{4}{5}}$

(a) $\log_a(2x+7) = \log_a x + 2\log_a 3$

$\Rightarrow \log_a(2x+7) = \log_a x = \log_a 9$

$\Rightarrow \log_a(2x+7) = \log_a(9x)$

$\Rightarrow 2x+7 = 9x$

$\Rightarrow 7 = 7x$

$\Rightarrow x = 1$

(b) $\log_a(3y+10) - \log_a y = 2\log_a 3$

$\Rightarrow \log_a\left(\frac{3y+10}{y}\right) = \log_a 9$

$\Rightarrow \frac{3y+10}{y} = 9$

$\Rightarrow 3y + 10 = 9y$

$\Rightarrow 10 = 6y$

$\Rightarrow \frac{5}{3} = y$

(c) $\log_2(2z+1) = 2 + \log_2 z$

$\Rightarrow \log_2(2z+1) = 2\log_2 2 + \log_2 z$

$\Rightarrow \log_2(2z+1) = \log_2 4 + \log_2 z$

$\Rightarrow \log_2(2z+1) = \log_2(4z)$

$\Rightarrow 2z+1 = 4z$

$1 = 2z$

$z = \frac{1}{2}$

(d) $\log_3(4w+1) - \log_3(w-1) = 2$

$\Rightarrow \log_3\left(\frac{4w+1}{w-1}\right) = 2\log_3 3$

$\Rightarrow \log_3\left(\frac{4w+1}{w-1}\right) = \log_3 9$

$\Rightarrow \frac{4w+1}{w-1} = 9$

$\Rightarrow 4w+1 = 9w-9$

$10 = 5w$

$w = 2$

(e) $\log_2(3t+4) - \log_2 t = 3$

$\Rightarrow \log_2\left(\frac{3t+4}{t}\right) = 3\log_2 2$

$\Rightarrow \log_2\left(\frac{3t+4}{t}\right) = \log_2 8$

$\Rightarrow \frac{3t+4}{t} = 8$

$\Rightarrow 3t+4 = 8t$

$\Rightarrow 4 = 5t$

$\Rightarrow t = \frac{4}{5}$

Question 3

Solve each of the following logarithmic equations.

a) $\log_5(125x) = 4$

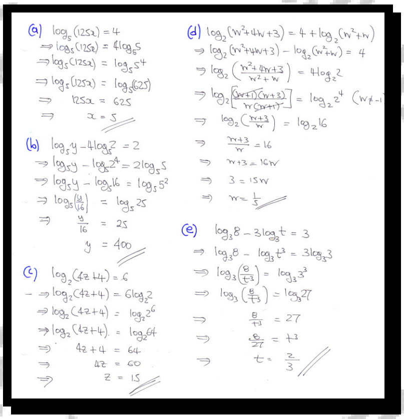
b) $\log_5 y - 4\log_5 2 = 2$

c) $\log_2(4z+4) = 6$

d) $\log_2(w^2+4w+3) = 4 + \log_2(w^2+w)$

e) $\log_3 8 - 3\log_3 t = 3$

$\boxed{x=5}$, $\boxed{y=400}$, $\boxed{z=15}$, $\boxed{w=\frac{1}{5}, w \neq -1}$, $\boxed{t=\frac{2}{3}}$


(a) $\log_5(125x) = 4$

$\Rightarrow \log_5(125x) = 4\log_5 5$

$\Rightarrow \log_5(125x) = \log_5 5^4$

$\Rightarrow \log_5(125x) = \log_5(625)$

$\Rightarrow 125x = 625$

$\Rightarrow x = 5$

(b) $\log_5 y - 4\log_5 2 = 2$

$\Rightarrow \log_5 y - \log_5 2^4 = 2\log_5 5$

$\Rightarrow \log_5 y - \log_5 16 = \log_5 5^2$

$\Rightarrow \log_5\left(\frac{y}{16}\right) = \log_5 25$

$\Rightarrow \frac{y}{16} = 25$

$y = 400$

(c) $\log_2(4z+4) = 6$

$\Rightarrow \log_2(4z+4) = 6\log_2 2$

$\Rightarrow \log_2(4z+4) = \log_2 2^6$

$\Rightarrow \log_2(4z+4) = \log_2 64$

$\Rightarrow 4z+4 = 64$

$\Rightarrow 4z = 60$

$\Rightarrow z = 15$

(d) $\log_2(w^2+4w+3) = 4 + \log_2(w^2+w)$

$\Rightarrow \log_2(w^2+4w+3) - \log_2(w^2+w) = 4$

$\Rightarrow \log_2\left(\frac{w^2+4w+3}{w^2+w}\right) = 4\log_2 2$

$\Rightarrow \log_2\left[\frac{\cancel{(w+1)}(w+3)}{w\cancel{(w+1)}}\right] = \log_2 2^4 \quad (w \neq -1)$

$\Rightarrow \log_2\left(\frac{w+3}{w}\right) = \log_2 16$

$\Rightarrow \frac{w+3}{w} = 16$

$\Rightarrow w+3 = 16w$

$\Rightarrow 3 = 15w$

$\Rightarrow w = \frac{1}{5}$

(e) $\log_3 8 - 3\log_3 t = 3$

$\Rightarrow \log_3 8 - \log_3 t^3 = 3\log_3 3$

$\Rightarrow \log_3\left(\frac{8}{t^3}\right) = \log_3 3^3$

$\Rightarrow \log_3\left(\frac{8}{t^3}\right) = \log_3 27$

$\Rightarrow \frac{8}{t^3} = 27$

$\Rightarrow \frac{8}{27} = t^3$

$\Rightarrow t = \frac{2}{3}$