2022.M27

2022L003G1EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2022

Mathematics

Paper 1

Ordinary Level

Friday 10 June   Afternoon 2:00 – 4:30

220 marks

Examination Number	
Day and Month of Birth	

For example, 3rd February is entered as 0302

Centre Stamp

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	120 marks	6 questions
Section B	Contexts and Applications	100 marks	4 questions

Answer questions as follows:

- any **four** questions from Section A – Concepts and Skills
- any **two** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet.
If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

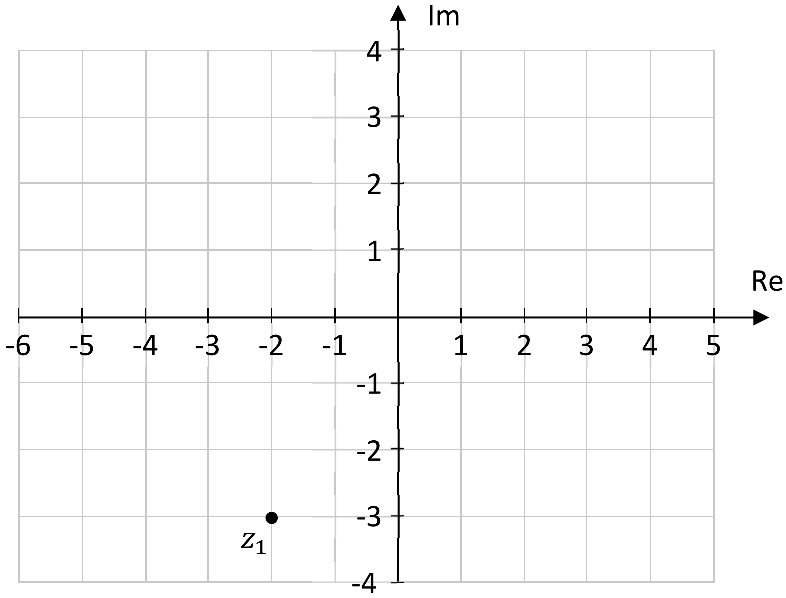
Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 120 marks

Answer **any four questions** from this section.

Question 1 **(30 marks)**

The complex number z_1 is shown on the Argand diagram below.



(a) Using the Argand diagram:

(i) write down the values of z_1 and $\overline{z_1}$, where $\overline{z_1}$ is the complex conjugate of z_1

$z_1 =$ $\qquad$ $\overline{z_1} =$

(ii) plot and label $\overline{z_1}$ on the Argand diagram above.

z_2 and z_3 are two other complex numbers.

$z_2 = -5 + 3i$ and $z_3 = 4 - 2i$, where $i^2 = -1$.

(b) Plot and label z_2 and z_3 on the Argand diagram on the previous page.

(c) Write $z_2 - z_3$ in the form $a + bi$, where $a, b \in \mathbb{R}$, $i^2 = -1$, **and** hence find $|z_2 - z_3|$.

$z_2 - z_3 =$ ______________

$|z_2 - z_3| =$ ______________

(d) Investigate if $z_3 = 4 - 2i$ is a solution of the equation $z^2 + 2iz - 7i = 0$.

Conclusion: ___________________________________

Question 2 **(30 marks)**

(a) Solve the following equation in x:

$$2(3x-5)+8=4x-5$$

(b) Write $\frac{(3^4)^5}{3^6}$ in the form 3^k, where $k \in \mathbb{R}$.

(c) Solve the simultaneous equations below to find the value of x and the value of y.

$$3x + 2y = 1$$
$$7x + 5y = -2$$

$x =$ ____________ $y =$ ____________

Question 3 **(30 marks)**

Joe, Émile, and Wei are all PAYE workers.
Each of them has an annual tax credit of €3300.
Their tax rates and bands are shown in the table below.
Assume that no other deductions are made from their income.

Annual Income	Tax Rate
First €35 300	20%
Balance	40%

(a) Joe's gross annual income is €27 500. Joe only pays tax at the lower rate.
Work out Joe's net annual income.

(b) Émile's gross annual income is €43 450.
Work out Émile's net annual income.

(c) Wei's gross annual income is over €$35\ 300$, so she pays tax at both rates.

Wei is looking for a pay rise.
She wants her net income to increase by €80 each month.

Work out how much her **gross annual income** will need to increase by, in order for this to happen.

