2014 Leaving Cert Ordinary Level
Official Sample Paper 1

Section A	**Concepts and Skills**	**150 marks**

Question 1 **(25 marks)**

(a) Write 6^{-2} and $81^{\frac{1}{2}}$ without using indices.

Given the relation $x^{-1} = \frac{1}{x}$, we have $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

Since $x^{\frac{1}{2}} = \sqrt{x}$, we have $81^{\frac{1}{2}} = \sqrt{81} = \pm 9$

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(b) Express 2^{24} in the form $a \times 10^n$, where $1 \leqslant a < 10$ and $n \in \mathbb{Z}$, correct to three significant figures.

Using a calculator, we find that $2^{24} = 16,777,216$ so we can write this as 1.68×10^7 correct to three significant figures.
Note that the answer is required to 3 significant figures, not 3 decimal places.

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(c) Show that $\frac{(a\sqrt{a})^3}{a^4}$ simplifies to $\sqrt{a}$.

Firstly we simplify the top line. Note that, as in part **(a)** we can write $a\sqrt{a} = a \cdot a^{\frac{1}{2}} = a^{\frac{3}{2}}$. Now we have

$$
\begin{aligned}
\frac{(a\sqrt{a})^3}{a^4} &= \frac{\left(a^{\frac{3}{2}}\right)^3}{a^4} \\
&= \frac{a^{3\left(\frac{3}{2}\right)}}{a^4} \\
&= \frac{a^{\frac{9}{2}}}{a^4} && (b^x)^y = b^{x \cdot y} \\
&= a^{\frac{9}{2}-4} && \frac{b^x}{b^y} = b^{x-y} \\
&= a^{\frac{1}{2}}
\end{aligned}
$$

which is equal to $\sqrt{a}$.

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(d) Solve the equation $49^x = 7^{2+x}$ and verify your answer.

We can rewrite the left hand side of the equation as $49^x = \left(7^2\right)^x = 7^{2x}$ so our equation becomes $7^{2x} = 7^{2+x}$. We equate the indices, getting $2x = 2 + x$ which is true when $x = 2$

To verify this we check that $49^2 = 7^{2+2}$, which is indeed true.

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Question 2 **(25 marks)**

(a) A sum of €5,000 is invested in an eight-year government bond with an annual equivalent rate (AER) of 6%. Find the value of the investment when it matures in eight years' time.

We know that the future value F of a present investment P invested for n years at a rate of $i\%$ is given by

$$F = P\left(1+\frac{i}{100}\right)^n$$

In our case, $P =$€5000, $n = 8$ and $i = 6$. This gives $F = 5000(1.06)^8$ which is €7,989.24 correct to the nearest cent.

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(b) A different investment bond gives 20% interest after 8 years. Calculate the AER for this bond.

We know that the future value of our investment is the original amount plus the 20% interest, so if our investment is P then the value after 8 years is $1.2P$

We wish to find a value for i such that

$$1.2P = P\left(1+\frac{i}{100}\right)^8$$

We divide across by P to get $1.2 = \left(1+\frac{i}{100}\right)^8$. We can then solve for i:

$$1.2 = \left(1+\frac{i}{100}\right)^8 \quad \Leftrightarrow \quad \sqrt[8]{1.2} = 1+\frac{i}{100}$$
$$\Leftrightarrow \quad \sqrt[8]{1.2} - 1 = \frac{i}{100}$$
$$\Leftrightarrow \quad 100\left(\sqrt[8]{1.2}-1\right) = i$$

From a calculator, this gives that the equivalent AER for this bond is $i = 2.305\%$ correct to three decimal places.

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Question 3 **(25 marks)**

Two complex numbers are $u = 3+2i$ and $v = -1+i$ where $i^2 = -1$.

(a) given that $w = u - v - 2$ evaluate w.

