Senior Cycle

TEACHER HANDBOOK

Senior Cycle Ordinary Level

5th & 6th year

Project Maths
Tionscadal Mata
Development Team

Based on the 2015 syllabus

Project Maths Tionscadal Mata Development Team

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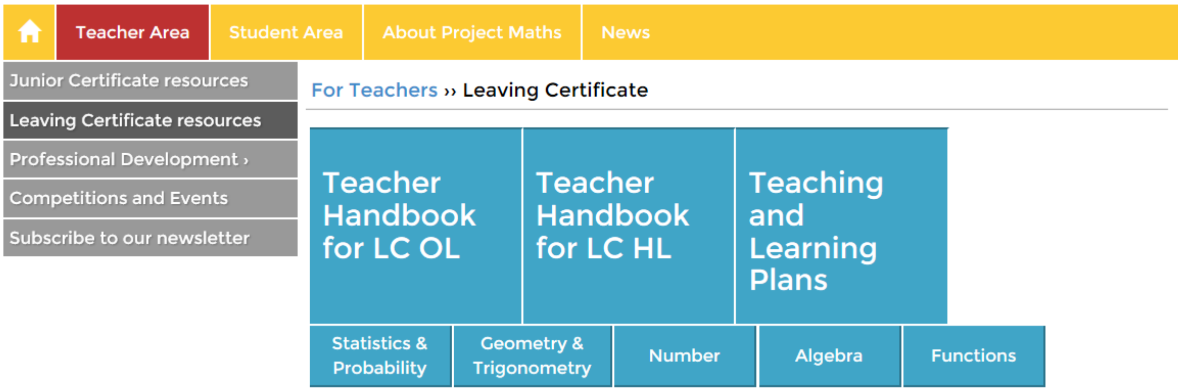
The strand structure of the syllabus should not be taken to imply that topics are to be studied in isolation. Where appropriate, connections should be made within and across the strands and with other areas of learning. (NCCA JC syllabus page 10 and LC syllabus page 8)

Resources which will allow teachers plan lessons, easily access specific learning outcomes in the syllabus and relevant support material such as "Teaching & Learning Plans" and suggested activities to support learning and teaching are available on the Project Maths website www.projectmaths.ie



Introduction

Student Learning

While this is a handbook for teachers, it must be emphasised that student learning and the process of mathematical thinking and building understanding are the main focus of this document.

Information and Communications Technologies are used whenever and wherever appropriate to help to support student learning. It is also envisaged that, at all levels, learners will engage with a dynamic geometry software package.

Students with mild general learning disabilities

Teachers are reminded that the NCCA Guidelines on mathematics for students with mild general learning disabilities can be accessed at

http://www.ncca.ie/uploadedfiles/PP_Maths.pdf

This document includes

- Approaches and Methodologies (from Page 4)
- Exemplars (from page 20).

Note: Synthesis and problem solving listed below must be incorporated into all of the Strands.
The list of skills below is taken from Strand 1of the syllabus but, an identical list is given at the beginning of each Strand in the syllabus.

At each syllabus level students should be able to

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- analyse information presented verbally and translate it into mathematical form
- devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Useful websites

	http://www.projectmaths.ie/ http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Literacy and Numeracy Strategy

The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020

Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems.

Literacy includes the capacity to read, understand and critically appreciate various forms of communication including spoken language, printed text, broadcast media, and digital media.

Colour coding used in the suggested sequence below:

Strand 1 **Statistics and Probability**	**Strand 2** **Geometry and Trigonometry**	**Strand 3** **Number**	**Strand 4** **Algebra**	**Strand 5** **Functions**

Suggested sequence of topics

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Proposed beginning of 6th year programme					
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Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson Idea	Page number	
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Lesson Ideas

Section 1: Number

Lesson Idea LCOL.1

Title
Number systems

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The operations of addition, multiplication, subtraction and division (emphasising the order of operations, including brackets) in the following domains:
  N, Z, Q and representing these numbers on a number line
- Decimals as special equivalent fractions and strengthening the connection between these numbers and fractions and place value understanding
- Fractions, decimals (that have a finite or a repeating decimal representation) and percentages as different representations of rational numbers
- Rounding of decimals
- Terminating and non-terminating decimals
- Irrational numbers $\mathbf{R}\setminus\mathbf{Q}$
  Number system $\mathbf{R}$, appreciating that $\mathbf{R}\setminus\mathbf{Q}$ and representing real numbers on a number line

Lesson Idea LCOL.2

Title
Rules for indices

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The rules for indices (where $a,b \in \mathbf{R}$, $p,q \in \mathbf{Q}$; $a^p, a^q \in \mathbf{Q}$; $a,b \neq 0$)
  $a^p a^q = a^{p+q}$
  $\frac{a^p}{a^q} = a^{p-q}$
  $a^0 = 1$
  $\left(a^p\right)^q = a^{pq}$

$$a^{-p} = \frac{1}{a^p}$$
$$(ab)^p = a^p b^p$$
$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

Lesson Idea LCOL.3 (Possibly for 6th year depending on the class)

Title
Adding, subtracting, multiplying and dividing real numbers

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Irrational numbers, including surds
- How to apply the following rules:
- $a^{\frac{1}{q}} = \sqrt[q]{a}, \quad q \in \mathbf{Z}, q \neq 0, a > 0$
  $a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p \quad p, q \in \mathbf{Z}, q \neq 0, a > 0$
- Adding, subtracting, multiplying and dividing surds

Lesson Idea LCOL.4

Title
Factors, multiples and primes

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Factors, multiples and prime numbers in $\mathbf{N}$
- Expressing numbers in terms of their prime factors
- Highest Common Factor and Lowest Common Multiple

Lesson Idea LCOL.5

Title
Percentages, estimating using real world contexts, % error

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Make estimates of measures in the physical world around them
- How to make and justify estimates and approximations of calculations
- How to check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result
- How to calculate percentage error and tolerance
- How to calculate accumulated error (due to addition or subtraction only)

Lesson Idea LCOL.6

Title
Financial Maths

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to solve problems involving
  - Cost price, selling price, loss, discount
  - Mark up (profit as a % of cost price)
  - Margin (profit as a % of selling price)
  - Income tax and net pay including other deductions
  - Compound interest investigated using multi-representations i.e. table, graph and formula (link to exponential functions and geometric sequences)
  - Compound interest rate terminology such as AER, EAR and CAR
  - Depreciation (reducing balance method) investigated using multi-representations i.e. table, graph and formula (link to exponential functions and geometric sequences)
  - Costing: materials, labour and wastage
  - Currency transactions

Lesson Idea LCOL.7

Title
Ratio and proportion

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Ratios as comparing two quantities of the same kind by division (no units)
- Contexts involving ratio and proportion
    - Metric system; change of units; everyday imperial units (conversion factors provided for imperial units)
    - Rates as the comparison of two quantities by division but with different units
    - Average rates of change with respect to time and units
    - Diagrams drawn to scale

Lesson Idea LCOL.8

Title
Scientific notation involving real world contexts

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to express non -zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \leq a < 10$ involving real world contexts
- How to enter very large numbers on the calculator
- How to enter very small numbers on the calculator
- How to perform arithmetic operations on numbers in scientific notation
- How to solve problems involving numbers in scientific notation

Section 2: Patterns, Functions and Algebra

Lesson Idea LCOL.9

Title
Relations approach to algebra- revision and extension of Junior Cycle material

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- That processes can generate sequences of numbers or objects
- How to investigate and discover patterns among these sequences
- How to use patterns to continue the sequence
- How to develop generalising strategies and ideas, present and interpret solutions, in the following:
    - The use of tables, diagrams, graphs and formulae as tools for representing and analysing **linear** patterns and relations
        - Discuss rate of change and the y - intercept. Consider how these relate to the context from which the relationship is derived and identify how they can appear in a table, in a graph and in a formula
        - Decide if two linear relations have a common value (decide if two lines intersect and where the intersection occurs).
        - Recognise that the distinguishing feature of a linear relationship is a constant rate of change
        - Recognise discrete linear relationships as arithmetic sequences
    - The use of tables, diagrams, graphs and formulae as tools for representing and analysing **quadratic** patterns and relations
        - Recognise that a distinguishing feature of quadratic relations is that the rate of change of the rate of change is constant
    - The concept of a function as a relationship between a set of inputs and a set of outputs where each input is related uniquely to only one output
    - The use of tables, diagrams, graphs and formulae as tools for representing and analysing **exponential** patterns and relations
        - Recognise that a distinguishing feature of exponential relations is a constant ratio between successive outputs
        - Recognise discrete exponential relationships as geometric sequences

- Recognise whether a sequence is arithmetic, geometric or neither

Lesson Idea LCOL.10

Title
Arithmetic sequences and series

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The link between linear relations and the formula for the general term (T_n) of an arithmetic sequence
- How to find the sum (S_n) of n terms of an arithmetic series
- How to apply the formula for the nth term of an arithmetic sequence and the formula for the sum to n terms of an arithmetic series to different contexts.