Question 4 **(30 marks)**

(a) $g(x) = x^3 - 7x^2 + x - 12$, where $x \in \mathbb{R}$.

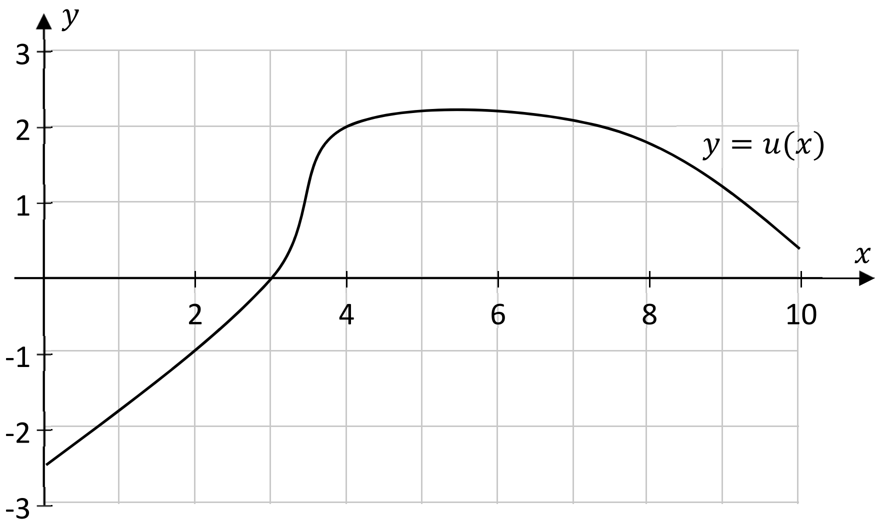
(i) Work out the value of $g(5)$.

(ii) Find $g'(x)$, the derivative of $g(x)$.

(iii) $g'(5) = 6$.

Use this to find the equation of the **tangent** to the curve $y = g(x)$ when $x = 5$.
Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{R}$.

(b) The graph of the function $y = u(x)$ is shown below, for $0 \leq x \leq 10$, $x \in \mathbb{R}$.



$u'(x)$ is the derivative of $u(x)$.

(i) Using the graph, write down a value of x for which $u'(x)$ is **negative**.

$x =$ ☐

(ii) On the diagram above, draw the tangent to $u(x)$ at the point $(4, 2)$ **and** use the tangent that you draw to work out an estimate for the value of $u'(4)$.

$u'(4) =$ ____________ , approximately

Question 5 **(30 marks)**

(a) Write each of the following values in the form $a \times 10^n$ where $1 \leq a < 10$ and $n \in \mathbb{Z}$.

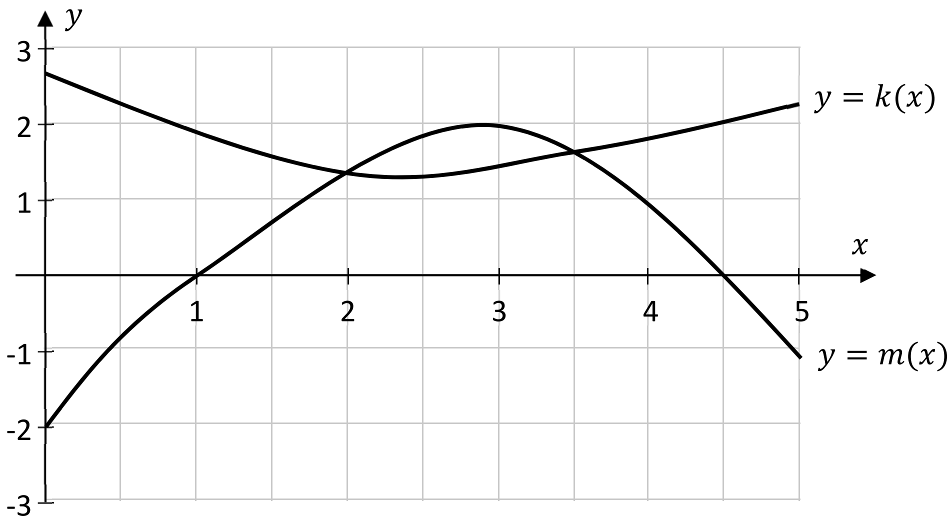
(i) 1200

(ii) $0{\cdot}27$

(b) A falcon can dive at a speed of up to 120 miles per hour.
1 mile is approximately $1{\cdot}6$ kilometres.

