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Chapter 1
Sets

This section will show you how to:

- use set language and notation, and Venn diagrams to describe sets and represent relationships between sets.

RECAP

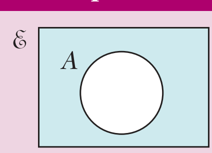
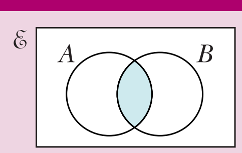
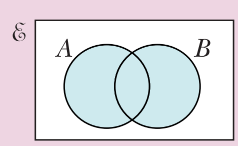
You should already be familiar with the following set notation:

$A = \{x : x \text{ is a natural number}\}$
$B = \{(x, y) : y = mx + c\}$
$C = \{x : a \leqslant x \leqslant b\}$
$D = \{a, b, c, \ldots\}$

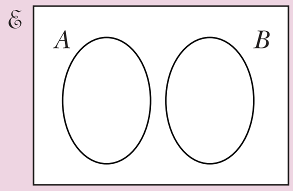
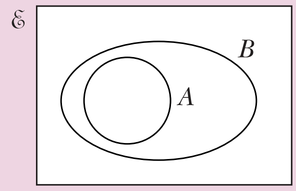
You should also be familiar with the following set symbols:

Union of A and B	$A \cup B$	The empty set	$\varnothing$
Intersection of A and B	$A \cap B$	Universal set	$\mathscr{E}$
Number of elements in set A	$\mathrm{n}(A)$	A is a subset of B	$A \subseteq B$
'… is an element of …'	$\in$	A is a proper subset of B	$A \subset B$
'… is not an element of …'	$\notin$	A is not a subset of B	$A \nsubseteq B$
Complement of set A	A'	A is not a proper subset of B	$A \not\subset B$

You should also know how to represent the complement, union and intersections of sets on a Venn diagram:

Complement	Intersection	Union
 A'	 $A \cap B$	 $A \cup B$

The special conditions $A \cap B = \varnothing$ and $A \subset B$ can be represented on a Venn diagram as:

Disjoint sets	Subsets
 $A \cap B = \varnothing$	 $A \subset B$

1.1 The language of sets

You have already studied sets (for either IGCSE or O level).

The worked examples and exercises in this chapter consolidate your earlier work.

It is important that you re-familiarise yourself with the set notation that is covered in the recap.

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There are some special number sets and symbols that represent these sets that you should also be familiar with:

the set of natural numbers $\{1, 2, 3, \ldots\}$	$\mathbb{N}$
the set of integers $\{0, \pm1, \pm2, \pm3, \ldots\}$	$\mathbb{Z}$
the set of real numbers	$\mathbb{R}$
the set of rational numbers	$\mathbb{Q}$

You have already met the set notation $\{x : -1 < x < 3\}$.

This is read as: the set of numbers x such that x lies between -1 and 3.

The set notation can also be written as $\{x : -1 < x < 3$, where $x \in \mathbb{R}\}$.

This is read as: the set of numbers x such that x lies between -1 and 3 where x is a real number.

WORKED EXAMPLE 1

$\mathscr{E} = \{x : 1 \leqslant x \leqslant 7$, where $x \in \mathbb{R}\}$

$A = \{x : 3 \leqslant x < 5\}$ $\quad B = \{x : 2x - 1 > 7\}$

Find the sets

a A' $\quad$ **b** $A \cap B$.

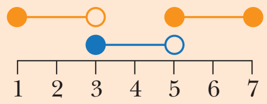
Answers

a Draw a number line from 1 to 7.

The set A is shown in blue.

The set A' is shown in orange.

$A' = \{x : 1 \leqslant x < 3 \cup 5 \leqslant x \leqslant 7\}$

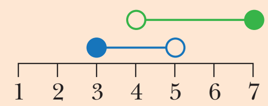


b The set A is shown in blue.

$2x - 1 > 7$

$2x > 8$

$x > 4$



$B = \{x : 4 < x \leqslant 7\}$ $\quad$ B is shown in green on the number line.

The intersection of A and B are the numbers that are common to the two sets.

$A \cap B = \{x : 4 < x < 5\}$

Exercise 1.1

1 $\mathscr{E} = \{x : 5 < x < 16$, x is a integer$\}$

$A = \{x : 9 < x \leqslant 12\}$

List the elements of A'.

2 $\mathscr{E} = \{x : 1 \leqslant x \leqslant 12$, x is a integer$\}$

$A = \{x : 5 \leqslant x \leqslant 8\}$

$B = \{x : x > 6\}$

$C = \{x : x$ is a factor of $8\}$

List the elements of

a $A \cap B$ **b** $(A \cap B)'$ **c** $(A \cap B)' \cap C$.