Lesson Idea LCOL.11

Title
Functions -interpreting and representing linear, quadratic and exponential functions in graphical form

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- That a function assigns a unique output to a given input
- Domain, co -domain and range of a function
- The use of function notation $f(x) =$, $f : x \rightarrow$, and $y =$
- How to graph functions of the form:
    - $ax + b$ where $a, b \in \mathbf{Q}, x \in \mathbf{R}$
    - $ax^2 + bx + c$, where $a, b, c \in \mathbf{Z}, x \in \mathbf{R}$
    - $ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbf{Z}, x \in \mathbf{R}$
    - ab^x where $a \in \mathbf{N}, b, x \in \mathbf{R}$
- How to interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions
- How to use graphical methods to find approximate solutions to

- $f(x) = 0$
- $f(x) = k$
- $f(x) = g(x)$ where $f(x)$ and $g(x)$ are of the above form, or where graphs of $f(x)$ and $g(x)$ are provided

- How to find local maximum and minimum values of quadratic functions from a graph
- The relationship between the graphs of $f(x) = x^2$, $g(x) = ax^2$, $h(x) = x^2 \pm c$, $\mathrm{k}(x) = ax^2 \pm c$, and $l(x) = (x \pm c)^2$

Lesson Idea LCOL.12

Title
Composition of functions

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to form composite functions (including notation used)
- Composite functions in context

Lesson Idea LCOL.13

Title
Revision of basic JC algebra - evaluating and expanding expressions

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The role of the variable (as unknown, representing quantities that vary and in expressing generality)
- Indices in algebra (exponents $\in \mathbf{N}$)
- Terms, coefficients and expressions
- How to generate algebraic expressions from simple contexts

- Evaluate expressions given the value of the variables
- Expand and regroup expressions
- How to add and subtract expressions of the form:

$$(ax+by+c)\pm\ldots\pm(dx+ey+f)$$
$$(ax^2+bx+c)\pm\ldots\pm(dx^2+ex+f) \text{ where } a,b,c,d,e,f\in\mathbf{Z}$$

- How to use the associative and distribution properties to simplify expressions of the form:
  - $a(bx\pm cy\pm d)\pm\ldots\pm e(fx\pm gy\pm h)$ where $a,b,c,d,e,f,g,h\in\mathbf{Z}$
  - $(x\pm y)(w\pm z)$
- How carry out operations of the form : $ax^2+bx+c\div dx+e$, where $a,b,c,d,e\in\mathbf{Z}$

Lesson Idea LCOL.14

Title
Factorising in Algebra - revision and extension of JC material

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Factors and multiples
- Factorising expressions of the form:

  ax, axy where $a\in\mathbf{Z}$

  $abxy+ay$ where $a,b\in\mathbf{Z}$

  ax^2+bx where $a,b\in\mathbf{Z}$

  $sx-ty+tx-sy$ where s,t,x,y are variable
- Factorise expressions of order 2

Idea LCOL.15

Title
Rearrangement of formulae

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to rearrange formulae using familiar context

Lesson Idea LCOL.16

Title
Adding algebraic fractions - revision and extension of JC material

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Operations of the form:
  $\dfrac{ax+b}{c} \pm \dfrac{dx+e}{f}$ where $a, b, c, d, e, f \in \mathbf{Z}$
  $\dfrac{a}{bx+c} \pm \dfrac{q}{px+r}$ where $a, b, c, p, q, r \in \mathbf{Z}$

Lesson Idea LCOL.17

Title
Solving linear and quadratic equations

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form:

- $f(x) = g(x)$
  with $f(x) = ax + b, \quad g(x) = cx + d$ and where $a, b, c, d \in \mathbf{Q}$

- $f(x) = g(x)$ with
  $f(x) = \frac{a}{bx + c} \pm \frac{p}{qx + r}$
  $g(x) = \frac{e}{f}$
  where $a, b, c, e, f, p, q, r \in \mathbf{Z}$

- $f(x) = k$ with $f(x) = ax^2 + bx + c$ (and not necessarily factorisable), $a, b, c \in \mathbf{Q}$ and interpret the result
- How to solve simultaneous linear equations with two unknowns and interpret the results
- How to solve simultaneous equations where one is linear and the other is of order two with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is ± 1 in the linear equation) and interpret the results.

- How to form quadratic equations given whole number roots

Lesson Idea LCOL.18

Title
Inequalities

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

How to select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form:

- $g(x) \leq k,\ g(x) \geq k,$

  $g(x) < k,\ g(x) > k$ where $g(x) = ax + b, a, b, k \in \mathbf{Q}$

- Graph solution sets on the number line for linear inequalities in one variable

Lesson Idea LCOL.19

Title
Relations without formulae (See 4.5 JCOL)

Resources
Online resources on the Project Maths website
Dynamic software package
Motion sensor

Content
These lessons will involve the students in investigating and understanding:

- Graphs of motion
- Quantitative graphs and drawing conclusions from them
- The connections between the shape of a graph and the story of a phenomenon
- Quantity and change of quantity on a graph

Section 3: Co-ordinate Geometry of the line

Lesson Idea LCOL.20

Title
Review of Junior Cycle ordinary-level coordinate geometry - coordinating the plane, distance and midpoint formula

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The coordination of the plane
- The distance formula
- The midpoint formula

Lesson Idea LCOL.21

Title
Review of slope including application to parallel and perpendicular lines

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The idea of slope as $\frac{Rise}{Run}$
- The slope formula
- The meaning of *positive, negative, zero and undefined slope.*
- Use of slopes to investigate if two lines are parallel
- The use of slopes to investigate if two lines are perpendicular or not
- That 3 points on the coordinate plane are collinear if and only if the slope between any two of them is the same

Lesson Idea LCOL. 22

Title
Area of a triangle

Resources
Online resources on the Project Maths website
Dynamic software package

Content
This lesson will involve the students in investigating and understanding:

- How to calculate the area of a triangle using coordinates
- The connection between this formula ,the geometric approach to the area of a triangle and the formula used in trigonometry for finding the area of a triangl**e**

Lesson Idea LCOL.23

Title
Equation of Line

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The equation of a line in the forms :

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

$$ax + by + c = 0$$

  (The significance of the variables m and c)
- Whether or not a point is on a line
- Where a line intersects the axes and why these points might be of interest to someone trying to interpret or plot a graph
- The interpretation of the intercepts in context
- How to find the slope of a line given its equation
- How to solve problems involving slopes of lines
- The link between coordinate geometry of the line and linear functions

Lesson Idea LCOL.24

Title
Intersection of two lines

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- A graphical approach to the intersection of two lines
- An algebraic approach to the intersection of two lines, using simultaneous equations
- The comparison and verification of both of these methods above

Section 4: Synthetic Geometry 1

The following is a suggested sequence for teaching the Leaving Certificate Course. In teaching these lessons, teachers and students can draw from the Teaching and Learning Plans and student activities on the website at www.projectmaths.ie

As outlined at the workshops, the use of learning materials such as "geostrips", "anglegs", geo-boards etc. can make the learning so much more enjoyable for students of all perceived abilities.

While proofs are not the issue as regards informal introduction, it is important that students are kept aware that the theorems build logically.

The lesson divisions which follow are for guidance only. The initial lesson ideas give the students a chance to revisit the material they met in the Junior Cycle. This can be done at a pace that is appropriate to the student's needs. It is recommended that new activities and challenges be introduced during this revision so that students do not see it as too much repetition and that they can see new ways of investigating familiar situations.

Note on experimentation and experimental results:
With experimentation, involving measurement, the results are only approximations and won't agree exactly. It is important for students to report faithfully what they find e.g. for a triangle they could find the sum of the angles to be 179^0 or 181° etc. The conclusion is that the angles appear to add up to 180^0 .This is a plausible working assumption. There is a distinction between what you can discover and what you can prove.

See below: Section 8.2 (From Discovery to Proof) of *Geometry for Post-primary School Mathematics*"

8.2 From Discovery to Proof

It is intended that all of the geometrical results on the course would first be encountered by students through investigation and discovery. As a result of various activities undertaken, students should come to appreciate that certain features of certain shapes or diagrams appear to be independent of the particular examples chosen. These apparently constant features therefore seem to be general results that we have reason to believe might always be true. At this stage in the work, we ask students to accept them as true for the purpose of applying them to various contextualised and abstract problems, but we also agree to come back later to revisit this question of their truth. Nonetheless, even at this stage, students should be asked to consider whether investigating a number of examples in this way is sufficient to be convinced that a particular result always holds, or whether a more convincing argument is required. Is a person who refuses to believe that the asserted result will always be true being unreasonable? An investigation of a statement that appears at first to be always true, but in fact is not, may be helpful, (e.g. the assertion that $n^2 + n + 41$ is prime for all $n \in \mathbb{N}$). Reference might be made to other examples of conjectures that were historically believed to be true until counterexamples were found.

Informally, the ideas involved in a mathematical proof can be developed even at this investigative stage. When students engage in activities that lead to closely related results, they may readily come to appreciate the manner in which these results are connected to each other. That is, they may see for themselves or be led to see that the result they discovered today is an inevitable logical consequence of the one they discovered yesterday. Also, it should be noted that working on problems or "cuts" involves logical deduction from general results.

Later, students at the relevant levels need to proceed beyond accepting a result on the basis of examples towards the idea of a more convincing logical argument. Informal justifications, such as a dissection-based proof of Pythagoras' theorem, have a role to play here. Such justifications develop an argument more strongly than a set of examples. It is worth discussing what the word "prove" means in various contexts, such as in a criminal trial, or in a civil court, or in everyday language. What mathematicians regard as a "proof" is quite different from these other contexts. The logic involved in the various steps must be unassailable. One might present one or more of the readily available dissection-based "proofs" of fallacies and then probe a dissection-based proof of Pythagoras' theorem to see what possible gaps might need to be bridged.