Use this to work out how long it would take the falcon to travel 100 metres, when diving at this speed. Give your answer in seconds, correct to one decimal place.

(c) The diagram below shows the graphs of the functions $k(x)$ and $m(x)$, for $0 \leq x \leq 5$, $x \in \mathbb{R}$.



Use the graphs to estimate each of the following, for $0 \leq x \leq 5$:

(i) the two values of x for which $m(x) = 0$

$x =$ $\boxed{}$ or $x =$ $\boxed{}$

(ii) the range of values of x for which $k(x)$ is less than $m(x)$.

Question 6 **(30 marks)**

The nth term of an arithmetic sequence is given by the following expression, for $n \in \mathbb{N}$:

$$T_n = -254 + (n-1)(4)$$

(a) **(i)** Find the value of T_1 , the first term of this sequence.

(ii) Find the value of the common difference for this sequence (that is, $T_2 - T_1$).

(b) Find the smallest value of $n \in \mathbb{N}$ for which

$$-254 + (n-1)(4) > 0$$

Answer: $n =$ ___________

(c) The sum of the first n terms of this sequence is given by $S_n = \frac{n}{2}[2(-254) + 4n - 4]$.

Solve the following equation for $n \in \mathbb{N}$. Note that $n \neq 0$.

$$\frac{n}{2}[2(-254) + 4n - 4] = 0$$

Answer: $n =$ ____________

Section B   Contexts and Applications   100 marks

Answer **any two questions** from this section.

Question 7 **(50 marks)**

Joseph is doing a training session. During the session, his heart-rate, $h(x)$, is measured in beats per minute (BPM). For part of the session, $h(x)$ can be modelled using the following function:

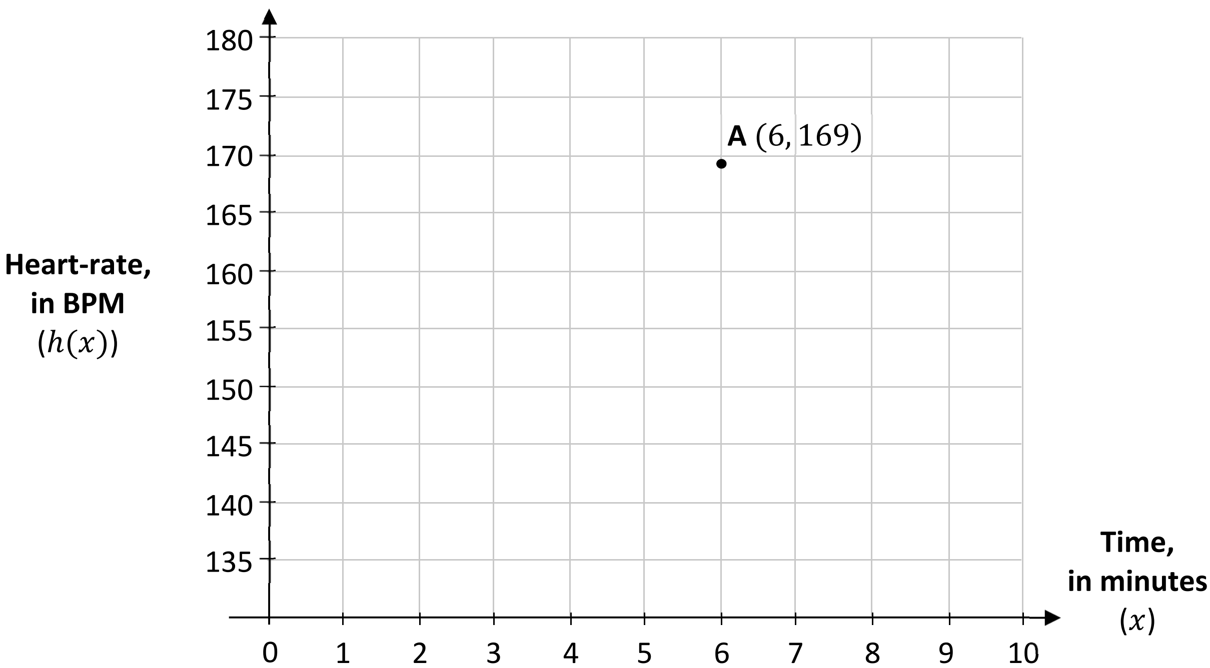
$$h(x) = -\,0{\cdot}38x^3 + 2{\cdot}6x^2 - 0{\cdot}13x + 158$$

where x is the time, in minutes, from the start of the session, and $0 \leq x \leq 6$, $x \in \mathbb{R}$.