We calculate this by grouping the real and imaginary parts of w together:

$$w = (3+2i) - (-1+i) - 2 = 3+2i+1-i-2 = 2+i$$

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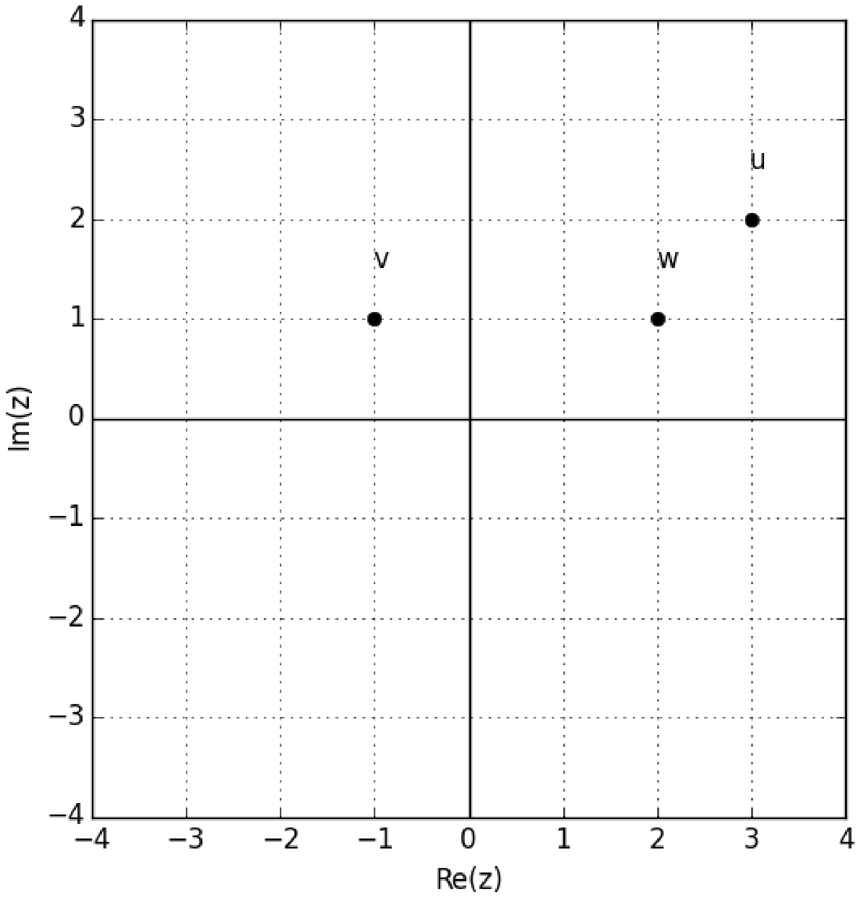
(b) Plot u, v and w on the Argand diagram.

We can plot points on the Argand diagram by considering the real and imaginary parts of the complex number as x and y coordinates respectively.



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(c) Find $\dfrac{2u+v}{w}$.

Firstly, we'll simplify the top line. $2u+v = 2(3+2i)+(-1+i) = 6+4i-1+i = 5+5i$. To divide these two complex numbers, we'll multiply the fraction top and bottom by the conjugate of the denominator, w.

$$\begin{aligned}
\frac{2u+v}{w} &= \frac{5+5i}{2+i} \\
&= \frac{(5+5i)(2-i)}{(2+i)(2-i)} \\
&= \frac{10-5i+10i-5(i^2)}{4-2i+2i-(i^2)} \\
&= \frac{15+5i}{5} \\
&= 3+i
\end{aligned}$$

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Question 4 **(25 marks)**

(a) Solve the equation $x^2 - 6x - 23 = 0$. Give your answer in the form $a \pm b\sqrt{2}$ where $a, b \in \mathbb{Z}$.

We will use the quadratic formula to solve this equation.

$$\begin{aligned}
x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad &\Leftrightarrow \quad x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-23)}}{2(1)} \\
&\Leftrightarrow \quad x = \frac{6 \pm \sqrt{36+92}}{2} \\
&\Leftrightarrow \quad x = \frac{6 \pm \sqrt{128}}{2} \\
&\Leftrightarrow \quad x = \frac{6 \pm \sqrt{64}\sqrt{2}}{2} \\
&\Leftrightarrow \quad x = \frac{6 \pm 8\sqrt{2}}{2} \\
&\Leftrightarrow \quad x = 3 \pm 4\sqrt{2}
\end{aligned}$$

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(b) Solve the simultaneous equations:

$$\begin{aligned}
2r - s &= 10 \\
rs - s^2 &= 12
\end{aligned}$$

We will need to write one variable in terms of the other from one of the two equations and substitute that value into the second equation. We will write r in terms of s from the first equation, and substitute that value of r into the second equation and then solve for s. The first equation tells us that $2r = 10 + s$ or $r = 5 + \frac{1}{2}s$.