As these concepts of argument and proof are developed, students should be led to appreciate the need to formalise our idea of a mathematical proof to lay out the ground rules that we can all agree on. Since a formal proof only allows us to progress logically from existing results to new ones, the need for axioms is readily identified, and the students can be introduced to formal proofs.

Lesson Idea LCOL.25

Title
Revision of preliminary concepts - Plane and points

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Plane, points, lines, rays, line segments, collinear points, length of a line segment
- What is meant by the terms "axiom" and "implies"
- Axiom 1: [Two points axiom]: There is exactly one line through any two given points.
- Axiom 2: [Ruler Axiom]: The properties of the distance between points
- Angles as a rotation, angles in different orientations
- How to estimate angles in degrees, naming angles
- Terms: Perpendicular, parallel, vertical, horizontal

Lesson Idea LCOL.26

Title
Revision - Angles, Axiom 3, Theorem 1, Constructions 8 & 9

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Measurement of angles using a protractor

  (Possible misconceptions: Students thinking that size of an angle varies with arm or arc-length; failure to recognise equal angles in different orientations
  Common error: Reading from the incorrect scale on the protractor)

- Axiom 3 (The protractor axiom)
- That a straight angle measures 180^0
- Supplementary angles
- Vertically opposite angles
- What is meant by the term "theorem"
- Theorem 1: Vertically opposite angles are equal in measure.

- What is meant by "proof"
- The use of the compass
- Construction 8: Line segment of a given length on a given ray
- Construction 9: Angle of a given number of degrees with a given ray as one arm

Lesson Idea LCOL.27

Title
Revision of JC synthetic geometry

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Construction 5: Line parallel to a given line, through a given point;
- Axiom 5: Given any line l and a point P, there is exactly one line through P parallel to l.
- Construction 6: Division of a line segment into 2 or 3 equal segments without measuring it
- Triangles and congruent triangles
    - Triangles: scalene, isosceles, equilateral, right-angled
    - Construction 10: Triangle given SSS – Congruent triangles (Axiom 4)
    - Construction 11: Triangle given SAS - Congruent triangles (Axiom 4)
    - Construction 12: Triangle given ASA - Congruent triangles (Axiom 4)
    - By construction, show that AAA and ASS are not sufficient conditions for congruence.
    - What is meant by the term "converse"
    - Theorem 2: (i) In an isosceles triangle the angles opposite the equal sides are equal.
      (ii) Conversely, if two angles are equal, then the triangle is isosceles

Suggested class activities

Students might engage in activities in relation to scalene, equilateral, isosceles, right-angled and obtuse-angled triangles

- Describe each in words
- Draw three examples of each in different orientations
- Determine the sum of the angles in each
- Determine which triangles, if any, contain two or more equal angles
- Establish if they can say for certainty the size of the angles in each

- Establish if any of the triangles can belong to more than one category

Lesson Idea LCOL.28

Title
Revision of JC synthetic geometry

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Alternate angles by examples and measuring
- Theorem 3:
  (i) If a transversal makes equal alternate angles on two lines, then the lines are parallel.
  (ii) Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.
- Theorem 4: The angles in any triangle add to 180^0.
- Corresponding angles explained by examples and measuring
- Theorem 5: Two lines are parallel if and only if for any transversal, corresponding angles are equal.
- Theorem 6: Each exterior angle of a triangle is equal to the sum of the interior opposite angles.

Lesson Idea LCOL.29

Title
Theorem 7 and Theorem 8

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Theorem 7: In a triangle, the angle opposite the greater of two sides is greater than the angle opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.

- Theorem 8: Two sides of a triangle are together greater than the third.

Lesson Idea LCOL.30

Title
Revision - Constructions 1, 2, & 4

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Construction 1: Bisector of a given angle, using only compass and straight angle
- Construction 2: Perpendicular bisector of a segment using only compass and straightedge
- Construction 4: Line perpendicular to a given line l, passing through a given point on l

Lesson Idea LCOL.31

Title
Revision of translations, central symmetries, axial symmetries and rotations

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Translations
- Axial symmetry
- Central symmetry
- Rotations
- **Suggested class activities**
  Students might engage in the following investigations:
- Does a translation preserve length?
  Does a translation preserve angle size?
  Does a translation map a line onto a parallel line?
  Does a translation map a triangle onto a congruent triangle?
- Does an axial symmetry preserve length?
  Does an axial symmetry preserve angle size?
  Does an axial symmetry maps a line onto a parallel line?
  Does an axial symmetry map a triangle onto a congruent triangle?
  How many axes of symmetry does an isosceles triangle have?
  How many axes of symmetry does an equilateral triangle have?

How many axes of symmetry does a circle have?
(Draw examples of the above.)

- Does a central symmetry preserve length?
  Does a central symmetry preserve angle size?
  Does a central symmetry map a line onto a parallel line?
  Does a central symmetry map a triangle onto a congruent triangle?
  Does an isosceles triangle have a centre of symmetry?
  Does an equilateral triangle have a centre of symmetry?
  Which types of triangle have a centre of symmetry?
  Does a circle have a centre of symmetry?

Note: quadrilaterals are investigated in the lessons following.

Lesson Idea LCOL.32

Title
Revision of quadrilaterals, parallelograms, Theorems 9 & 10, Construction 20

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Theorem 9: In a parallelogram, opposite sides are equal, and opposite angles are equal
  Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.

  Remark 1: Sometimes it happens that the converse of a true statement is false. For example, it is true that if a quadrilateral is a rhombus, then its diagonals are perpendicular. But it is not true that a quadrilateral whose diagonals are perpendicular is always a rhombus.
  Remark 2: The converse of Corollary 1 is false: it may happen that a diagonal divides a convex quadrilateral into two congruent triangles, even though the quadrilateral is not a parallelogram.
- Theorem 10: The diagonals of a parallelogram bisect one another.

  Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.
- Construction 20: Parallelogram, given the length of the sides and the measure of the angles
- The properties of different quadrilaterals

Suggested class activities

Students might engage in the following activities which lead to an informal proof of theorem 9:
Draw a parallelogram ABCD which is not a rectangle or a rhombus
Draw in one diagonal BD

Mark in all the alternate angles – they should find 2 pairs
Establish that triangles ABD and BCD are congruent and explain their reasoning
Establish what this means about the opposite sides of parallelogram ABCD
Make a deduction about the opposite angles of parallelogram ABCD

The students can determine:

If the diagonal bisects the angles at the vertex

The sum of the four angles of parallelogram ABCD
The result if two adjacent angles of the parallelogram are added together

Students might engage in the following activities which lead to an informal proof of theorem 10 (In all instances they should be encouraged to explain their reasoning):
Draw a parallelogram ABCD which is not a rectangle or a rhombus
Draw in the two diagonals AC and BD intersecting at E
Determine if the two diagonals equal in length. (Measure)
Mark in all the equal sides and angles in the triangles AED and BEC
Explain why triangles ADE and BEC are congruent (Give a reason.)

Possible further investigations:

The students can determine:

If the triangles AEB and DEC are congruent
If the diagonals perpendicular
If the parallelogram contains 4 two pairs of congruent triangles
If the diagonals bisect the vertex angles of the parallelogram
The number of axes of symmetry the parallelogram has
If the parallelogram has a centre of symmetry and its location if it does exist

Students might engage in the following activities about a square, rhombus, parallelogram and rectangle: (In all instances they should be encouraged to explain their reasoning.)

Describe each of them in words.
Draw three examples of each in different orientations.
Determine which sides are equal in length.
Determine the sum of the angles in each case.
Determine which angles are equal.
Determine the sum of two adjacent angles in each case.
Establish whether or not a diagonal bisects the angles it passes through.
Establish whether or not the diagonals are perpendicular.
Determine whether or not a diagonal divides it into two congruent triangles.
Calculate the length of a diagonal given the length of its sides, where possible.
Establish whether or not the two diagonals are equal in length.
Determine whether or not the diagonals divide the different shapes into 4 congruent triangles.
Establish if the diagonals bisect each other.

The students should determine the number of axes of symmetry each of the shapes has and which ones have a centre of symmetry.
An interesting option would be to conduct the activities above on a KITE.

Lesson Idea LCOL.33

Title
Revision: More Quadrilaterals – Investigating a Square

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The properties of a square
- The properties of a rectangle(which is not a square)

Suggested class activities

Students might engage in the following activities relating to a square: (In all instances they should be encouraged to explain their reasoning.)

Draw a square ABCD.
Draw in the two diagonals AC and BD intersecting at E.
Determine whether or not the two diagonals are equal in length.
Mark in all the equal sides and angles in the triangles AED and BEC.
Establish that triangles ADE and BEC are congruent.
Determine if the triangles AEB and DEC are congruent.
Determine if there are two pairs of congruent triangles in the square.
Show that the diagonals perpendicular. Give a reason.
Establish whether or not the diagonals bisect the vertex angles of the square.
Find how many axes of symmetry the square has.
Determine whether or not the square has a centre of symmetry and if it does, what is its location.

Students might engage in the following activities about a rectangle:

(In all instances they should be encouraged to explain their reasoning.)