3 $\mathscr{E} = \{x : 0 < x < 10, x \text{ is an integer}\}$

$P = \{x : x^2 - 6x + 5 = 0\}$

$Q = \{x : 2x - 3 < 7\}$

Find the values of x such that

a $x \in P$ **b** $x \in Q$ **c** $x \in P \cap Q$ **d** $x \in (P \cup Q)'$.

4 $\mathscr{E} = \{x : 1 \leqslant x \leqslant 10, x \in \mathbb{R}\}$

$A = \{x : 3 < x \leqslant 8\}$

$B = \{x : 5 < x < 9\}$

Find the sets

a A' **b** B' **c** $A \cap B$ **d** $A \cup B$.

5 $\mathscr{E}$ = {members of an outdoor pursuits club}

C = {members who go cycling}

R = {members who go running}

W = {members who go walking}

Write the following statements using set notation.

a There are 52 members of the club.

b There are 35 members who go running.

c There are 21 members who go running and cycling.

d Every member who goes running also goes walking.

6 $\mathscr{E}$ = {members of a youth club}

M = {members who like music}

R = {members who like rock-climbing}

S = {members who like sailing}

Describe the following in words

a $M \cup R$ **b** $M \cap S$ **c** R' **d** $R \cap S = \varnothing$.

7 $\mathscr{E}$ = {students in a school}

A = {students studying art}

M = {students studying mathematics}

P = {students studying physics}

a Express the following statements using set notation

i all physics students also study mathematics

ii no student studies both art and physics.

b Describe the following in words

i $A \cap M \cap P'$ **ii** $A' \cap (M \cup P)$.

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8 $\mathscr{E} = \{x : 1 \leqslant x \leqslant 50$, where x is an integer$\}$

$C = \{$cube numbers$\}$

$P = \{$prime numbers$\}$

$S = \{$square numbers$\}$

Express the following statements using set notation

a 17 is a prime number

b 30 is not a cube number

c there are 3 cube numbers between 1 and 50 inclusive

d there are 35 integers between 1 and 50 inclusive, that are not prime

e there are no square numbers that are prime.

9 $\mathscr{E} = \{$students in a school$\}$

$A = \{$students in the athletics team$\}$

$C = \{$students in the chess team$\}$

$F = \{$students in the football team$\}$

$G = \{$students who are girls$\}$

Express the following statements using set notation

a all students in the chess team are girls

b all students in the football team are boys

c there are no students who are in both the athletics team and the chess team

d there are 3 people who are in both the athletics team and the football team.

10 Illustrate each of the following sets on a graph.

a $\{(x, y) : y = x + 2\}$

b $\{(x, y) : x + y = 3\}$

c $\{(x, y) : y = 2x - 1\}$

d $\{(x, y) : x + y \geqslant 2\}$

11 $A = \{(x, y) : y = 2x + 3\}$

$B = \{(x, y) : y = 3\}$

$C = \{(x, y) : x + y = 6\}$

$D = \{(x, y) : y = 2x + 4\}$

a List the elements of

i $A \cap B$ **ii** $A \cap C$.

b Find

i $\mathrm{n}(B \cap D)$ **ii** $\mathrm{n}(A \cap D)$.

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12 In each of the following sets, $x \in \mathbb{R}$.

$A = \{x : 7 - 2x = 3\}$

$B = \{x : x^2 - 3x - 10 = 0\}$

$C = \{x : x^2 + 6x + 9 = 0\}$

$D = \{x : x(x + 2)(x - 7) = 0\}$

$E = \{x : x^2 + 4x + 5 = 0\}$

a Find

i $n(A)$ **ii** $n(B)$ **iii** $n(C)$ **iv** $n(E)$.

b List the elements of the sets

i $B \cup D$ **ii** $B \cap D$.

c Use set notation to complete the statement: $C \cap D = \ldots$

1.2 Shading sets on Venn diagrams

When an expression is complicated, you may need to use some diagrams for your working out before deciding on your answer.

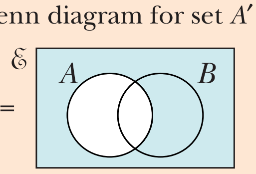
WORKED EXAMPLE 2

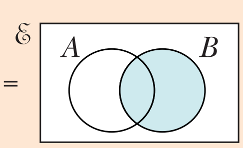
On a Venn diagram shade the regions:

a $A' \cap B$ **b** $A' \cup B$

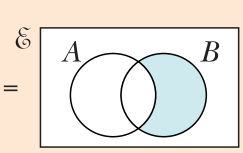
Answers

First shade a Venn diagram for set A' and a Venn diagram for set B:

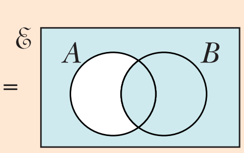
$A' =$


$B =$


a $A' \cap B$ is the region that is in both A' and B.