The second equation now reads $\left(5 + \frac{1}{2}s\right)s - s^2 = 12$ which expands to $5s + \frac{1}{2}s^2 - s^2 = 12$, or $5s - \frac{1}{2}s^2 = 12$. We can multiply both sides by 2 and rearrange the equation to get $s^2 - 10s + 24 = 0$. This factorises into $(s-4)(s-6) = 0$, i.e. $s = 4$ or $s = 6$.

We finally use these values to solve for r. If $s = 4$, then $r = 5 + \frac{1}{2}(4) = 7$. If $s = 6$, then $r = 5 + \frac{1}{2}(6) = 8$. Thus our solutions are

$$r = 7, s = 4 \qquad \text{or} \qquad r = 8, s = 6$$

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Question 5 **(25 marks)**

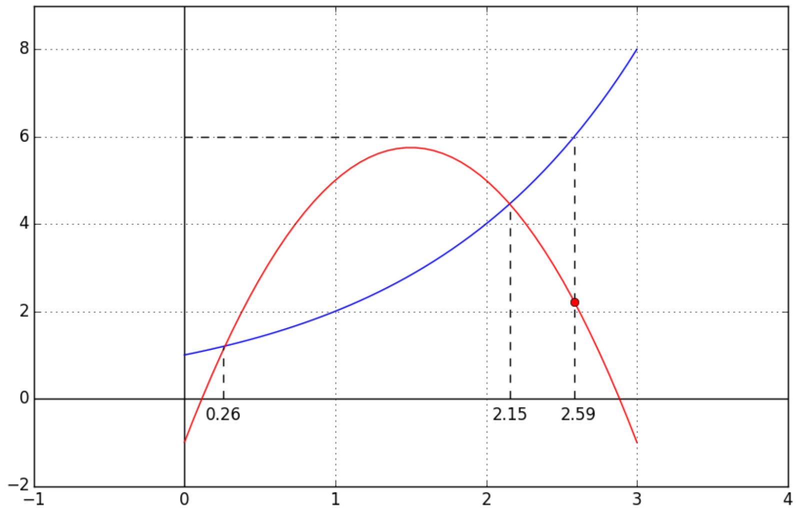
Two functions f and g are defined for $x \in \mathbb{R}$ as follows:

$$f : x \mapsto 2^x$$
$$g : x \mapsto 9x - 3x^2 - 1$$

(a) Complete the table below and use it to draw the graph of f and g.

By direct calculation we have:

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	1	1.414	2	2.828	4	5.657	8
$g(x)$	-1	2.75	5	5.75	5	2.75	-1



Here, $f(x)$ is represented by the blue graph and $g(x)$ is represented by the red graph. There is some extra information in the above graph which applies to the following parts.

(b) Use your graphs to estimate the value(s) of x for which $2^x + 3x^2 - 9x + 1 = 0$.

We can rearrange the above equation as $2^x = 9x - 3x^2 - 1$, which happens when $f(x) = g(x)$. These are the points at which the graphs for the two functions intersect.

From the above graph, the values of x at these two points are $x = 0.26, 2.15$ correct to two decimal places.

(c) Let k be the number such that $2^k = 6$. Using your graph(s), or otherwise, estimate $g(k)$.

Firstly we need to estimate a value for k which has to satisfy $6 = 2^k = f(k)$. We can draw the line $y = 6$ and see the point at which this line intersects the function $f(x)$. From our graph, this happens when $k = 2.59$ correct to two decimal places. We can now evaluate $g(k)$:

$$g(k) = 9(2.59) - 3(2.59)^2 - 1 = 2.1857$$

We can verify this by noting that the line $x = k$ intersects the graph of $g(x)$ when $y = 2.1857$ approximately.