Draw a rectangle ABCD which is not a square
Draw in the two diagonals AC and BD intersecting at E and establish if the two diagonals are equal in length
Mark in all the equal sides and angles in the triangles AED and BEC.
Establish that triangles ADE and BEC are congruent.
Determine whether or not the triangles AEB and DEC are congruent.
Determine whether or not there are two pairs of congruent triangles in the rectangle.

Show that the diagonals are perpendicular.
Determine whether or not the diagonals bisect the vertex angles of the rectangle.
Find how many axes of symmetry the rectangle has.
Determine whether or not the rectangle has a centre of symmetry and if it does, find its location.

Possible extra activity:
Repeat these activities for the rhombus ABCD

Lesson Idea LCOL.34

Title
Theorem 11

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Theorem 11: If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

Lesson Idea LCOL.35

Title
Theorem 12 and revision of Theorem 13

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Theorem 12: Let ΔABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio $s:t$, then it also cuts [AC] in the same ratio.
  Converse to Theorem 12: Let ΔABC be a triangle. If a line l cuts the sides AB and AC in the same ratio, then it is parallel to BC.

- The meaning of similar triangles and the difference between similar and congruent triangles.

- Theorem 13: If two triangles ΔABC and $\Delta A'B'C'$' are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$$

Converse: If

$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$,then the two triangles ΔABC and $\Delta A'B'C'$' are similar .

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Lesson Idea LCOL.36

Title

Revision of right-angled triangles and Pythagoras' Theorem

Resources

Online resources on the Project Maths website

Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

- Construction 13: Right-angled triangle, given length of hypotenuse and one other side.
- Construction 14: Right-angled triangle, given one side and one of the acute angles (several cases)
- Construction 15: Rectangle, given side lengths
- Theorem 14: Theorem of Pythagoras
- Theorem 15: [Converse to Pythagoras] If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.
- Proposition 9: If two right-angled triangles each have hypotenuse and one other side equal in length respectively, then they are congruent. (RHS)

Lesson Idea LCOL.37

Title

Introduction to area

Resources

Online resources on the Project Maths website

Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

- Theorem 16: For a triangle, base times height does not depend on the choice of base.
- Definition 38**:** The **area** of a triangle is half the base multiplied by the height.
- Theorem 17: A diagonal of a parallelogram bisects the area.
- Theorem 18: The area of a parallelogram is the base by the height.

Suggested class activities
Students might engage in the following activities:

In the case of each of these types of triangles: equilateral, isosceles, right-angled and obtuse-angled: draw three diagrams for each type of triangle showing each side as a base and the corresponding perpendicular height.

Students investigate the validity of the following statement and its converse: "Congruent triangles have equal areas".

Lesson Idea LCOL.38

Title

Enlargements

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Enlargements, paying attention to
  - Centre of enlargement
  - Scale factor k, $0 < k < 1$, $k > 1$, $k \in \mathbf{Q}$
  - How to draw an enlargement of a figure given a scale factor when the centre of enlargement is outside the figure to be enlarged
  - How to draw an enlargement of a figure given a scale factor when the centre of enlargement is inside the figure to be enlarged
  - How to draw an enlargement of a figure given a scale factor when the centre of enlargement is a vertex of the figure to be enlarged or is a point on the figure
  - How to find the scale factor
- That when a figure is enlarged by a scale factor k, the area of the image figure is increased by a factor k^2
- How to solve problems involving enlargements

Section 5: Probability and Statistics 1

Lesson Idea LCOL.39

Title
Fundamental Principle of Counting and Permutations

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The Fundamental Principle of Counting
- How to count the arrangements of n distinct objects ($n!$)
- How to count the number of ways of arranging r objects from n distinct objects

Lesson Idea LCOL.40

Title
Concepts of probability

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- JC learning outcomes for probability

Lesson Idea LCOL.41

Title
Rules of probability

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

How to use set theory to discuss experiments, outcomes, samples spaces
- The basic rules of probability (AND/OR), mutually exclusive events, through the use of Venn diagrams
- The use of the formulae:
  1. Addition Rule (for mutually exclusive events only): $P(A \cup B) = P(A) + P(B)$
  2. Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  3. Multiplication Rule(for independent events): $P(A \cap B) = P(A) \times P(B)$
- Use of tree diagrams

Lesson Idea LCOL.42

Title
The purpose of Statistics and the Data Handling Cycle

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The purpose and uses of statistics and possible misconceptions and misuses of Statistics
- How to design a plan and collect data on the basis of the above knowledge
- The data handling cycle (Pose a question, collect data, analyse data, interpret the result and refine the original question if necessary)
- The Census at School (**CAS**) questionnaire as a means of collecting data
- Questionnaire designs
- Populations and samples
- That sampling variability influences the use of sample information to make statements about the population
- The importance of representativeness so as to avoid biased samples
- Sample selection (Simple Random Sample)
- The extent to which conclusions can be generalised
- Primary sources of data (observational (including sample surveys) and experimental studies) and secondary sources of data
- The different ways of collecting data

- How to summarise data in a diagrammatic form

The students will also engage in analysing spreadsheets of data for example the spreadsheet of class data returned from the Census at School questionnaire to include:

  - Recognising different types of data – category (nominal /ordinal), numerical (discrete/ continuous)
  - Recognising univariate/bivariate data
  - Discussing possible questions which might be answered with the data

Lesson Idea LCOL.43

Title
Analysing data graphically and numerically, interpreting and drawing inferences from data

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The concept of the distribution of data and frequency distribution tables
- The selection and use of appropriate graphical methods to describe the sample(univariate data only) taking account of data type: bar charts, pie charts, line plots, histograms(equal class intervals), stem and leaf plot (including back to back)
- The distribution of numerical data in terms of **shape** (concepts of symmetry, clustering, gaps, skewness)
- The selection and use of appropriate numerical methods to describe the sample:
  - The distribution of data in terms of **centre** (mean, median and mode and the advantages and disadvantages of each)
  - The relative positions of mean and median in symmetric and skewed data
  - The distribution of numerical data in terms of **spread** (range, inter-quartile range)
    - The concept of inter-quartile range as a measure of spread around the median
  - The distribution of data in terms of **spread** (standard deviation)
    - The concept of standard deviation as a measure of spread around the mean
    - The use of a calculator to calculate standard deviation
- How to analyse plots of data to explain differences in measures of centre and spread
- How to interpret a histogram in terms of distribution of data and make decisions based on the empirical rule (based on a normal distribution)

- The effect on the mean and of adding or subtracting a constant to each of the data points and of multiplying or dividing the data points by a constant
- Outliers and their effect on measures of centre and spread
- The effect on the mean of adding or subtracting a constant to each of the data points and of multiplying or dividing the data points by a constant

Proposed end of fifth year

Section 6: Length, Area and Volume

Lesson Idea LCOL.44

Title
Length of the perimeter and area of plane figures, nets of solids, surface area and volume

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- How to solve problems involving the length of the perimeter and the area of plane figures: disc, triangle, rectangle, square, parallelogram, trapezium, sectors of discs and figures made from combinations of these

- The nets of prisms, cylinders and cones

- How to solve problems involving the surface area and volume of the following sold figures: rectangular block, cylinder, right cone, triangular based prism (right angle, isosceles and equilateral), sphere, hemisphere and solids made from combinations of these

Lesson Idea LCOL.45

Title
Trapezoidal rule

Resources

Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- How to use the trapezoidal rule to approximate area
- How to calculate percentage error involved in using trapezoidal rule in e.g. the circle

Section 7: Functions and differential calculus

Lesson Idea LCOL.46

Title
Relations without formulae

Resources
Online resources on the Project Maths website
Dynamic software package
Motion sensor
Workshop 4 booklet (graph matching)

Content
These lessons will involve the students in investigating and understanding:

- Graphs of motion
- Quantitative graphs and drawing conclusions from them
- The connections between the shape of a graph and the story of a phenomenon
- Quantity and change of quantity on a graph

Lesson Idea LCOL.47

Title
Revision of function concepts

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Revision of function concepts

Lesson Idea LCOL.48

Title
Calculus

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

Rate of change, average rate of change, instantaneous rate of change, the derivative

This will include:

- Calculus as the study of mathematically defined change How to use graphs and real life examples to analyse rates of change for:
    - Linear functions $f(x) = k$ where k is a constant
    - Linear functions in general - links should be established to the slope of a line from coordinate geometry
    - Functions where the rate of change varies

- Instantaneous rate of change as opposed to average rate of change say over the course of a journey
- The equality of the instantaneous and average rates of change for linear functions
- How to find the rate of change in situations where it is not constant and the need to define it at every point
    - The idea of average rate of change between two points on $f(x)= x^2$ and its calculation as the slope of the secant connecting the two endpoints of the interval under consideration
    - That the instantaneous rate of change is not the same as the average rate of change between two points for $f(x)= x^2$
    - That the average rate of change approaches the instantaneous rate as the interval under consideration approaches zero (the concept of a limit)
    - That the instantaneous rate of change is the slope of the tangent at the point
- The meaning of the first derivative as the instantaneous rate of change of one quantity relative to another and the use and meaning of the terms "differentiation" and notation such as $\frac{dy}{dx}$ and $f'(x)$
- How to find the first derivatives of linear functions using the equation $y = mx + c$ and observing that the slope m the first derivative
- Use the observation of patterns of change in the slopes of quadratic and cubic functions at points on their graphs to come up with a rule for differentiating these functions.
- How to find the first derivative of linear, quadratic and cubic functions by rule
- How to apply differentiation to real life examples of rates of change
- What it means when a function is increasing/decreasing/constant in terms of the slope, rate of change, derivative