$A' \cap B =$


b $A' \cup B$ is the region that is in A' or B or both so you need all the shaded regions.

$A' \cup B =$


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CLASS DISCUSSION

$(A \cap B)'$	$(A \cap B')'$	$A' \cap B$	$A' \cup B$
$(A \cup B)'$	$(A \cup B')'$	$A' \cup B'$	$A' \cap B'$
$(A' \cup B)'$	$(A' \cap B)'$	$A \cup B'$	$A \cap B'$

Each blue expression has an equivalent orange expression.

By considering the Venn diagrams for each expression, find the six pairs of equivalent expressions.

Discuss these matching pairs with your classmates.

Describe any rules that you have discovered.

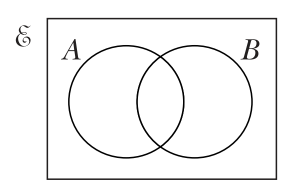
Now copy and complete the following:

$(A \cap B)' = A' \ldots B'$

$(A \cup B)' = A' \ldots B'$

Exercise 1.2

1 On copies of this diagram shade the following regions.



a $A \cap B'$ **b** $(A \cup B)'$

c $(A' \cap B)'$ **d** $(A \cap B)' \cap B$

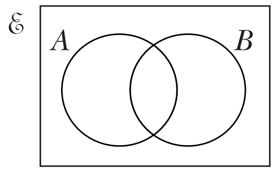
2 $F \cap G = \varnothing$

Show sets F and G on a Venn diagram.

3 $Q \subset P$

Show sets P and Q on a Venn diagram.

4



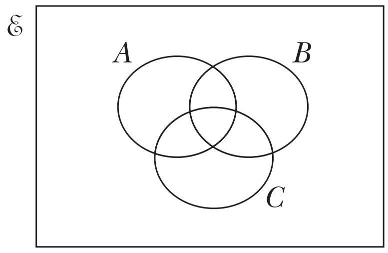
a Copy the Venn diagram and shade the region $A \cup B'$.

b Use set notation to express the set $A \cup B'$ in an alternative way.

5 Investigate whether the following statements are true or false:

a $A \cup B = A' \cap B'$ **b** $A \cap B = A' \cup B'$

6 On copies of this diagram, shade the following regions.



a $(A \cap B) \cup C$ **b** $(A \cup B) \cap C$

c $A \cup (B' \cap C)$ **d** $A \cup (B' \cap C')$

e $A \cap B \cap C'$ **f** $A' \cap (B \cup C)$

g $(A \cup B) \cap C'$ **h** $(A \cup B \cup C)'$

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7 In the class discussion you discovered that

$(A \cap B)' = A' \cup B'$

$(A \cup B)' = A' \cap B'$.

Investigate whether the following statements are true or false:

a $(P \cap Q \cap R)' = P' \cup Q' \cup R'$

b $(P \cup Q \cup R)' = P' \cap Q' \cap R'$

8 $(A \cup B) \subset C$

Show sets A, B and C on a Venn diagram.

9 $A \cap B = \varnothing$ and $(A \cup B) \subset C$

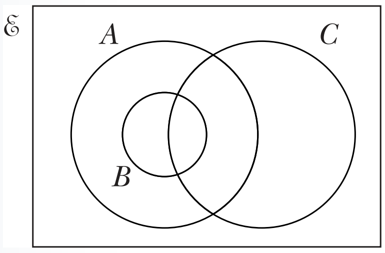
Show sets A, B and C on a Venn diagram.

10 Investigate whether the following statement is true or false:

$(P \cap Q) \cup (P \cap R) = P \cap (Q \cup R)$

CHALLENGE Q

11 Copy the diagram and shade the region representing $(A \cap C) \cap B'$.



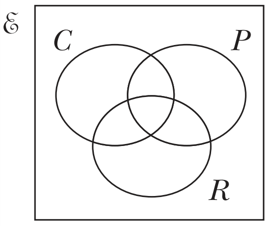
CHALLENGE Q

12 $\mathscr{E}$ = {students in a class}

C = {students who have a calculator}

P = {students who have a protractor}

R = {students who have a ruler}



a Draw a copy of the Venn diagram. Shade the region which represents those students who have a calculator and a ruler, but no protractor.

b Draw a second copy of the Venn diagram. Shade the region which represents those students who have either a calculator or a ruler or both, but not a protractor.