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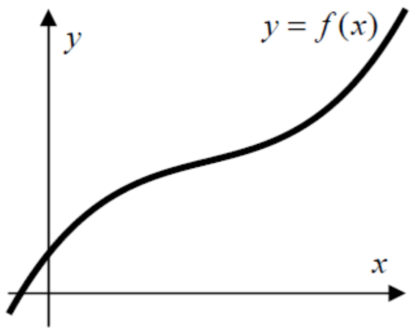
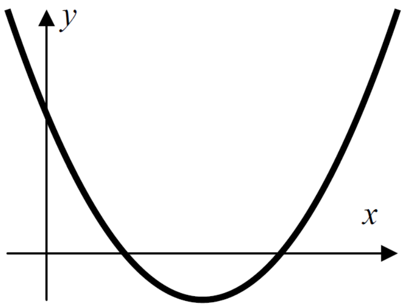
Question 6 **(25 marks)**

The graph of a cubic function f is shown on the right.

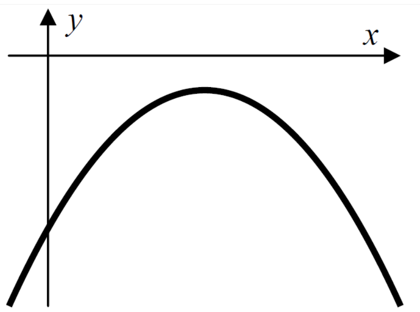
One of the four diagrams **A, B, C, D** below shows the graph of the derivative of f.

State which one it is and justify your answer.

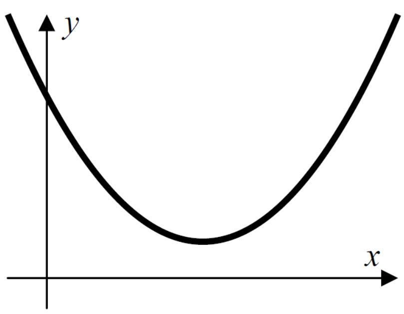




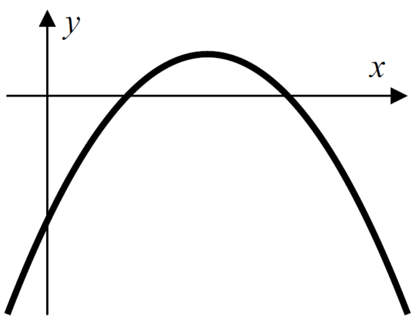
A



B



C

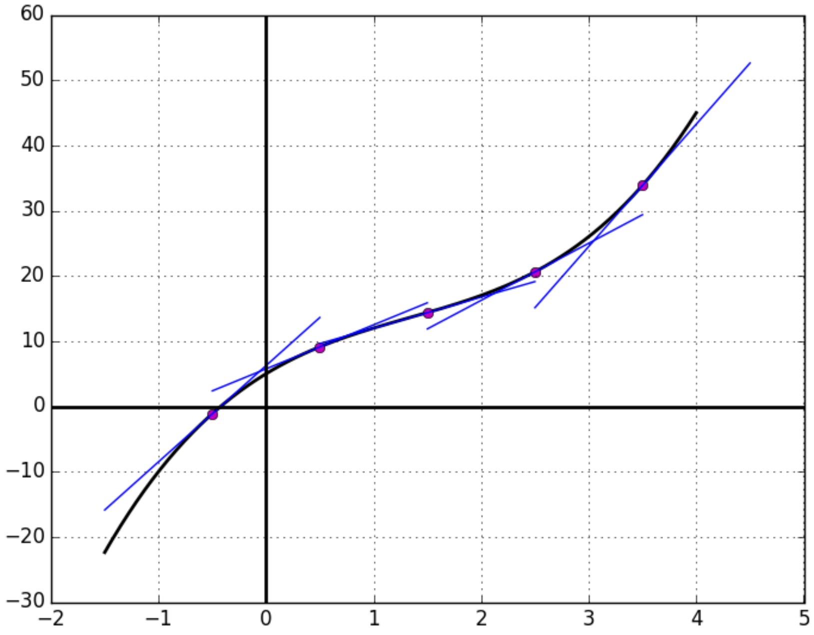


D

Recall that the derivative of a function at a point is the slope of the tangent line to the function at that point. Thus, if we consider the tangent lines of the function $f(x)$, that should help us to determine which is the correct derivative. There are three different cases for a tangent line:

1. **Sloping upwards:** this means that the derivative will be positive.
2. **Sloping downwards:** this means that the derivative will be negative.
3. **Horizontal line:** this means that the derivative will be equal to zero.

Let's look at some of the tangent lines for the function $f(x)$:



There is no line which is horizontal. This means that the derivaive is never equal to zero in this region. This means that graph **A** and **D** must be discarded, since both of these graphs cross the x-axis twice.

The tangent lines are all sloping upwards, so the derivative must always be positive in this region. Since graph **B** is always below the x-axis, it must also be discarded. Thus the correct graph is **C**.

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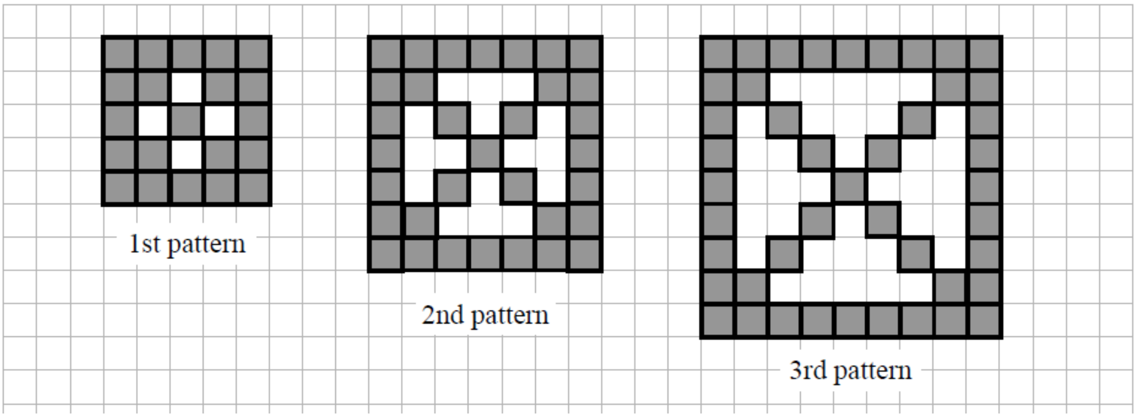
Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section

Question 7 **(50 marks)**

Síle is investigating the number of grey square tiles needed to make patterns in a sequence. The first three patterns are shown below, and the sequence continues in the same way. In each pattern, the tiles form a square and its two diagonal. There are no tiles in the white areas in the pattern - there are only the grey tiles.



(a) In the table below, write the number of tiles needed for each of the first five patterns.

In pattern 1 the number of diagonal tiles is 9. This comes from 5 on one diagonal and 4 on the other, since the one in middle has already been counted. In pattern 2 the number of diagonal tiles is 13: 7 on one diagonal and 6 on the other. In pattern 3 the number of diagonal tiles is 17: 9 on one diagonal and 8 on the other.

For pattern 4, we should have 11 on one diagonal and 10 on the other, so a total of 21 diagonal tiles. Similarly, for pattern 5, we will have 13 on one diagonal and 12 on the other, giving a total of 25 diagonal tiles.

When we have counted the two diagonals, we need to look at the four edges of the square. Remember though that the corner tiles of the square have already been counted when we considered the diagonals. In pattern 1, each edge has 3 remaining tiles, so a total of 12 tiles. In pattern 2, each edge has 5 remaining tiles, so a total of 20 tiles. In pattern 3, each edge has 7 remaining tiles, so a total of 28 tiles.

For pattern 4, each edge will have 9 remaining tiles, so a total of 36 tiles. For pattern 5, each edge will have 11 remaining tiles, so a total of 44 tiles.