- How to apply an understanding of the change in $\frac{dy}{dx}$ from positive to zero to negative around a local maximum in order to identify a local maximum
- How to apply an understanding of the change in $\frac{dy}{dx}$ from negative to zero to positive around a local minimum in order to identify a local minimum
- The meaning of the second derivative as the rate of change of a rate of change at any instant

- Real life examples of the rate of change of a rate of change, for example acceleration as a rate of change of velocity
- How to match a function with its first and second derivatives
- How to find second derivatives of linear, quadratic and cubic functions by rule
- The application of the second derivative to identify local maxima and local minima
- The second derivative and its connection with "concave up" and "concave down" sections of curves
- How to apply differentiation to curve sketching

Section 8: Probability and Statistics 2

Lesson Idea LCOL.49

Title
Revision of counting and probability concepts from fifth year

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- The Fundamental Principle of Counting
- How to count the arrangements of n distinct objects (n!)
- How to count the arrangements of n distinct object taking r at a time
- Concepts of probability
- Rules of probability
- Use of tree diagrams, set theory

Lesson Idea LCOL.50

Title
Bernoulli Trials

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Find the probability that two independent events both occur
- Bernoulli trials
  (A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: "success" or "failure". How to solve problems involving up to 3 Bernoulli trials
- How to calculate the probability that the 1st success occurs on the n^{th} Bernoulli trial where n is specified

Lesson Idea LCOL.51 (option to switch lessons 51 and 50 as perhaps random variables should be introduced before Bernoulli trials. See HL lessons 48 and 49.)

Title
Random variables and expected value

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Random variables, discrete and continuous, which lead to discrete and continuous probability distributions
- Expected value $E(X)$ of probability distributions
- The calculation of expected value and the fact that this does not need to be one of the outcomes
- The role of expected value in decision making and the issue of fair games

Lesson Idea LCOL.52

Title
Revision of statistics concepts from 5th year

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in participating in investigating and understanding:

- Revision of statistics concepts from 5th year

Lesson Idea LCOL.53

Title
Bivariate data, scatter plots, correlation

Resources

Online resources on the Project Maths website

Dynamic software package

A mathematical instruments set

Content

These lessons will involve the students in investigating and understanding:

- Bivariate data versus univariate data
- Different types of bivariate data
- The use of scatter plots to determine the relationship between numeric variables
- That correlation always has a value from -1 to +1 inclusive, and that it measures the extent of the **linear relationship** between two variables
- How to match correlation coefficients values to appropriate scatter plots
- That correlation does not imply causality

Lesson Idea LCHL.54

Title

Drawing inferences from data, margin of error, the concept of a hypothesis test,

Resources

Online resources on the Project Maths website

Dynamic software package

A mathematical instruments set

Content

These lessons will involve the students in investigating and understanding:

- How sampling variability influences the use of sample information to make statements about the population
- How to use appropriate tools to describe variability drawing inferences about the population from the sample
- How to interpret the analysis and relate the interpretation to the original question
- How to interpret a histogram in terms of the distribution of data
- The use of the empirical rule
- How to calculate the margin of error for a population proportion ($\frac{1}{\sqrt{n}}$)
- The concept of a hypothesis test
- The distinction between a null and an alternative hypothesis

- How to conduct a hypothesis test on a population proportion using the margin of error

Note: The margin of error referred to here is the maximum value of the 95% confidence interval.

Section 9: Synthetic Geometry 2

Lesson Idea LCOL.55

Title
Revision of fifth year synthetic geometry

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- The synthetic geometry from 5th year

Lesson Idea LCOL.56

Title
Corollaries 3 and 4

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- The term "corollary"
- Corollary 3: Each angle in a semi-circle is a right angle.
- Corollary 4:If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.

Lesson Idea LCOL.57

Title
Theorem 20, Corollary 6 and Construction 19

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Theorem 20: (i) Each tangent to a circle is perpendicular to the radius that goes to the point of contact.

  (ii) If P lies on the circle s, and a line l through P is perpendicular to the radius to P, then l is a tangent to s.

- Corollary 6: If two circles share a common tangent line at one point, then the two centres and that point are collinear.
- Construction 19: Tangent to a given circle at a given point on it.

Lesson Idea LCOL.58

Title
Construction 18

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
This lesson will involve the students in investigating and understanding:

- Construction 18: Angle of 60^0 without using a protractor or set square

Lesson Idea LCOL.59

Title
Theorem 21

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Theorem 21: (i) The perpendicular from the centre of a circle to a chord bisects the chord.

  (ii) The perpendicular bisector of a chord passes through the centre of a circle.

Lesson Idea LCOL.60

Title
Construction 21

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Definition 45: medians and centroid
- Construction 21: Centroid of a triangle

Suggested class activities
Students might engage in the following activities:

Draw the medians and centroid for an acute-angled triangle, a right-angled triangle and an obtuse-angled triangle.
In which instances is the centroid inside the triangle?

Lesson Idea LCOL.61

Title
Constructions 16 and 17

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Construction 16: Circumcentre and circumcircle of a given triangle, using only straight edge and compass.

- Construction 17: Incentre and incircle of a given triangle, using only straight edge and compass.

Suggested class activities
Students might engage in the following activities:

Draw the circumcentre and incentre of an acute-angled triangle, a right-angled triangle, an obtuse-angled triangle.

They should then answer the following questions explaining their reasoning in each case:

In which instances is the circumcentre inside the triangle?

Where is the circumcentre in a right-angled triangle? (see Theorem 19, corollary 3)

In which instances is the incentre inside the triangle?

Section 10: Co-ordinate geometry of the circle

Lesson Idea LCOL.62

Title
Co-ordinate geometry of the circle

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- That $x^2 + y^2 = r^2$ represents the equation of a circle centre (0,0) and radius of length r (Link to Pythagoras' Theorem - distance from any point $p(x, y)$ on the circle to the centre of the circle is equal to the length of the radius of the circle.)
- That $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle wit centre $(h$, k) and radius r
  (Link to Pythagoras' Theorem - distance from any point $p(x, y)$ on the circle to the centre of the circle is equal to the length of the radius of the circle.)

- How to solve problems involving a line and a circle with centre (0,0)

Section11: Trigonometry

Lesson Idea LCOL.63

Title
Trigonometry 1

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- The use of Pythagoras' Theorem to solve problems (2D only)
- Trigonometric ratios in a right-angled triangle
- The use of the trigonometric ratios to solve problems involving right-angled triangles
- The use of similar triangles to find unknowns in right-angled triangles
- The use of the clinometer
- How to use trigonometry to calculate the area of a triangle
- How to solve problems using the sine and cosine rules (2D)
- How to define $\sin\theta$ and $\cos\theta$ for all values of θ
- How to use the unit circle to solve equations such as $\sin\theta = \frac{1}{2}, \cos\theta = 0$ etc.
- How to define tan θ
- How to solve problems involving the area of a sector of a circle and the length of an arc
- Work with trigonometric ratios in surd form

Section 12: Complex numbers

Lesson Idea LCOL.64

Title
Complex numbers

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set
Content
These lessons will involve the students in investigating and understanding:

- The origin and need for complex numbers
- The use of complex numbers to model two dimensional systems as in computer games, alternating current and voltage etc.
- How to interpret multiplication by i as a rotation of 90^0 anticlockwise
- How to express complex numbers in rectangular form $a+ib$ and illustrate them on the Argand diagram
- How to investigate the operations of addition and subtraction of complex numbers in the rectangular form $(a+ib)$ using the Argand diagram
- How to interpret the modulus as distance from the origin on an Argand diagram
- How to multiply a complex number by another complex number
- Interpret the results of the above multiplication on the Argand diagram
- How to interpret the complex conjugate as a reflection in the real axis
- Division of complex numbers in the $(a+ib)$ rectangular form and representation on the Argand diagram
- How to solve quadratic equations having complex roots and how to interpret the solutions

Appendix A

Geometry: Thinking at Different Levels
The Van Hiele Theory

The **Van Hiele model** describes how students learn geometry. Pierre van Hiele and Dina van Hiele-Geldof, mathematics teachers from the Netherlands, observed their geometry students in the 1950's.The following is a brief summary of the Van Hiele theory. According to this theory, students progress through five levels of thinking starting from merely recognising a shape to being able to write a formal proof. The levels are as follows:

***Visualisation (Level 0)**

The objects of thought are shapes and what they look like.

Students have an overall impression of a shape. The appearance of a shape is what is important. They may think that a rotated square is a “diamond” and not a square because it is different from their visual image of a square. They will be able to distinguish shapes like triangles, squares, rectangles etc but will not be able to explain, for example, what makes a rectangle a rectangle. **Vocabulary**: Students use visual words like “pointy”, “curvy”, “corner” as well as correct language like angle, rectangle and parallelogram.

***Analysis (Level 1)**

The objects of thought are “classes” of shapes rather than individual shapes.