(a) **(i)** Complete the table below to show the values of $h(x)$ for the given values of x. Give each value of $h(x)$ correct to the nearest whole number.

Time (minutes)	0	1	2	3	4	5	6
Heart-rate (BPM)		160		171			169

(ii) Draw the graph of $y = h(x)$ on the axes below, for $0 \leq x \leq 6$, $x \in \mathbb{R}$. Note that the point **A** $(6, 169)$ is on the graph.



(b) Explain what the co-ordinates of the point **A** $(6, 169)$ represent, in the context of Joseph's heart-rate.

(c) Using the same axes and scales, continue your graph on the previous page to show the following information. From the point represented by **A** $(6, 169)$, Joseph's heart-rate:

- stays at the same level for the next 2 minutes, and then
- decreases at a steady rate of 10 BPM per minute for 2 minutes.

(d) During his training session, the number of calories per minute that Joseph is burning after x minutes can be modelled by $c(x)$ as follows, where $h(x)$ is Joseph's heart-rate at that time:

$$c(x) = 0{\cdot}1\,h(x) - 7$$

Using the information in the table or graph, work out $c(6)$, the number of calories per minute that Joseph is burning 6 minutes after the start of the session.

This question continues on the next page.

(e) Joseph has a smart watch that beeps every 15 seconds during the session.
It beeps for the first time at exactly $2{:}\,55$ p.m., as Joseph starts his session.
It beeps for the last time at exactly $3{:}\,23$ p.m., as Joseph finishes his session.

Work out how many times, in total, the smart watch beeps during the session, including the first and last beep.

(f) Solve the equation

$$h'(x) = -\,1{\cdot}14x^2 + 5{\cdot}2x - 0{\cdot}13 \;\; = 0$$

to find how long after the start of the session Joseph's heart-rate is at a maximum, for $0 \le x \le 6$, $x \in \mathbb{R}$. Give your answer in minutes, correct to 2 decimal places.

Question 8 **(50 marks)**

(a) Jessica is a scientist. Jessica is making up a solution of acid.
She has two different bottles, each with the following concentration of the acid:

Bottle A	**Bottle B**
Concentration: 12%	Concentration: 5%

This means that, for example, in every 100 ml of liquid in Bottle **A**, there are 12 ml of acid.

(i) Work out how many ml of acid are in 200 ml of liquid from Bottle **A**.

(ii) Jessica mixes 200 ml of liquid from Bottle **A** with 300 ml of liquid from Bottle **B**.
Work out the overall concentration of the acid in Jessica's mixture.
Give your answer as a percentage.

(iii) Explain why Jessica could **not** make a solution with a 4% concentration of acid by mixing liquid from Bottle **A** and Bottle **B**.

This question continues on the next page.

(iv) When she is making another mixture, Jessica makes a mistake in measuring. She wants to measure out 250 ml but she measures out 260 ml instead.

Work out the percentage error in this measurement.

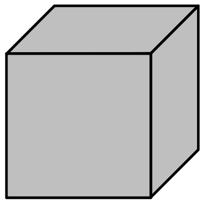
(b) If a solid is made up of faces with straight edges, then the following identity is often true:

$$C - E + F = 2$$

where: C is the number of corners,
E is the number of edges, and
F is the number of faces.

(i) Write down the values of C, E, and F for a cube, **and** show that $C - E + F = 2$ for these values.

The value for E has already been filled in.



$C =$ $\boxed{}$ $E =$ $\boxed{12}$ $F =$ $\boxed{}$

(ii) Each of the faces of a **different** solid is in the shape of a triangle of area 5 cm^2.

This solid has 12 corners (C) and 30 edges (E), and $C - E + F = 2$ for this solid.

Work out the surface area of this solid, in cm^2.