Adding the tiles in the diagonals to the remaining tiles on the edges of the square gives us the total tiles needed.

Pattern	1	2	3	4	5
No. of Tiles	21	33	45	57	69

(b) Find, in terms of n, a formula that gives the number of tiles needed to make the nth pattern.

Consider the above sequence of the number of tiles needed: 21, 33, 45, 57, 69. There is a constant common difference of $d = 12$, so this series is an arithmetic one. The first term is $a = 21$, so the general expression for the nth term is given by

$$T_n = a + (n-1)d = 21 + (n-1)12 = 12n + 9$$

(c) Using your formula, or otherwise, find the number of tiles in the tenth pattern.

Setting $n = 10$ in the above formula gives us that $T_{10} = 12(10) + 9 = 129$. Thus, the tenth pattern will need 129 tiles.

(d) Síle has 399 tiles. What is the biggest pattern in the sequence that she can make?

We wish to find the largest positive whole number n such that $T_n \leqslant 399$. Substituting our formula, we get

$$12n + 9 \leqslant 399 \quad \Leftrightarrow \quad 12n \leqslant 390 \quad \Leftrightarrow \quad n \leqslant \frac{390}{12} = 32.5$$

The largest positive whole number which satisfies this inequality is $n = 32$. Thus, the biggest pattern that Síle can make is pattern 32.

We can verify this by noting that $T_{32} = 12(32) + 9 = 393$ and $T_{33} = 12(33) + 9 = 405$.

(e) Find in terms of n a formula for the total number of tiles in the first n patterns.

To find the total number of tiles need to make the first n pattens, we need to find the sum of the first n terms in the sequence T_n. The formula for this S_n is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a and d are the first term and common difference of T_n respectively. From part **(b)**, we know that $a = 21$ and $d = 12$, so

$$S_n = \frac{n}{2}[2(21) + (n-1)12] = \frac{n}{2}[30 + 12n] = 6n^2 + 15n$$

(f) Síle starts at the beginning of the sequence and makes as many of the patterns as she can. She does not break up the earlier patterns to make the new ones. For example, after making the first two patterns, she has used up 54 tiles $(21 + 33)$. How many patterns can she make in total with her 339 tiles?

As with part **(d)** above, we wish to find the greatest positive whole number such that $S_n \leqslant 399$. Substituting our formula for S_n, we get

$$6n^2+15n \leqslant 399 \quad \Leftrightarrow \quad 6n^2+15n-399 \leqslant 0 \quad \Leftrightarrow \quad 2n^2+5n-133 \leqslant 0$$

We will consider the function $2x^2+5x-133$, and find the largest positive whole number for which the function is negative. We can factorise the function as $2x^2+5x-133 = (2x+19)(x-7)$, and so the function will cross the x-axis when $x = -\frac{19}{2}, 7$. Thus, the function will be negative when $-\frac{19}{2} \leqslant x \leqslant 7$.

The largest positive whole number which satisfies this inequality is $n = 7$. Thus, the largest number of patterns that Síle can make is 7 patterns. We can verify this by noting that $S_7 = 6(7)^2 + 15(7) = 399$.

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Question 8 **(50 marks)**

John is given two sunflower plants. One plant is 16 cm high and the other is 24 cm high. John measures the height of each plant at the same time every day for a week. He notes that the 16 cm plant grows 4 cm each day, and the 24 cm plant grows 3·5 cm each day.

(a) Draw up a table showing the heights of the two plants each day for the week, starting on the day that John got them.

We will refer to the plant which was initially 16 cm as plant A and the other as plant B. In the table below, day 0 will refer to the day that John bought the plants.