- Students think about what makes a rectangle a rectangle and can separate the defining characteristics of a rectangle from irrelevant information like size and orientation. They recognize its parts (sides, diagonals and angles) and compare their properties (similar, congruent)
- They understand that if a shape belongs to a class like “rectangle”, then it has all the properties of that class (2 pairs of equal sides, right angles, 2 equal diagonals, 2 axes of symmetry).
- **Vocabulary:** words like parallel, perpendicular and congruent relating to properties within a figure and the words all, always, sometimes, never, alike, different.
- A concise definition of a figure, using a sufficient rather than an exhaustive list of properties is not possible at this level.
- They do not deal with questions like “Is a square a parallelogram?” but just look at the properties of each class of shape, without comparing the classes.

*Some visualisation and analysis is covered in Primary School.

Relational/ Ordering/Informal Deduction (Level 2)

The objects of thought are the properties of shapes.

- Students are ready to understand interrelationships of properties within figures and between figures. Opposite sides of a parallelogram are parallel and opposite angles are equal.
- A rectangle is a parallelogram since it has all the properties of a parallelogram as well as having all 90^0 angles.

- Students can recognise the difference between a statement and its converse. All squares are rectangles (true) is different from all rectangles are squares (not true).
- Capable of **"if –then"** thinking – if a shape is a rectangle then all the angles in it are right angles. If |<A |= |<B |and |<B| = |<C| then |< A| =|<C|
- They can select one or two properties to define a figure rather than an exhaustive list. If a quadrilateral has 4 equal sides and one right angle it must be a square.
- Students can discover new properties by simple deduction. The two acute angles in a right angled triangle add to 90^0 because all the angles in a triangle add up to 180^0. They can explain logically without having to measure everything.

Formal deduction (Level 3)

Students learn how to use an axiomatic system to establish geometric theory. This is the level at which proof of Theorems is learned. The sequence of theorems given in the appendix is arranged in such a manner that each theorem builds on the previous theorem(s).

Rigor (Level 4)

Comparing different axiomatic systems – not done at secondary level

Characteristics of these levels: Students cannot function at any particular level unless they are competent at all previous levels. The teacher's role is crucial in structuring activities to bring students from one level to the next.

How does the teacher bring students from any one level to the next?

5 phases of learning:

1. In an informal discussion of the topic, students are asked to give their initial observations.
2. The teacher provides structured activities such as drawing, making and measuring.
3. The students then verbalise and write down what they have learned and report back in groups to the class, which leads to a class discussion.
4. The teacher then provides an activity which will require students to apply what they have discovered
5. In the last stage students are required to summarise all they have learned and should be able to remember it as they have discovered it through guidance.

A PowerPoint presentation of the Van Hiele theory can be got at www.projectmaths.ie

2 examples are given on the PowerPoint slides

(1) Using similar triangles to show advancement between levels and

(2) Using an investigation of the rhombus to show how to progress from level 0 to level 1 with this figure using the 5 teaching phases.

A mind map of Van Hiele can be found at
http://agutie.homestead.com/files/mindmap/van_hiele_geometry_level.html

Appendix B

Guide to Theorems, Axioms and Constructions at all Levels*

This is intended as a quick guide to the various axioms, theorems and constructions as set out in the ***Geometry Course for Post-Primary School Mathematics.*** You can get this from the project maths website: www.projectmaths.ie

It is not intended as a replacement for this document, merely as an aid to reading at a glance which material is required to be studied at various levels. The sequence of theorems as given must be followed.

As stated in the heading, these theorems and constructions are underpinned by 46 definitions and 20 propositions which are all set out in the ***Geometry Course for Post-Primary School Mathematics***, along with many undefined terms and definable terms used without explicit definition.

*An **axiom** is a statement accepted without proof, as a basis for argument

*A **theorem** is a statement deduced from the axioms by logical argument. Theorems can also be deduced from previously established theorems.

* A **proposition** is a useful or interesting statement that could be proved at this point, but whose proof is not stipulated as an essential part of the programme. Teachers are free to deal with them as they see fit, but they should be mentioned, at least (Appendix p. 20, footnote).

*The instruments that may be used for **constructions** are listed and described on page 38 of the Appendix and are a straight edge, compass, ruler, protractor and set-square.

Terms Students at **Junior Certificate Higher level and Leaving Certificate Ordinary level** will be expected to understand the meanings of the following terms related to logic and deductive reasoning:
Theorem, proof, axiom, corollary, converse, implies.

In addition, students at **Leaving Certificate Higher level** will be expected to understand the meanings of the following terms related to logic and deductive reasoning:

Is equivalent to, if and only if, proof by contradiction.

Synthetic Geometry

Guide to Axioms, Theorems and Constructions for all Levels

Interactive files are available in the **Student Area** on the Project Maths website.

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL only ** proof required for LCHL only ♦ These results are required as background knowledge for constructions and/or applications of trigonometry.	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
	Axiom 1: There is exactly one line through any two given points	✓	✓	✓	✓♦	✓	✓
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	✓	✓	✓	✓♦	✓	✓
	Axiom 3: Protractor Axiom (The properties of the degree measure of an angle).	✓	✓	✓	✓♦	✓	✓
1	Vertically opposite angles are equal in measure.	✓	✓	✓		✓	✓
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	✓	✓	✓		✓	✓
2	In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	✓	✓	✓	✓♦	✓	✓
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	✓	✓	✓	✓♦	✓	✓
3	If a transversal makes equal alternate angles on two lines then the lines are parallel. Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.	✓	✓	✓		✓	✓
4*	The angles in any triangle add to 180^0.	✓	✓	✓	✓♦	✓	✓
5	Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	✓	✓	✓		✓	✓
6*	Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	✓	✓	✓		✓	✓
7	The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.					✓	✓
8	Two sides of a triangle are together greater than the third.					✓	✓
9*	In a parallelogram, opposite sides are equal, and opposite angles are equal. Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.		✓	✓		✓	✓
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.			✓			✓
10	The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.		✓	✓		✓	✓

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL only ** proof required for LCHL only ♦ These results are required as background knowledge for constructions and/or applications of trigonometry.	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
11**	If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.			✓		✓	✓
12**	Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio. Conversely, if the sides of two triangles are in proportion, then the two triangles are similar.			✓		✓	✓
13**	If two triangles are similar, then their sides are proportional, in order (and converse)		✓	✓		✓	✓
14*	[Theorem of Pythagoras]In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.		✓	✓	✓♦	✓	✓
15	[Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.		✓	✓		✓	✓
	Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent.		✓	✓		✓	✓
16	For a triangle, base x height does not depend on the choice of base.					✓	✓
	Definition 38: The area of a triangle is half the base by the height.					✓	✓
17	A diagonal of a parallelogram bisects the area.					✓	✓
18	The area of a parallelogram is the base x height.					✓	✓
19*	The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.			✓			✓
	Corollary 2†: All angles at points of a circle, standing on the same arc are equal (and converse).			✓			✓
	Corollary 3: Each angle in a semi-circle is a right angle.		✓	✓		✓	✓
	Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.		✓	✓		✓	✓
	Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180^0(and converse).			✓			✓
20	(i) Each tangent is perpendicular to the radius that goes to the point of contact. (ii) If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.					✓	✓
	Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear.					✓	✓
21	(i) The perpendicular from the centre to a chord bisects the chord. (ii) The perpendicular bisector of a chord passes through the centre.					✓	✓

† The corollaries are numbered as in the *Geometry for Post-primary School Mathematics*; corollary 2 is the first one relating to theorem 19 ♦ These results are required as background knowledge for constructions and/or applications of trigonometry

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	CMN Introd. Course	JC ORD	JC HR	LC FN	LC ORD	LC HR
1	Bisector of an angle, using only compass and straight edge.	✓	✓	✓		✓	✓
2	Perpendicular bisector of a segment, using only compass and straight edge.	✓	✓	✓		✓	✓
3	Line perpendicular to a given line l, passing through a given point not on l.			✓			✓
4	Line perpendicular to a given line l, passing through a given point on l.	✓	✓	✓	✓	✓	✓
5	Line parallel to given line, through a given point.	✓	✓	✓	✓	✓	✓
6	Division of a line segment into 2 or 3 equal segments without measuring it.	✓	✓	✓		✓	✓
7	Division of a line segment into any number of equal segments, without measuring it.			✓			✓
8	Line segment of a given length on a given ray.	✓	✓	✓		✓	✓
9	Angle of a given number of degrees with a given ray as one arm.		✓	✓		✓	✓
10	Triangle, given lengths of 3 sides.		✓	✓	✓	✓	✓
11	Triangle, given SAS data.		✓	✓		✓	✓
12	Triangle, given ASA data		✓	✓		✓	✓
13	Right-angled triangle, given length of hypotenuse and one other side		✓	✓	✓	✓	✓
14	Right-angled triangle, given one side and one of the acute angles.		✓	✓		✓	✓
15	Rectangle given side lengths.		✓	✓	✓	✓	✓
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.					✓	✓
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.					✓	✓
18	Angle of 60^0 without using a protractor or set square.					✓	✓
19	Tangent to a given circle at a given point on it.					✓	✓
20	Parallelogram, given the length of the sides and the measure of the angles.					✓	✓
21	Centroid of a triangle.					✓	✓
22	Orthocentre of a triangle.						✓

Appendix C Investigations of quadrilaterals and triangles

Investigating Quadrilaterals

Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Describe it in words.					
Draw three examples in different orientations.					
How many axes of symmetry does it have? Show on a diagram.					
Does it have a centre of symmetry? Show on a diagram.					
Which sides are equal?					
What is the sum of all the angles?					
Are all angles equal?					
Which angles are equal?					
What is the sum of two					

Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
adjacent angles?					
Does a diagonal bisect the angles it passes through?					
Does a diagonal divide it into two congruent triangles?					
Given the length of its sides, can you calculate the length of a diagonal?					
Are the two diagonal s equal in length?					
Do the diagonals divide it into four congruent triangles?					
Do the diagonals divide it into four triangles of equal area?					
Are the diagonals perpendicular?					