(iii) The surface of a **third** solid is made up of h hexagons and p pentagons, where $h, p \in \mathbb{N}$. For this solid:

$$\frac{6h + 5p}{3} - \frac{6h + 5p}{2} + h + p = 2$$

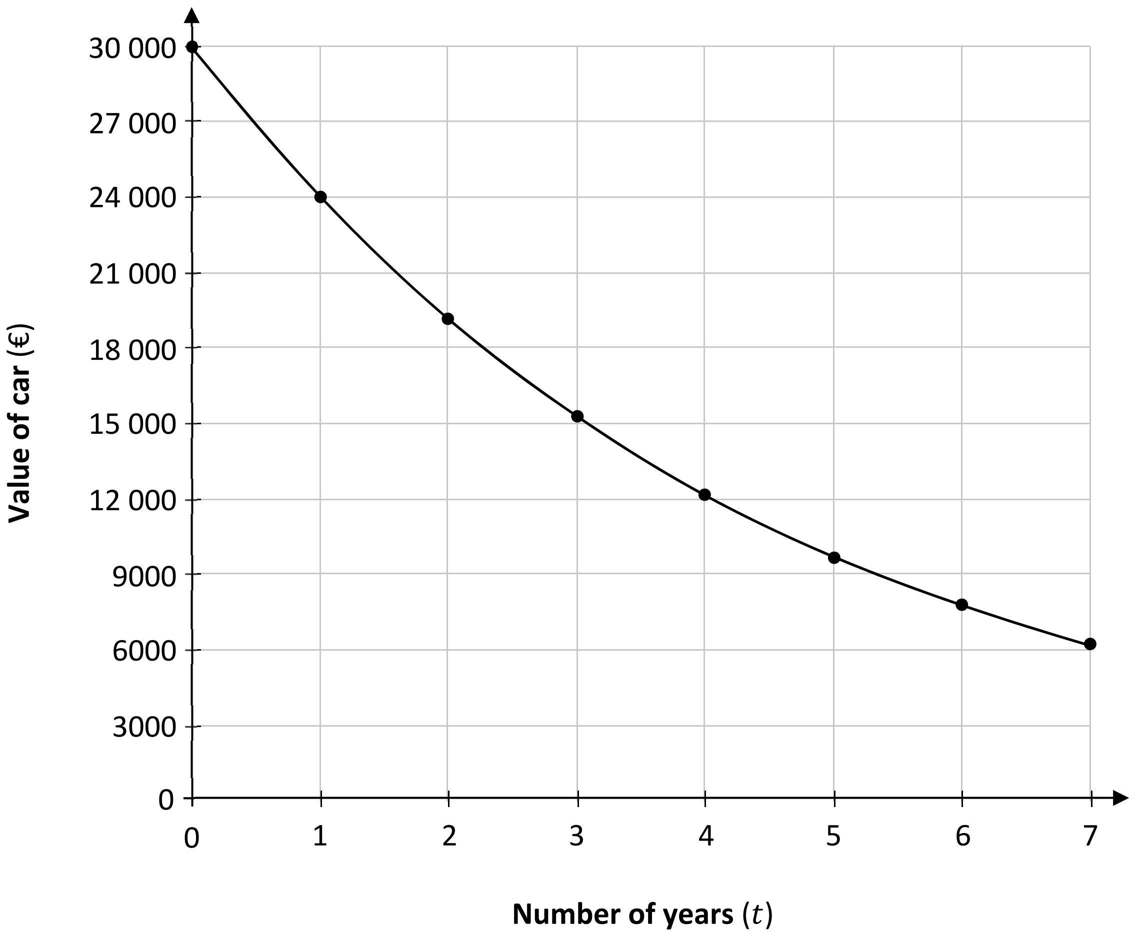
Use this equation to find the number of pentagons in the surface of this solid (that is, the value of p).

$p =$ ____________

Question 9 **(50 marks)**

Brian buys a new car.

The graph below represents a model that can be used to predict the value of this car, V, for the next number of years. This model assumes that the value of the car reduces (depreciates) by a fixed **percentage** each year.



(a) **(i)** Use the graph to write down $V(0)$, the initial value of Brian's car, **and** $V(1)$, the value of Brian's car after 1 year.

$V(0) =$ € $\qquad\qquad$ $V(1) =$ €

(ii) Show that the value of the car will reduce by 20% in its first year, according to this model.