Day	0	1	2	3	4	5	6	7
A	16cm	20cm	24cm	28cm	32cm	36cm	40cm	44cm
B	24cm	27.5cm	31cm	34.5cm	38cm	40.5cm	44cm	48.5cm

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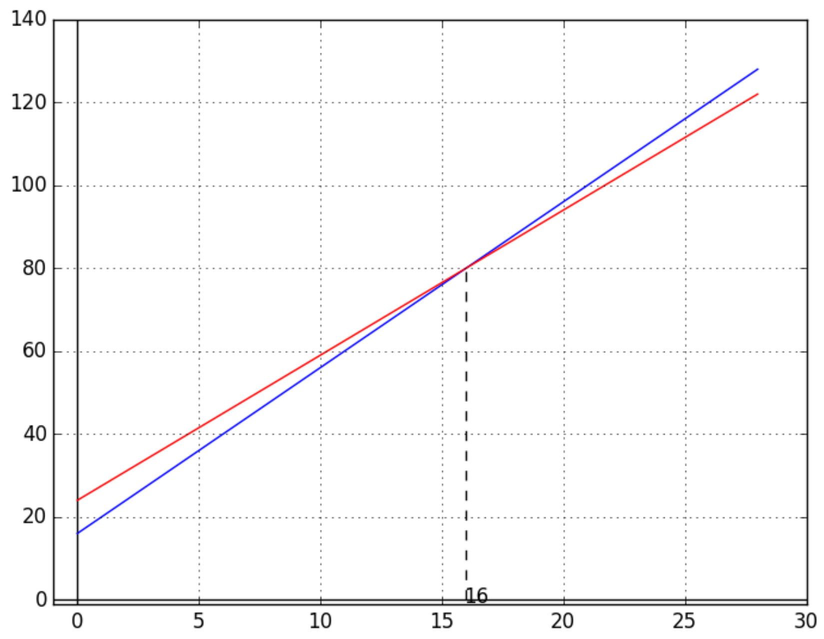
(b) Write down two formulas – one for each plant – to represent the heights of the two plants on any given day. State clearly the meaning of any letters used in your formulas.

Let $f(x)$ represent the height in centimetres of plant A, x days after John bought it. Similarly, let $g(x)$ represent the height in centimetres of plant B, x days after John bought it. We know the initial values $f(0) = 16$ and $g(0) = 24$. As we increase x by one, f increases by 4 and g increases by 3.5, so our general formulae are

$$f(x) = 16 + 4x \qquad \text{and} \qquad g(x) = 24 + 3.5x$$

We could also determine these by noting that the graphs will be straight lines. So for f, for example, two points on the line will be $(0,16)$ and $(1,20)$. We could then use the formula $(y - y_1) = m(x - x_1)$, where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and solve for $y = f(x)$. A similar approach would give g.

(c) John assumes that the plants will continue to grow at the same rates. Draw graphs to represent the heights of the two plants over the first *four weeks*.



Here, the blue line represents $f(x)$ (the height of plant A) and the red line represents $g(x)$ (the height of plant B).

(d) **(i)** From your diagram, write down the point of intersection of the two graphs.

The two graphs intersect when $x = 16$. We can verify this by noting that $f(16) = 16+4(16) = 80$ and $g(16) = 24+3.5(16) = 80$.

(ii) Explain what the point of intersection means, with respect to the two plants. Your answer should refer to the meaning of *both* co-ordinates.

The point of intersection $(16, 80)$ tells us the day at which the heights of the two plants will be equal. Since the x coordinate represents the day and the y coordinate represents the height in centimetres of each plant, on day 16, both plants will be 80 cm tall.

(e) Check your answer to part **(d)(i)** using your formulas from part **(b)**.

We wish to determine the point x for which $f(x) = g(x)$, i.e.

$$16+4x = 24+3.5x \quad \Leftrightarrow \quad 0.5x = 8 \quad \Leftrightarrow \quad x = 16$$

as expected.

(f) The point of intersection can be found either by reading the graph or by using algebra. State one advantage of finding it using algebra.

By using algebra, we can find the point of intersection exactly with no errors. When reading from a graph, the point has to be estimated, which can introduce errors.

(g) John's model for the growth of the plants might not be correct. State one limitation of the model that might affect the point of intersection and its interpretation.

This model assumes that both plants grow at constant rates. This may not be practically true, as plants may grow at varying speeds. Thus, the graphs for the functions may realistically not be straight lines, which would affect where the point of intersection lies.