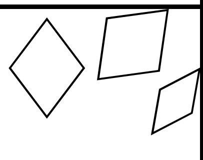
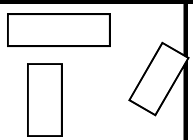
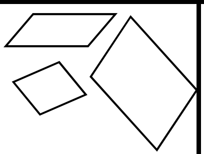
Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Do the two diagonals bisect each other?					
What information do you need to calculate its area? How do you calculate it?					
Does a diagonal bisect its area?					

Investigating triangles

Triangles	Equilateral	Isosceles	Right angled	Obtuse Angled
Describe it in words.				
Draw three examples				

Triangles	Equilateral	Isosceles	Right angled	Obtuse Angled
in different orientations.				
How many axes of symmetry does it have? Show on a diagram.				
Does it have a centre of symmetry? Show on a diagram.				
What is the sum of the three angles?				
Are all angles equal?				
Are there any equal angles? Where?				
Can you say for certain what size the angles are?				
Apart from the isosceles triangles themselves which of the others could also be isosceles?				
What information do you need to calculate its area? How do you calculate it?				
Draw 3 diagrams for each type of triangle showing each side as a base and the corresponding perpendicular height?				
How do you calculate the area?				
Is the centroid inside				

Triangles	Equilateral	Isosceles	Right angled	Obtuse Angled
the triangle always?				
Is the circumcentre inside the triangle always?				
Is the incentre inside the triangle always?				

Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Describe it in words.	A square is a quadrilateral in which all sides are equal in length and all angles are 90^0. (need only say that one angle is 90^0)	A rhombus is a quadrilateral with all sides equal and opposite angles equal. (a parallelogram with all sides equal in length.)	A rectangle is a quadrilateral with opposite sides equal and parallel and all interior angles equal to 90^0.	A parallelogram is a quadrilateral with opposite sides equal and parallel and opposite angles equal.	A trapezium is a quadrilateral which has 1 pair of parallel sides.
Draw three examples in different orientations.					
How many axes of symmetry does it have? Show on a diagram.	4	2	2	None	None if the non parallel sides are not equal in length.

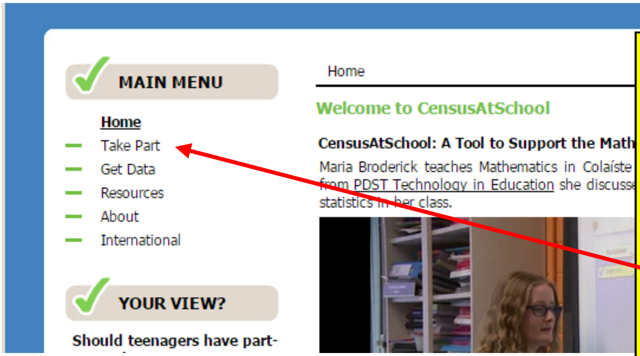
Quadrilaterals	Square	Rhombus (not a square)	Rectangle (not a square)	Parallelogram (not a rectangle or a rhombus)	Trapezium (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Does it have a centre of symmetry? Show on a diagram.	✓	✓	✓	✓	No
Which sides are equal?	All	All	Opposite	Opposite	none
What is the sum of all the angles?	360^0	360^0	360^0	360^0	360^0
Are all angles equal?	✓	x	✓	x	x
Which angles are equal?	All angles	Opposite angles	All angles	Opposite angles	
What is the sum of two adjacent angles?	180^0	180^0	180^0	180^0	180^0
Does a diagonal bisect the angles it passes through?	✓	✓	x	x	x
Does a diagonal divide it into two congruent triangles?	✓	✓	✓	✓	x
Given the length of its sides, can you calculate the length of a diagonal?	✓	No. Need to know an angle. Investigate using geostrips.	✓	No. Need to know an angle.	Need to know the lengths of two adjacent sides and the angle between them.

Quadrilaterals	**Square**	**Rhombus** (not a square)	**Rectangle** (not a square)	**Parallelogram** (not a rectangle or a rhombus)	**Trapezium** (not a parallelogram) and not an isosceles trapezium which has the non parallel sides equal in length)
Are the two diagonal s equal in length?	✓	x	✓	x	x
Do the diagonals divide it into four congruent triangles?	✓	✓	x	x	x
Do the diagonals divide it into four triangles of equal area?	✓	✓	✓	✓	x
Are the diagonals perpendicular?	✓	✓	x	x	x
Do the two diagonals bisect each other?	✓	✓	✓	✓	x
What information do you need to calculate its area? **How do you calculate it?**	One side length x. Area = x^2 (Base (b) and perpendicular height (h)from a vertex to that base Area = b x h)	Base (b) and perpendicular height (h)from a vertex to that base Area = b x h If you know the lengths of the diagonals x and y Area = ½ x y.	Lengths of 2 adjacent sides l and b. Area = l x b. (Base (b) and perpendicular height (h)from a vertex to that base Area = b x h)	Base (b) and perpendicular height (h)from a vertex to that base Area = b x h	The lengths of its parallel sides (a and b)and the perpendicular distance between them. Area = ½ (a+b) h
Does a diagonal bisect its area?	✓	✓	✓	✓	

Appendix D

How to register for *CensusAtSchool,* complete the online questionnaire and retrieve class data for analysis and interpretation

Part 1: Registration

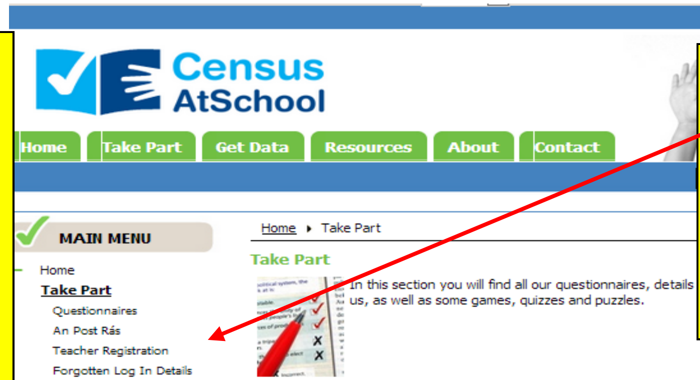


www.CensusAtSchool.ie

This is the Irish *CensusAtSchool* homepage.

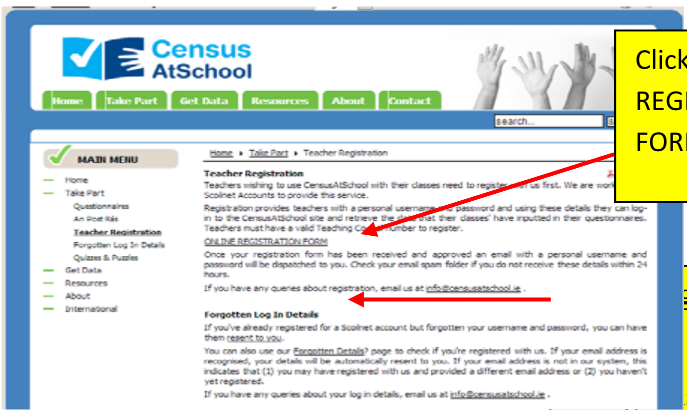
Click "Take Part" under the main menu.

1



Click "Teacher Registration" under "Take Part" in the main menu

2



Click "ONLINE REGISTRATION FORM"

gotten Log In

susatschool.ie

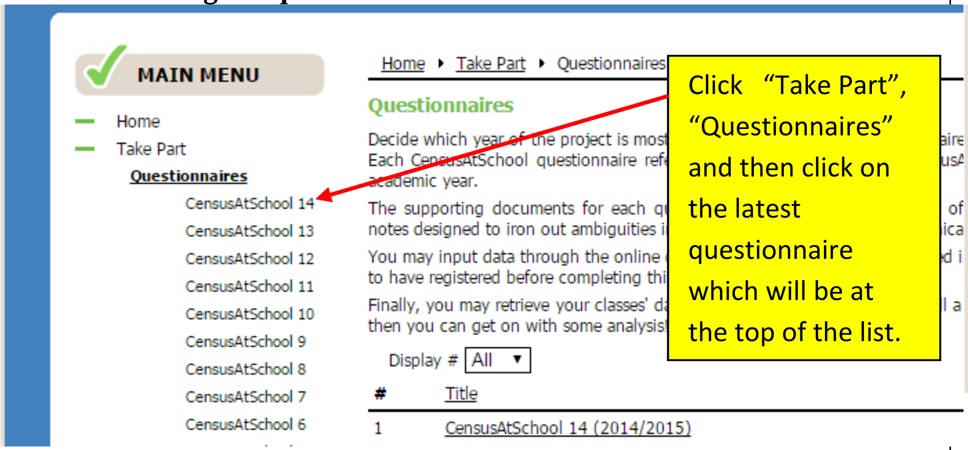
3

Fill out and click on "Submit"

Once your registration form has been received and approved an email with a personal username and password will be dispatched to you. Check your email spam folder if you do not receive these details within 48 hours.