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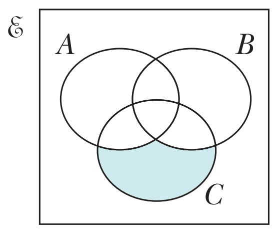
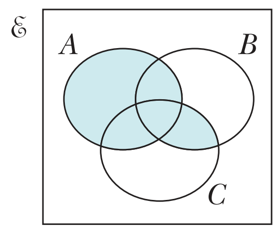
1.3 Describing sets on a Venn diagram

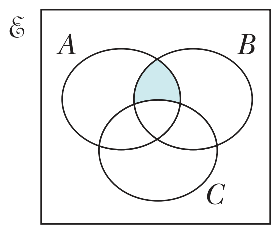
You may be given a Venn diagram and then be asked to use set notation to describe the shaded region. This is illustrated in the following example.

WORKED EXAMPLE 3

Describe the shaded regions using set notation.

a


b


c


Answers

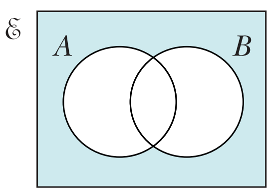
a The shaded region is outside $A \cup B$ and is inside C.
The region is $(A \cup B)' \cap C$.

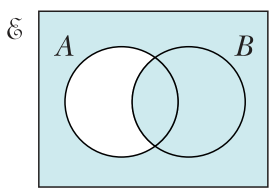
b The shaded region contains A and $B \cap C$
The region is $A \cup (B \cap C)$.

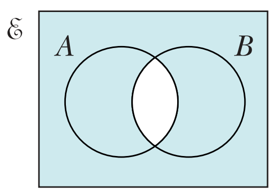
c The shaded region is inside $A \cap B$ and outside C.
The region is $(A \cap B) \cap C'$.

Exercise 1.3

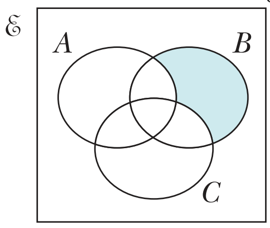
1 Describe the shaded regions using set notation.

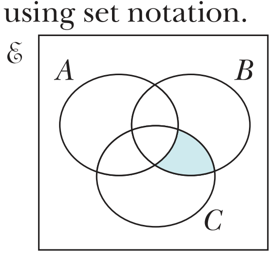
a


b


c


2 Describe the shaded regions using set notation.

a


b


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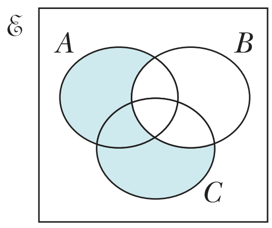
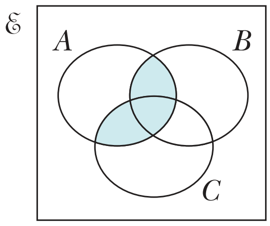
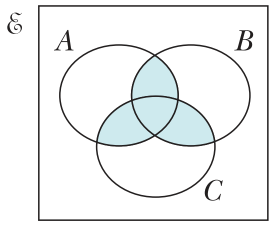
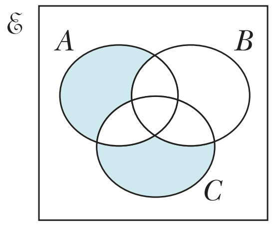


d


e


f


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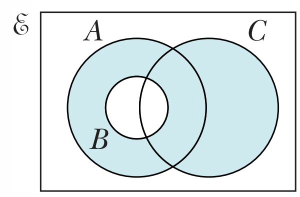
CHALLENGE Q

3 Describe the shaded region using set notation.



1.4 Numbers of elements in regions on a Venn diagram

You can use Venn diagrams to show the number of elements in each set.

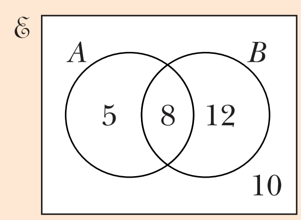
WORKED EXAMPLE 4

$n(\mathscr{E}) = 35 \qquad n(A) = 13 \qquad n(B) = 20 \qquad n(A \cap B) = 8$

Find $n(A \cup B)'$.

Answers

Draw a Venn diagram and put 8 in the intersection.



There are a total of 13 items in A, so 5 must go in the remaining part of A.

There are a total of 20 items in B, so 12 must go in the remaining part of B.

There are a total of 35 items in $\mathscr{E}$, so $\quad n(A \cap B)' = 35 - (5 + 8 + 12)$

$n(A \cup B)' = 10.$

CLASS DISCUSSION

Paul says that: $n(A \cup B) = n(A) + n(B)$

Discuss this statement with your classmates and decide if this statement is:

Always true	Sometimes true	Never true

You must justify your decision.

Discuss how you could adapt this statement so that it is always true.

Complete the rule: $n(A \cup B) = n(A) + n(B) - \ldots$