# Interest Problems with Exponential Growth & Decay

## Algebra 2

I. In Coordinate Algebra, you worked with the Compound Interest Formula $A = P(1+\frac{r}{n})^{nt}$

where $A$ = the amount of money in the account, $P$ is the principal (amount of money initially saved or borrowed), $r$ is the interest rate (be sure to convert percent to decimal), $n$ is the number of times interest is compounded per year and $t$ is the time in years

A. Determine the value of $n$ for the following scenarios

1. Interest is compounded quarterly
2. Interest is compounded semi-annually
3. Interest is compounded monthly
4. Interest is compounded daily

B. See if you can solve the following (you **can** work with a **partner**) by substituting the appropriate values into the compound interest formula

1. Find the value of $15,000 invested in a mutual fund that earned an annual percentage rate of 8.25%, compounded quarterly, for 8 years.

2. Find the value of $15,000 invested in a mutual fund that earned an annual percentage rate of 8.25%, compounded monthly, for 8 years.

3. Find the value of $2,000 invested in the stock market that earned an annual percentage rate of $13\frac{1}{4}\%$, compounded annually, for 22 years.

4. Find the value of $2,000 invested in a C.D. that earned an annual percentage rate of 4.25%, compounded daily, for 22 years.

5. Find the value of $8,000 invested in a savings account that earned 2% annual interest if compounded semi-annually (twice a year), for 9 years.

C. See if you can solve the following (you **can** work with a **partner**) by graphing appropriate functions on the graphing calculator. Be sure to view with an appropriate window

6. How long would it take $5,000 to double in value if it earned 11% interest, compounded quarterly?

7. How long would it take $6,000 to double in value if it earned 9% interest, compounded quarterly?

8. How long would it take $6,000 to double in value if it earned 6% interest, compounded quarterly?

II. $10,000 is invested for 20 years in a money market account that earns $5\frac{3}{4}\%$ annual interest.

A. Find the balance in the account if the interest is compounded:

A. quarterly

B. monthly

C. daily

D. hourly

E. every minute

III. When money is compounded continuously, you can imagine that the number of times it is compounded per year gets infinitely large: that is, *n* gets infinitely large. Use technology to investigate what happens to the expression $\left(1+\frac{1}{n}\right)^n$ as *n* increases in value. Record the value of $\left(1+\frac{1}{n}\right)^n$ for each value of *n*. (Some values of *n* are given in the table.)

| Frequency of Compounding | Number of times compounded in a year (*n*) | $\left(1+\frac{1}{n}\right)^n$ |
|---|---|---|
| Annually | 1 | |
| Semiannually | 2 | |
| Quarterly | | |
| Monthly | | |
| Weekly | | |
| Daily | | |
| Hourly | 8760 | |
| Every Minute | | |
| Every Second | 31536000 | |

Did you see that as *n* increases in value, the expression $\left(1+\frac{1}{n}\right)^n$ gets closer and closer to the value of approximately 2.71828182845904523536028747135 27?

Now, use you calculator to find the value of $e$.

**e (Euler's Number)**

The number **e** is a famous irrational number, and is one of the most important numbers in mathematics.

The first few digits are:

**2.71828182845904523536028747135 27** (and more ...)

*It is often called **Euler's number** after Leonhard Euler.*
*And Euler is spoken like "Oiler".*

**e** is the base of the Natural Logarithms (invented by John Napier).

**e** is found in many interesting areas, so it is worth learning about.

IV. One of the most commonly used applications with natural base $e$ is finding interest and amount of money when interest is compounded continuously. The formula for interest compounded continuously is $A = Pe^{rt}$

A. See if you can solve the following (you **can** work with a **partner**) by substituting the appropriate values into the continuous interest formula

1. Now let's find the balance in the account where \$10,000 is invested for 20 years in a money market account that earns $5\frac{3}{4}\%$ annual interest when the interest is compounded continuously. How does this compare to the answers you got in Part II of this task?

2. Find the value of \$15,000 invested in a mutual fund that earned an annual percentage rate of 8.25%, compounded continuously, for 8 years.

3. How long would it take \$5,000 to double in value if it earned 11% interest, compounded continuously?

B. Graph the following and state the domain, range, asymptote, and y-intercept. Are these functions exponential growth or decay?

1. $y = e^x$

2. $y = e^{x+2}$

3. $y = e^x - 2$

4. $y = e^{-x}$