4

Part 2: Accessing the questionnaire



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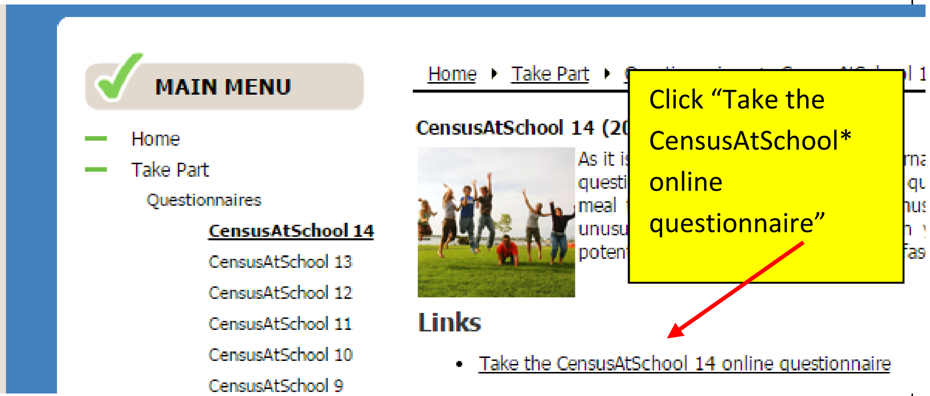
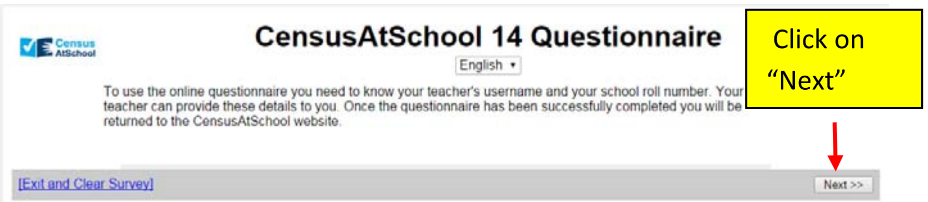


6

Part 3: Filling out the questionnaire



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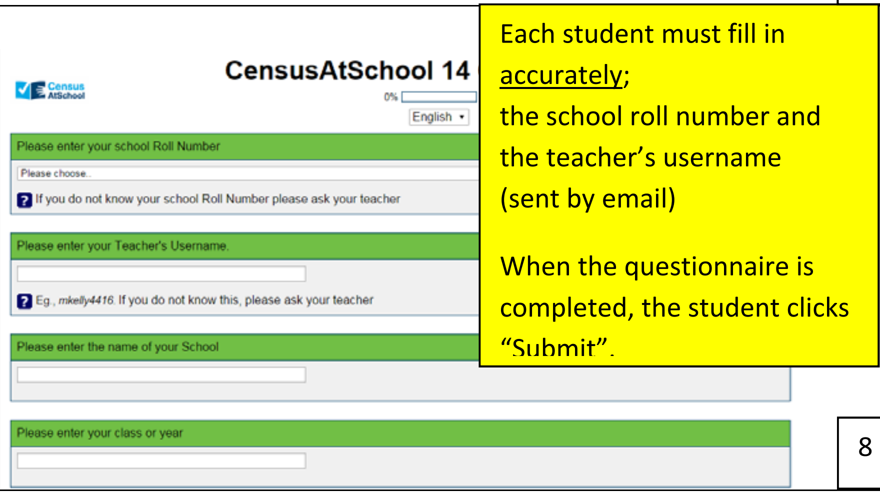
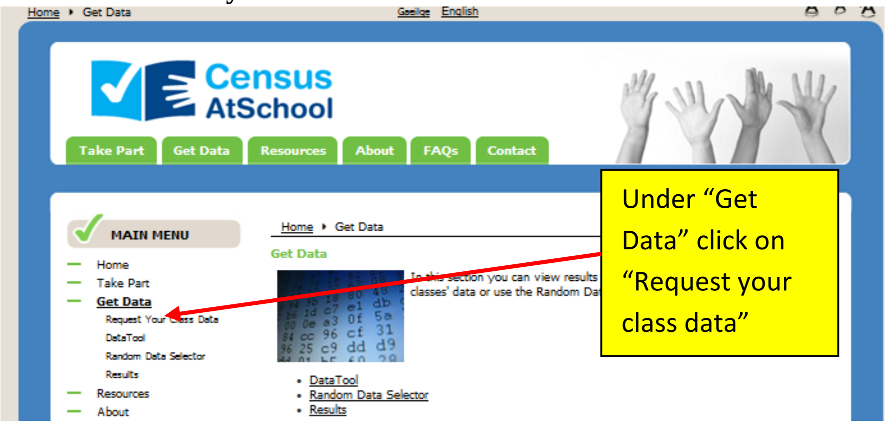


8

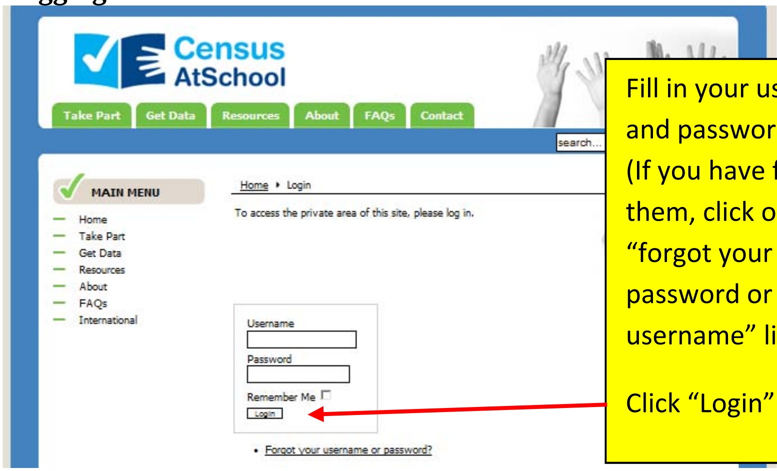
Part 4 Now that my students have filled in the data – what next?



Under "Get Data" click on "Request your class data"

9

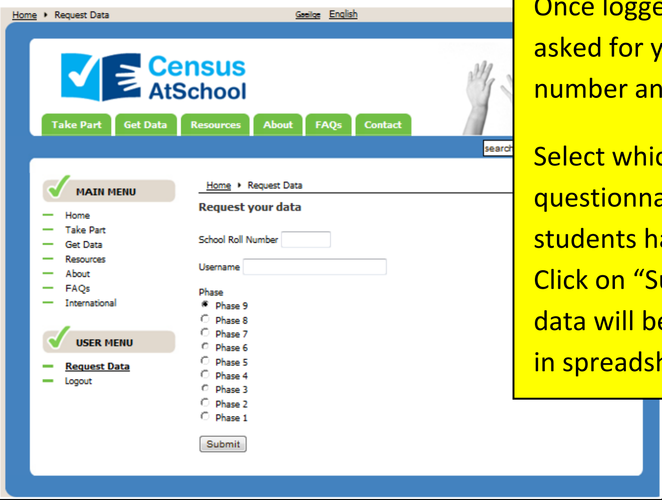
Logging in to retrieve data



Fill in your username and password. (If you have forgotten them, click on the "forgot your password or username" link.)

Click "Login".

10



Once logged in you will be asked for your school roll number and username.

Select whichever questionnaire (phase) your students have completed. Click on "Submit" and your data will be returned to you in spreadsheet form.

11

Your data will look similar to the spreadsheet below.

Sex	BrthMnth	BrthYear	BrthPlce	BedRms	PeoplHme	InetAxs	NumPts	PtsNone	PtsDog	PtsCat	PtsMouse	PtsRabt
Female	July	1992	New Sout	4	5	Yes - broa	4	No	Yes	Yes	No	No
Female	February	1996	Australian	4	5	Yes - broa	4	No	Yes	Yes	No	No
Male	Septembe	1994	Victoria	4	5	Yes - dial-	5	No	Yes	No	No	No
Female	Septembe	1991	Fiji	4	4	No - Interi	0	Yes	No	No	No	No
Male	August	1993	Queenslai	3	4	No - Interi	0	No	No	No	No	No
Female	Decembei	1993	Tasmania	3	5	Yes - dial-	3	No	No	Yes	No	No
Female	August	1990	Queenslai	4	5	Yes - broa	3	No	Yes	No	No	No
Female	Novembe	1991	New Sout	11	5	Yes - dial-	4	No	Yes	No	No	Yes
Male	June	1995	Victoria	3	5	Yes - othe	6	No	Yes	No	No	No
Female	March	1993	Western A	30	122	No - Interi	2	No	Yes	No	No	No
Female	Decembei	1989	Queenslai	3	3	Yes - broa	2	No	No	No	No	No
Female	April	1993	South Aus	3	5	No - Interi	4	No	Yes	Yes	No	No
Male	March	1996	New Sout	3	2	No - Interi	0	Yes	No	No	No	No
Male	Septembe	1993	Queenslai	4	4	Yes - dial-	18	No	Yes	No	No	No
Male	May	1995	Tasmania	3	4	Yes - broa	2	No	No	No	No	Yes
Female	March	1994	Western A	4	4	Yes - dial-	4	No	Yes	Yes	No	No

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