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Question 9 **(50 marks)**

(a) A farmer is growing winter wheat. The amount of wheat he will get per hectare depends on, among other things, the amount of nitrogen fertiliser that he uses. For his particular farm, the amount of wheat depends on the nitrogen in the following way:

$$Y = 7000 + 32N - 0.1N^2$$

where Y is the amount of wheat produced, in kg per hectare, and N is the amount of nitrogen added, in kg per hectare.

(i) How much wheat will he get per hectare if he uses 100 kg of nitrogen per hectare?

The amount of wheat produced is given by $Y(100) = 7000 + 32(100) - 0.1(100)^2 = 9,200$ kg per hectare.

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(ii) Find the amount of nitrogen that he must use in order to maximise the amount of wheat produced.

The first derivative test tells us that the stationary points occur when the first derivative is equal to zero. The second derivative test tell us that if the second derivative is negative when evaluated at a stationary point, then that point is a local maximum. Firstly we need to work out both first and second derivatives, which are

$$\frac{dY}{dN} = (0)(7000) + (1)(32)N^{1-1} - (2)(0.1)N^{2-1} = 32 - 0.2N$$

$$\frac{d^2Y}{dN^2} = (0)(32) - (1)(0.2)N^{1-1} = -0.2$$

Now we solve $\frac{dY}{dN} = 0$, which happens when $32 = 0.2N$, or $N = 160$. We substitute this value into the second derivative to get $\left.\frac{d^2Y}{dN^2}\right|_{N=160} = -0.2$ Since this value is negative, the maximum value of Y occurs when $N = 160$.

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(iii) What is the maximum possible amount of wheat produced per hectare?

From the previous part, we know that the output of wheat is at its maximum when $N = 160$. Thus, the maximum amount of wheat produced is $Y(160) = 7000 + 32(160) - 0.1(160)^2 = 9,560$ kg per hectare.

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(iv) The farmer's total costs for producing the wheat are €1,300 per hectare. He can sell the wheat for €160 per tonne. He can also get €75 per hectare for the leftover straw. If he achieves the maximum amount of wheat, what is his profit per hectare?

If the farmer achieves the maximum amount of wheat, the previous section tell us that he will have $9,560$ kg per hectare, which is 9.56 tonnes per hectare (note: 1 tonne $= 1000$ kg). On selling the wheat, the farmer will earn $160 \times 9.56 =$ €1,529.60 per hectare. The farmer's total revenue, including the leftover straw, will be $1,529.60 + 75 =$ €1,604.60 per hectare.

The total costs for producing the wheat are €1,300 so the profit will be $1604.60 - 1300 =$ €304.60 per hectare.

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(b) A marble is dropped from the top of a fifteen-storey building. The height of the marble above the ground, in metres, after t seconds is given by the formula:

$$h(t) = 44.1 - 4.9t^2$$

Find the speed at which the marble hits the ground. Give your answer:

(i) in metres per second

Firstly, we need to find the time at which the marble will hit the ground. This will be the value of t for which $h(t) = 0$, so

$$\begin{aligned} 44.1 - 4.9t^2 = 0 \quad &\Leftrightarrow \quad 44.1 = 4.9t^2 \\ &\Leftrightarrow \quad 9 = t^2 \\ &\Leftrightarrow \quad t = \pm 3 \end{aligned}$$

Since we are dealing with time, which is always positive, we take $t = 3$. Given the height $h(t)$, the speed after t seconds will be given by $|h'(t)|$. The derivative is given by $h'(t) = (2)(-4.9)t^{2-1} = -9.8t$

Thus, after 3 seconds (when the marble hits the ground), the speed will be $|h'(3)| = |-9.8(3)| = 29.4$ metres per second, downwards.

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(ii) in kilometres per hour

Given the speed in metres per second, we can convert it to kilometres per hour. To convert a speed from metres per second to kilometres per second, we divide by 1000. To convert a speed from metres per second to metres per hour, we multiply by 3600. Thus we will multiply our speed by $\frac{3600}{1000} = 3.6$. The speed at which the marble hits the ground will be $29.4 \times 3.6 = 105.84$ kilometres per hour.

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