Compound Interest

The Formulas for Compound Interest

The compound interest is calculated by the formula:

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A(t)$ –the amount after t years

P – the principal

r – the rate per year (written in decimal form)

n – the number of times the interest is compounded per year

t – the number of years

The continuously compounded interest is calculated by the formula:

$$A(t) = Pe^{rt}$$

$A(t)$ –the amount after t years

P – the principal

r – the rate per year (written in decimal form)

t – the number of years

Note: With continuous compounding, the number of compounding periods increases infinitely.

Simple Interest vs Compound Interest

Simple interest is calculated by the formula	Compound interest is calculated by the formula
$A(t) = P(1 + rt)$	$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$
Simple Interest is computed only on the original investment.	Compound Interest is computed on the original investment as well as on any accumulated interest.

An Example that Compares Simple Interest to Compound Interest Over 4 Years

At the end of the	\$100 is invested at 5% using simple interest	\$100 is invested at 5%. using compound interest compounded annually.
First year	$Interest = 0.05 \cdot \$100 = \$5$ $Total\ Amount = \$100 + \$5 = \$105$	$Interest = 0.05 \cdot \$100 = \$5$ $Total\ Amount = \$100 + \$5 = \$105$
Second Year	$Interest = 0.05 \cdot \$100 = \$5$ $Total\ Amount = \$105 + \$5 = \$110$	$Interest = 0.05 \cdot \$105 = \$5.25$ $Total\ Amount = \$105 + \$5.25 = \$110.25$
Third Year	$Interest = 0.05 \cdot \$100 = \$5$ $Total\ Amount = \$110 + \$5 = \$115$	$Interest = 0.05 \cdot \$110.25 = 5.51$ $Total\ Amount = \$110.25 + \$5.51 = \$115.76$
Fourth Year	$Interest = 0.05 \cdot \$100 = \$5$ $Total\ Amount = \$115 + \$5 = \$120$	$Interest = 0.05 \cdot \$115.76 = \$5.79$ $Total\ Amount = \$115.76 + \$5.79 = \$121.55$

Example of a Word Problem

$5,000 was invested for 7 years at an interest rate of 6%.
Find the accumulated amount of the investment, if the money was:

a. Invested using simple interest;
b. Compounded semiannually;
c. Compounded monthly;
d. Compounded daily;
e. Compounded continuously.

Solution

a. $A(t) = P(1 + rt) = \$5{,}000(1 + 0.06 \cdot 7) = \$7{,}100$

b. Compounded semiannually means compounded twice a year. $n = 2$

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} = \$5{,}000\left(1 + \frac{0.06}{2}\right)^{2 \cdot 7} = \$7{,}562.9486 \ldots \approx \$7{,}562.95$$

c. Compounded monthly means compounded twelve times a year. $n = 12$

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} = \$5{,}000\left(1 + \frac{0.06}{12}\right)^{12 \cdot 7} = \$7{,}601.8481 \ldots \approx \$7{,}601.85$$

d. Compounded daily means compounded 365 times a year. $n = 365$

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} = \$5{,}000\left(1 + \frac{0.06}{365}\right)^{365 \cdot 7} = \$7{,}609.5451 \ldots \approx \$7{,}609.55$$

e. Compounded continuously means the number of compounding periods increases infinitely.
$A(t) = Pe^{rt} = \$5{,}000 \cdot e^{0.06 \cdot 7} = \$7{,}609.8077 \ldots \approx \$7{,}609.81$