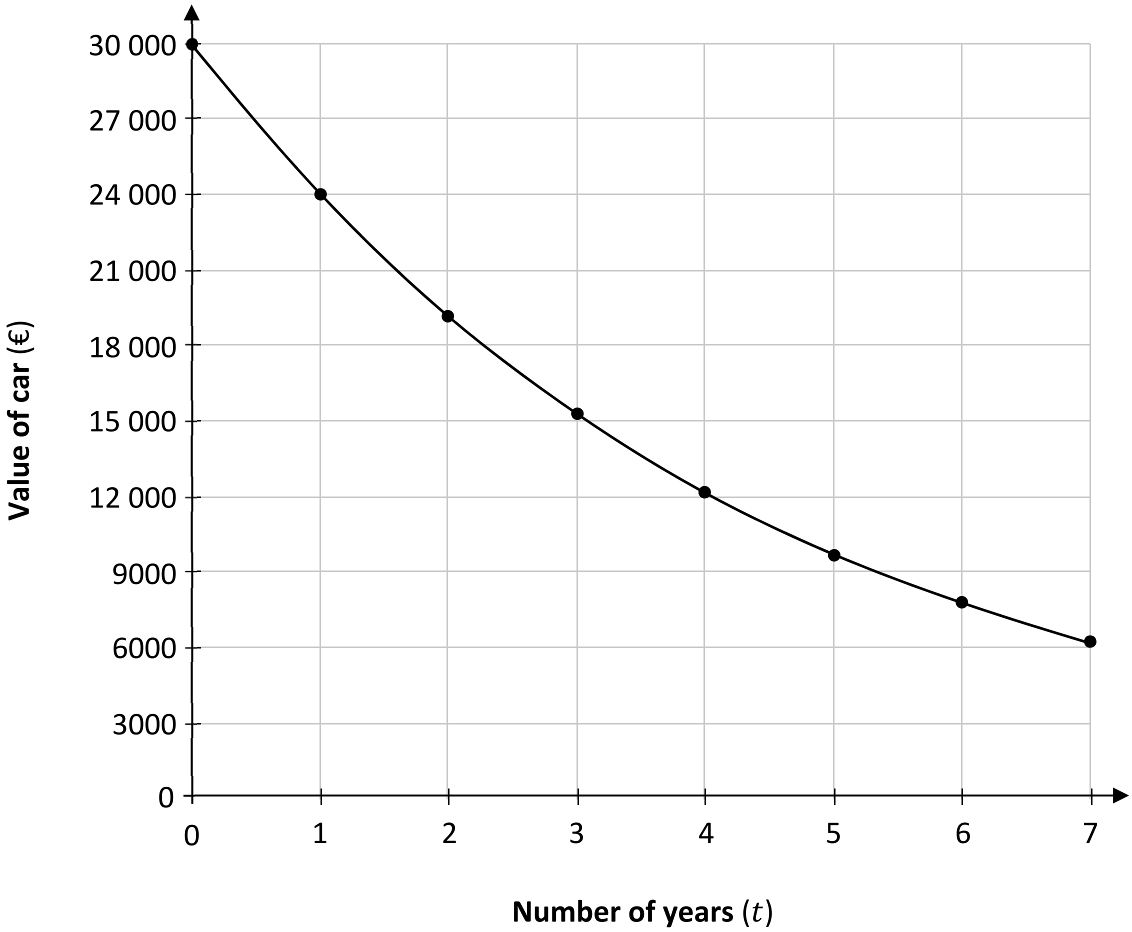
(b) **(i)** Based on this model, write a formula for $V(t)$, the value of Brian's car after t years, in terms of the age of the car (t).

Use the fact that the value decreases by 20% each year.

(ii) Hence, or otherwise, work out the value of Brian's car after 4 years, according to this model. Show your working out.

This question continues on the next page.

(c) The graph from **part (a)** is shown again below.



A different (linear) model assumes that the value of the car reduces (depreciates) by a fixed **amount** each year. The value of the car will also reduce by 20% in its first year, according to this model.

(i) Draw a line on the diagram above, passing through the first two points on the graph with whole-number values of t ($t = 0$ and $t = 1$). Continue your line until it reaches the horizontal axis.

(ii) Hence, or otherwise, estimate T, the age of Brian's car when its value would be €0, according to this new model.

(d) Eva buys a new car that has a price of €$19\,445$.

She pays 30% of this price as a deposit and makes repayments of €$206{\cdot}97$ each month for the following 3 years. At the end of the 3 years, she pays an additional lump sum of €7389.

Work out the total cost of the car for Eva.

(e) Eva drives her car home from the garage, a distance of 12 km.

Eva usually drives this journey at an average speed of 60 km/hr.
On this day, there are roadworks, so her average speed is only 40 km/hr for the journey.

Work out the percentage increase in the time it takes Eva to drive home, because of the roadworks.

Question 10 **(50 marks)**

Keith plays hurling.

(a) During a match, Keith hits the ball with his hurl.
The height of the ball could be modelled by the following quadratic function:

$$h = -2t^2 + 5\text{t} + 1{\cdot}2$$

where h is the height of the ball, in metres, t seconds after being hit, and $t \in \mathbb{R}$.

(i) How high, in metres, was the ball when it was hit (when $t = 0$)?

(ii) The ball was caught after $2{\cdot}4$ seconds.
How high, in metres, was the ball when it was caught?

(iii) When the ball passed over the halfway line, it was at a height of 3·2 metres and its height was decreasing.

How many seconds after it was hit did the ball pass over the halfway line?
Remember that $h = -2t^2 + 5\text{t} + 1{\cdot}2$.

Answer: ____________

(iv) Find $\frac{dh}{dt}$ **and** hence find how long it took the ball to reach its greatest height.

Give your answer in seconds.

$\frac{dh}{dt} =$ ________________________________

Length of time = _______________

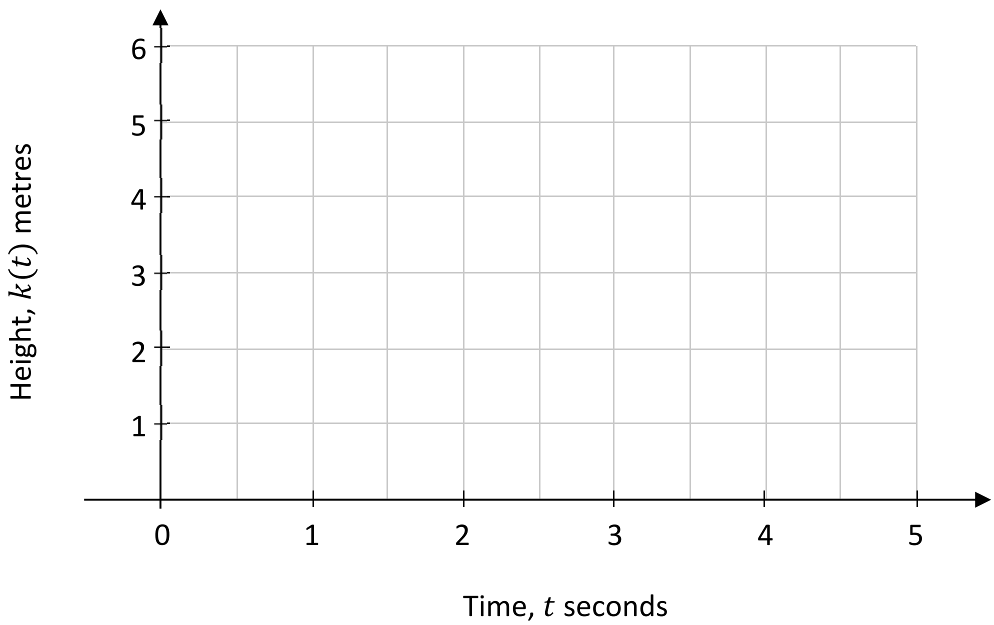
This question continues on the next page.

(b) Later in the game, Keith hit the ball again. This time, the height of the ball t seconds after it was hit could be modelled by a different quadratic function, $y = k(t)$, where k is in metres.

This time, the ball was 1 metre high when Keith hit it.
Its greatest height was 5 metres, which it reached after 2 seconds.
It hit the ground without being caught.

Using the information above, write down the co-ordinates of three points that **must** be on the graph of $y = k(t)$, **and** draw the graph of $y = k(t)$ on the axes below, from when the ball is hit until it hits the ground.

Points: $(\quad , \quad)$, $(\quad , \quad)$, and $(\quad , \quad)$



(c) **(i)** Keith buys a new hurl. It usually costs €33.
Keith gets a student discount of 15%.
Work out the price Keith pays for the hurl.

(ii) Keith also buys a jersey. This costs €$49{\cdot}50$, including VAT at 23%.
Work out the **VAT** on this jersey. Give your answer correct to the nearest cent.

Page for extra work.
Label any extra work clearly with the question number and part

Page for extra work.
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